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Mini course

Small inhomogeneities, cloaking and approximate cloaking

Michael Vogelius
Rutgers University (New Jersey, USA)

In this set of lectures I shall first attempt to give a brief overview of a wide body of work concerning representation formulas for the electromagnetic effect of small inhomogeneities inside a known reference medium. Such representation formulas have a wide range of applications, and I shall try to illustrate some of these. One particular recent application concerns the area of "cloaking". In this context I shall describe in detail a particular scheme of "approximate cloaking by mapping" and show how estimates for the effect of small inhomogeneities lead to very precise estimates for the degree of approximate cloaking.

Plenary talks

Parameter identification for a simplified model of the respiratory tract

Muriel Boulakia
Pierre and Marie Curie University (Paris, France)

In this lecture, we are interested by the Stokes system. We assume that we have measurements of the solution on a part of the boundary where Neumann conditions are prescribed and we want to recover a Robin coefficient which appears in the Robin boundary conditions prescribed on some non accessible part of the boundary. This model can be viewed as a simplified model for the respiratory tract and the Robin coefficient corresponds to the resistance of the airways. We will first study the identifiability of the Robin coefficient and then establish a stability estimate of logarithm type.
This work is a common work with Anne-Claire Egloffe and Céline Grandmont.

An "exterior approach" to solve the inverse obstacle problem for the Laplace equation/Stokes system

Laurent Bourgeois
ENSTA (Paris, France)

We consider in this talk the problem of finding a Dirichlet obstacle in a homogeneous medium governed by the Laplace equation or the Stokes system, from a single pair of Cauchy data on a subpart of the boundary of such medium. In the case of the Stokes system, for example, such problem models the identification of a hard obstacle immersed in a fluid from the knowledge of both the velocity and the normal stress on a subpart of the fluid surface.
Our iterative approach consists in coupling a method of quasi-reversibility and a level set method:

- For an obstacle obtained at a given iteration, the method of quasi-reversibility is used to update the solution outside the obstacle.
- For a solution obtained at a given iteration, the level set method is used to update the boundary of the obstacle.

The main feature of such approach is that it does not rely on a minimization process.
During the talk, we will first introduce the two main ingredients of the above approach, this is the quasi-reversibility method and a new type of level set method based on a simple Poisson equation. Then we will prove the convergence of the method of quasi-reversibility on the one hand and the convergence of the level set method on the other hand. Lastly, the efficiency of our exterior approach will be illustrated by some numerical results based on a finite element method.
This work is a collaboration with Jérémi Dardé. Some references concerning the case of Laplace equation are given below while a paper in the case of the Stokes system is in preparation.

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Controllability and Lipschitz stability for Grushin-type operators

Piermarco Cannarsa
University of Rome Tor Vergata (Italy)

The Baouendi-Grushin operator is an important example of a degenerate elliptic operator that has strong connections with almost-Riemannian structures. It is also the infinitesimal generator of a strongly continuous semigroup on Lebesgue spaces with very interesting properties from the point of view of control theory. Such properties will be discussed in this lecture, starting with approximate and null controllability for parabolic control systems associated with Grushin-type operators on a bounded two-dimensional domain. We will then address the inverse source problem for these operators deriving a Lipschitz stability result.

Systems of linear and nonlinear parabolic equations: two different approaches for the reconstruction of coefficients

Michel Cristofol
LATP, Aix-Marseille University (France)

In this talk I am interested to give an overview of recent results concerning the reconstruction of one or several coefficients associated to systems of linear and non linear parabolic equations.
The main goal is to obtain these results minimizing the observations. The first results (see [1], [2], [3] and [4]), involve Carleman inequalities and give Lipschitz stability results, but a measurement of the components of the system on all the domain is necessary. The last result (see [5]) avoids this constraint and concerns a uniqueness result for a strong non linear parabolic system (Lotka Volterra type). The use of the initial condition $u_0 = u(0, .)$ is a partial answer to a longstanding open problem.

