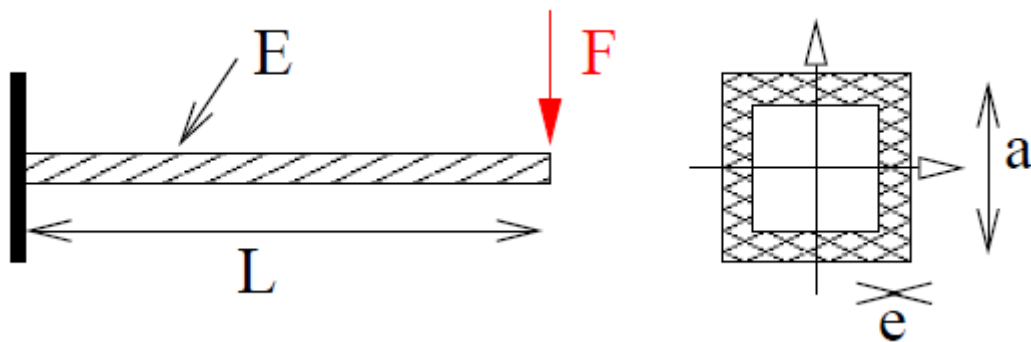


# TD3 - Exercises in R on uncertainty quantification and propagation

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## 1. A diving board model

We are interested in the vertical deflection of a beam fixed at one end and at the other end subjected to a vertical point load.



The vertical deflection of the beam end is expressed as follows:

$$y(E, F, L, I) = \frac{FL^3}{3EI}$$

where:

- E is the Young modulus of the material
- F is the applied punctual load
- L is the beam length
- I is the quadratic moment, which can be written as:

$$I = \frac{a^4 - (a - e)^4}{12}$$

For deterministic studies, the values are :

$$\begin{aligned} F &= 300 \text{ N} \\ E &= 3.0 \times 10^9 \text{ Pa} \\ L &= 2.5 \text{ m} \\ I &= 4.0 \times 10^{-6} \text{ m}^4 \end{aligned}$$

It corresponds to the nominal point (30000, 3.0e7, 250, 400) when the length is expressed in cm and not m. These values correspond to a diving board, 2.5m long, with a load of 30kg placed at its end. One could imagine, for example, a diving pool plastic, on which stands a child of 30kg.

Uncertainties in the variables (E, F, L, I) are due, for example, to:

E: manufacturing (stochastic uncertainty),

F: measurement uncertainty of the load (epistemic uncertainty),  
L and I: geometrical defect of manufacturing (stochastic uncertainty).

## 2 Uncertainty quantification, fitting a parametric law on a sample

Some data are available on F, coming from measurements. We want to fit a parametric distribution model on these data, in particular in order to use it for the propagation step uncertainties.

The data are contained in the file **sampleF.csv**.

1. Compute the mean, standard deviation, minimum and maximum of the sample. How much data do we have?
2. We want to visualize the histogram of the sample. From the shape of the histogram, begin to propose some families of laws that could match.
3. For different families of laws, estimate the parameters that best fits the data, using the method of moments or the method of maximum likelihood (fitdistr() function of the package “MASS”).
4. Perform, for each of the studied laws, a statistical test to reject (or not) the hypothesis of adequacy of the law to the data. View the adequacy of probability densities on the histogram. Draw the graphs quantile-quantile (qqplot).
5. Sort using Kolmogorov tests, laws that were not rejected by the test of adequacy. What is the law and the corresponding parameters that best fit the available data?

## 3 Uncertainty propagation, central tendency study

We consider in the following that the input variables are independent and follow the probabilistic model below:

| Variable | Law        | Parameters                              |
|----------|------------|---|
| F        | Normal     | $\mu = 30000$ ; $\sigma = 9000$         |
| E        | Triangular | $a = 2.8e7$ ; $b = 4.8e7$ ; $c = 3.0e7$ |
| L        | Uniform    | $a = 250$ ; $b = 260$                   |
| I        | Triangular | $a = 310$ ; $b = 450$ ; $c = 400$       |

We want to propagate these uncertainties through the beam model, in order to study the variability of output. For example you want to get the mean value of the beam end deflection, as its standard deviation, minimum, maximum, ... The objective is to get a good idea of the impact of input uncertainties in model output uncertainty.

1. Plot the probability densities of the input variables. For the triangular distribution, we use the “triangle” package.

2. Write a function for calculating the end beam deflection.
  3. By random Monte-Carlo, we wish to obtain a sample of 1000 values of the output. From this sample, proceed as in the first part to obtain the mean, standard deviation, minimum, maximum. Visualize the convergence of the estimates of the mean and variance as a function of sample size.
  4. Compute the the first and second order moments via the quadratic summation method. To compute the derivatives (at a nominal point), we can:
    - Either use finite differences (for example using a perturbation of 1%)
    - Either calculate exact derivatives by hand;
    - Either use the formal derivatives given by an R function, using the function `deriv()` and recovering the derivatives with the function `eval()`.We recommend using the third method because it is less cumbersome to implement.
- Is this method gives satisfactory results for the estimated moments? How many calls did we need?
5. We try to get the 95%-quantile of the output. Give the empirical quantile from the previous sample. Then calculate the quantile using the method of Wilks (`WilksFormula()` function from package “mistrall”) for a confidence level of 90% and two different order values: 1 and 3. Compare the results.

## **4 Uncertainty propagation, probability of exceeding a treshold**

We study now the risk of beam rupture. Here, the failure criterion is: "The vertical deflection of the beam exceeds the value of 30 cm". Note that the threshold value is greater than the 95% quantile.

1. Write the failure function (failure occurs if the value is negative).
2. We first want to evaluate this probability by the Monte Carlo method. We choose for this algorithm two stopping criteria: a maximum number of calls and a limit coefficient of variation of the estimator. Choose some values that seem relevant. You can iterate these values until you get a satisfactory result. What criterion was met first? How many simulations have been carried out? What is the coefficient of variation of our estimator?
3. We now use the method FORM (`FORM()` function from package “mistrall”) to assess the probability of failure of the beam. Edit the function to see how it is parameterized. Do you obtain good results? By FORM, you can also obtain some sensitivity indices, called important factors, which measure the sensitivity of the output to the input variables around the design point. What input variable has the most influence?
4. We now want to use the importance sampling method after the FORM method (`FORM()` function). The idea is that the importance density is put around the design point obtained by the FORM method. Compare the results with the previous ones (from Monte Carlo and FORM).