Feuille de TD 3

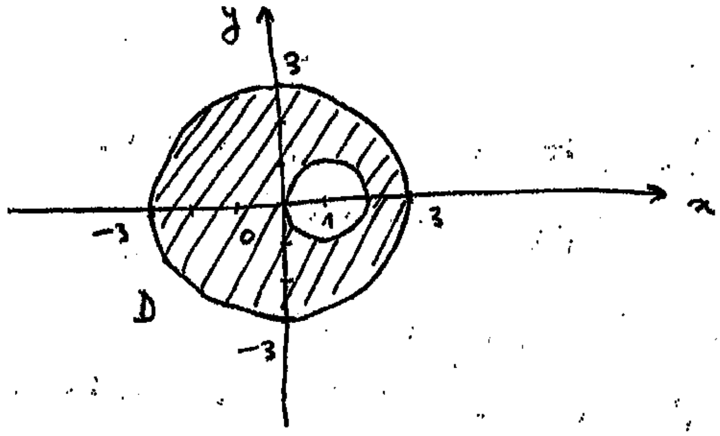
Exercice 5

$D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2-9<0 ,\ (x-1)^2+y^2 \geq 1\}$

1) $\forall \begin{smallmatrix} x\in\mathbb{R} \\ y\in\mathbb{R}\end{smallmatrix}$, on a

$(x,y) \in D \iff (x,-y) \in D$

$\rightarrow$ L'axe des x est un axe de symétrie de D.



2) $$x_G = \frac{\int_D x\,dx\,dy}{\int_D dx\,dy}.$$

$D \cup C = \Omega$, où $C = \{(x,y)\in\mathbb{R}^2 \mid (x-1)^2+y^2 \leq 1\}$

disjoints $\qquad \Omega = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2-9<0\}$

$$\rightarrow \int_D dx\,dy = \int_\Omega dx\,dy - \int_C dx\,dy = \text{Aire}(\Omega) - \text{Aire}(C) = 9\pi - \pi = \boxed{8\pi}$$

$$\int_D x\,dx\,dy = \int_\Omega x\,dx\,dy - \int_C x\,dx\,dy.$$

$\Omega = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2 < R^2\}$, avec $\underline{R=3}$

- Chang. de coordonnées polaires $\Psi : \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$$\int_\Omega x\,dx\,dy = \int_0^R\int_0^{2\pi} r\cos\theta \cdot r\,dr\,d\theta = \left(\int_0^R r^2\,dr\right)\left(\int_0^{2\pi}\cos\theta\,d\theta\right)$$

$$= \frac{R^3}{3}\cdot[\sin\theta]_0^{2\pi} = \boxed{0}$$

De même, avec le chang. de var. $\Psi \begin{cases} x = u+1 \\ y = v\end{cases}$, $\det\Psi'(u,v) = 1$

$A = \{(u,v)\in\mathbb{R}^2 \mid u^2+v^2 \leq 1\}$

$$\int_C x\,dx\,dy = \int_A (u+1)\cdot du\,dv,$$

$$= \int_A u\,du\,dv + \int_A du\,dv \overset{\text{comme pour } \Omega\ (R=1)}{=} 0 + \text{Aire}(A) = \underline{\pi}$$

$$\hookrightarrow \int_D x\,dx\,dy = -\pi \quad \rightarrow \quad \underline{\underline{x_G}} = \frac{-\pi}{8\pi} = \boxed{-\frac{1}{8}}$$

À cause de la symétrie de D par rapport à l'axe des x, $\boxed{y_G = 0}$.

En effet, si on note $D^+ = D \cap \{(x,y) \in \mathbb{R}^2 \,/\, y > 0\}$
$D^- = D \cap \{(x,y) \in \mathbb{R}^2 \,/\, y < 0\}$,

alors $\displaystyle\int_D y \, dx\, dy = \int_{D^+} y \, dx\, dy + \int_{D^-} y \, dx\, dy$.

Mais $\displaystyle\int_{D^-} y \, dx\, dy = -\int_{D^-} (-y) \, dx\, dy = -\int_{D^+} v \, du\, dv$

avec le chang. de var.
$\Psi : \begin{cases} x = u \\ y = -v \end{cases}$, $|\det \Psi'(u,v)| = |-1| = 1$.
$\Psi(D^+) = D^-$

Lis $\displaystyle\int_D y \, dx\, dy = 0 \Rightarrow y_G = 0$.