

# Workshop franco-marocain d'Analyse et Géométrie complexe

Projet ANR MACK

Marrakech, 24-27.10.2011

## Monday 24th october 2011

- 9h30-10h45: Cyril Imbert (ENS Paris, France)  
*Introduction to fully non-linear parabolic equations 1.*
- *Coffee break* [10h45-11h15]
- 11h15-12h30: Vincent Guedj (Institut Math. de Toulouse, France)  
*Convergence of the normalized Kähler-Ricci flow 1.*
- *Lunch* [12h30-14h30]
- 14h30-15h45: Sébastien Boucksom (Institut Math. de Jussieu, France)  
*Tian's properness theorem for Kähler-Einstein Fano manifolds 1.*
- 16h00-17h00: Jean-Pierre Demailly (Institut Fourier, France)  
*Fano manifolds with nef tangent bundles.*

## Tuesday 25th october 2011

- 9h30-10h45: Simone DiVerio (Institut Math. Jussieu, France)  
*Non-deformability of Kähler-Einstein metrics 1.*
- *Coffee break* [10h45-11h15]

- 11h15-12h30: Cyril Imbert (ENS Paris, France)  
*Introduction to fully non-linear parabolic equations 2.*
- *Lunch* [12h30-14h30]
- 14h30-15h45: Vincent Guedj (Institut Math. de Toulouse, France)  
*Convergence of the normalized Kähler-Ricci flow 2.*
- 16h00-17h00: Said Asserda (Université de Kénitra, Maroc)  
*Complete Ricci-flat metrics in  $\mathbb{C}^2$ .*

## Wednesday 26th october 2011

- 9h30-10h45: Sébastien Boucksom (Institut Math. de Jussieu, France)  
*Tian's properness theorem for Kähler-Einstein Fano manifolds 2.*
- *Coffee break* [10h45-11h15]
- 11h15-12h30: Cyril Imbert (ENS Paris, France)  
*Introduction to fully non-linear parabolic equations 3.*
- *Lunch* [12h30-14h30]
- 14h30-15h45: Amael Broustet (Université de Lille, France)  
*Non-deformability of Kähler-Einstein metrics 2.*
- 16h00-17h00: Stéphane Lamy (Institut Math. Toulouse, France)  
*Sarkisov links arising from the blow-up of a smooth space curve.*

## Thursday 27th october 2011

- 9h30-10h45: Vincent Guedj (Institut Math. de Toulouse, France)  
*Convergence of the normalized Kähler-Ricci flow 3.*
- *Coffee break* [10h45-11h15]

- 11h15-12h30: Sébastien Boucksom (Institut Math. de Jussieu, France)  
*Tian's properness theorem for Kähler-Einstein Fano manifolds 3.*
  - *Lunch* [12h30-14h30]
  - 14h30-15h00: Ngoc Cuong Nguyen (Jagiellonian University, Poland)  
*Low regularity of solutions to degenerate Monge-Ampère equations.*
  - 15h30-16h30: Bo Berndtsson (Göteborg University, Sweden)  
*Generalized Moser-Trudinger inequalities.*
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# Abstracts

## **Introduction to fully non-linear parabolic equations**

These lectures will provide a brief introduction to fully non-linear parabolic equations, with some emphasize on

- the maximum principle,
- the concept of viscosity solution for degenerate equations,
- the short time existence for smooth initial data.

## **Convergence of the normalized Kähler-Ricci flow**

The normalized Kähler-Ricci flow exists for all time on a Fano manifold, as was shown by H.D.Cao (Inventiones 1985). When there exists a unique Kähler-Einstein metric, it follows from Tian's properness result that a Moser-Trudinger inequality holds. This -together with his fundamental estimates on the Ricci deviation- allowed Perelman to show that in this case the normalized Kähler-Ricci flow converges to the unique Kähler-Einstein metric. We will explain the main steps of this result, following Tian-Zhu (JAMS 2007) and a recent survey of Phong-Sturm (Adv. Lect. Math. 4, Int. Press, 2010).

## **Tian's properness theorem for Kähler-Einstein Fano manifolds**

The Kähler-Einstein equation has the remarkable property of being in a natural way the Euler-Lagrange equation of a functional, first introduced and studied by Ding and Tian. They established in particular that the properness of this functional, understood in an appropriate way, implies the existence of a Kähler-Einstein metric. A remarkable result of Tian (Inventiones 1997) subsequently proved the converse implication, which is probably more surprising from a variational point of view, and also has an important analytico-geometric interpretation as a Moser-Trudinger inequality. In a more recent joint work, Phong, Song, Sturm and Weinkowe (Am.J.Math 2008) strengthened Tian's result in the form of a linear growth estimate for the Ding-Tian functional, while also streamlining Tian's original arguments. The goal of this talk is to present their approach to the properness theorem.

### **Non-deformability of Kähler-Einstein metrics**

We shall describe the construction of a special class of Fano 3-folds introduced by Mukai and Umemura. These manifolds provide a fairly simple example of a small deformation of a Fano 3-fold admitting a Kähler-Einstein metric whose nearby fibers have no such a metric. Moreover, the generic element of this class of manifolds does not admit any Kähler-Einstein metric but has no non-zero holomorphic vector field. The existence of a Kähler-Einstein metric on the central fiber follows from a direct computation of its alpha-invariant. The content of these talks are taken from papers of S.K.Donaldson (arXiv 2007) and G. Tian (Inventiones 1997).

### **Fano manifolds with nef tangent bundles (after T.Peternell)**

It is conjectured that Fano manifolds with nef tangent bundles are precisely rational homogeneous manifolds  $G/P$ . We will present a strategy recently settled by Thomas Peternell, which reduces the problem to proving that the  $n$ -th Segre class is non zero. We will then indicate how this question could possibly be solved by studying certain modified Kähler-Einstein equations and related ad hoc semi-stability conditions. A use of regularization techniques for positive  $(1, 1)$ -currents proves at least the existence of approximate solutions, but the convergence of the process is still an open problem.

### **Complete Ricci-flat metrics in $\mathbb{C}^2$**

A celebrated result of Jorgens-Calabi-Pogorelov asserts that the only convex functions in  $\mathbb{R}^n$  whose real Monge-Ampère is identically one are the quadratic polynomials. There is no complex analogue of this result. We will explain how the euclidean Taub-Nut metric can be used to construct a complete Ricci-flat but non flat metric in  $\mathbb{C}^2$ . This yields in particular an example of a non trivial strictly plurisubharmonic function in  $\mathbb{C}^2$  whose complex Monge-Ampère is identically one.

### **Sarkisov links arising from the blow-up of a smooth space curve**

I will present a joint work with Jérémy Blanc (arXiv 2011) where we study smooth curves in  $\mathbb{P}^3$  whose blow-up produces a threefold with anticanonical divisor big and nef. In particular we are able to prove the existence of some

Sarkisov links which were previously only known as numerical possibilities, and which correspond to elements in the Cremona group  $Bir(\mathbb{P}^3)$  similar to the classical Geiser involution in dimension 2.

### **Low regularity of solutions to degenerate Monge-Ampère equations**

We will discuss the maximal regularity of solutions to degenerate complex Monge-Ampère equations on compact Kähler manifolds, following examples of S.Plis (Ann.Polon. 2005).

### **Generalized Moser-Trudinger inequalities**

I will present a recent joint work with R.Berman (2011) where we establish generalized Moser-Trudinger inequalities valid for any integer Kähler class.

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## Participants

1. Omar Alehyane (El Jedida, Maroc / 23-28.10.11)
2. Hicham Amal (Kénitra, Maroc / 23-28.10.11)
3. Nourredine Askour (Beni-Mellal, Maroc / Maroc / 23-28.10.11)
4. Saïd Asserda (Kénitra, Maroc / 23-28.10.11)
5. Mohammed Barraâ (Marrakech, Maroc / 23-28.10.11)
6. Slimane Benelkourchi (Kénitra, Maroc / 23-28.10.11))
7. Bo Berndtsson (Göteborg, Suède / 23-30.10.11)
8. Taïb Belghiti (Kénitra, Maroc / 23-28.10.11)
9. Elmostafa Bendib (Kénitra, Maroc / 23-28.10.11)
10. Allami Benyaïche (Kénitra, Maroc / 23-28.10.11)
11. Charaf Bensouda (Kénitra, Maroc / 23-28.10.11)
12. Sébastien Boucksom (Paris, France / 22.10-03.11.11)
13. Abdelhamid Boussejra (Kénitra, Maroc / 23-28.10.11)
14. Ammari Boutayeb (Kénitra, Maroc / 23-28.10.11)
15. Amael Broustet (Lille, France / 21-28.10.11)
16. Jean-Pierre Demailly (Grenoble, France / 23-29.10.11)
17. Simone DiVerio (Paris, France / 23-30.10.11)
18. Hakim Elyadini (Kénitra, Maroc / 23-28.10.11)
19. Philippe Eyssidieux (Grenoble, France / 23-28.10.11)
20. Abdelhak Faouzi (ElJadida, Maroc / 23-28.10.11)
21. Bousselham Ganbouri (Kénitra, Maroc / 23-28.10.11)
22. Allal Ghanmi (Rabat, Maroc / 23-28.10.11)

23. Vincent Guedj (Toulouse, France / 21-28.10.11)
24. Henri Guenancia (Paris, France / 23-29.10.11)
25. Cyril Imbert (Paris, France / 23-26.10.11)
26. Ahmed Intissar (Rabat, Maroc / 23-28.10.11)
27. Samir Kabbaj (Kénitra, Maroc / 23-28.10.11)
28. M'hamed Kassi (Kénitra, Maroc / 23-28.10.11)
29. Julien Keller (Marseille, France / 23-30.10.11)
30. Abdelhafid Khadiri (Kénitra, Maroc / 23-28.10.11)
31. Stéphane Lamy (Toulouse, France / 23-29.10.11)
32. Hoang Chinh Lu (Toulouse, 23-28.10.11)
33. Wadie Mansouri (Kénitra, Maroc / 23-28.10.11)
34. Rafik Meziani (Kénitra, Maroc / 23-28.10.11)
35. Mohammed Missouri (Kénitra, Maroc / 23-28.10.11)
36. Zouhair Mouayn (Beni-Mellal, Maroc / 23-28.10.11)
37. Ngoc Cuong Nguyen (Cracovie, Pologne / 23-28.10.11)
38. Dan Popovici (Toulouse, France / 23-30.10.11)
39. Hassan Sfouli (Kénitra, Maroc / 23-28.10.11)
40. Ragnar Sigurdsson (Reykjavik, Islande / 23-28.10.11)
41. Dror Varolin (Stony-Brook, Etats-Unis / 22.10-03.11.11)
42. Ahmed Zeriahi (Toulouse, France / 21.10-01.11.11)
43. Hicham Zoubeir (Kénitra, Maroc / 23-28.10.11)