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Cellular motility: mechanical bases and control-theoretic point of view

Antonio De Simone
SISSA (Trieste, Italy)

We will discuss the mechanical bases of cellular motility by swimming and crawling. Starting from observations of biological self-propulsion, we will analyze the geometric structure underlying motility at small scales, the swimming strategies available to microscopic swimmers, and recipes to optimize their strokes.

On the steady motion of a coupled system solid-liquid

Giovanni Paolo Galdi
University of Pittsburgh (Pennsylvania, USA)

The main topic of this talk is centered around the unconstrained (free) motion of an elastic solid, $\mathcal{B}$, in a Navier-Stokes liquid, $\mathcal{L}$, occupying the whole space outside $\mathcal{B}$, under the assumption that a constant body force is acting on $\mathcal{B}$. More specifically, we are interested in the steady motion of the coupled system $\{\mathcal{B}, \mathcal{L}\}$, which means that there exists a frame with respect to which the relevant governing equation possess a time-independent solution. We discuss the existence of such a frame and of corresponding steady solutions, and show that they actually exist, provided some smallness restrictions are imposed on the physical parameters, and the reference configuration of $\mathcal{B}$ satisfies suitable geometric properties.
Part of this work is in collaboration with Josef Bemelmans (RWTH Aachen) and Mads Keyd (TU Darmstadt).

Uniqueness of weak solutions of two-dimensional fluid-rigid body systems

Olivier Glass
Paris Dauphine University (Paris, France)

I will discuss two different systems describing the interaction of a rigid body and an incompressible fluid in the plane. In the first case, the fluid is inviscid and described by the incompressible Euler equation. In the second case, the fluid is viscous and described by the incompressible Navier-Stokes equation. I will explain uniqueness results for weak solutions of these systems: uniqueness of the "Yudovich" solutions for the first system, and of the "Leray" solutions for the second one.
This is a joint work with Franck Sueur.

Some existence results for fluid-structure interaction problems

Céline Grandmont
INRIA Paris-Rocquencourt (Le Chesnay, France)

In this talk we will review some known existence results concerning a fluid interacting with a thin structure located on one part of the fluid boundary domain. We will consider a three dimensional (resp. two dimensional) viscous incompressible fluid governed by the Navier-Stokes equations and interacting with an elastic plate or membrane (resp. beam or rod). The deformation of the fluid domain, which depends on the displacement of the structure, is not neglected, leading to geometrical non linearities. For this kind of non linear coupled problems, we will review existence results of strong or weak solutions for the steady case as well as the unsteady one. We will also discuss the need of damping terms for the elastic part as well as investigate the question of contact between the moving structure and the rigid boundary.

On the inverse problems for the coupled continuum pipe flow model for flows in karst aquifers

Shuai Lu
School of Mathematical Sciences, Fudan University (Shanghai, China)

We investigate two inverse problems for the coupled continuum pipe flow (CCPF) model which describes the fluid flows in karst aquifers. After generalizing the well-posedness of the forward problem to the anisotropic exchange rate case which is a space-dependent variable, we present the uniqueness of this parameter by measuring the Cauchy data. Besides, the uniqueness of the geometry of the conduit by the Cauchy data is verified as well. These results enhance the practicality of the CCPF model.
It is a joint work with Xinming Wu, Jin Cheng (Fudan University) and Philipp Kügler (Austrian Academy of Science).

Separation and bifurcation phenomena for flows interacting with a boundary

Marco Sammartino
University of Palermo (Italy)

In this talk we shall review some rigorous and numerical results on the evolution of boundary layers for the incompressible Navier-Stokes equations. The possibility of interpreting the phenomena leading to separation of the boundary layer in terms of bifurcation of equilibria will be explored.

Stability results for nonlinear parabolic systems. Application to fluid-structure interaction systems

Takéo Takahashi
INRIA Nancy - Grand Est (France)

We present some abstract results on the stabilization of nonlinear parabolic systems. More precisely, we show how a natural unique continuation property is sufficient to obtain the feedback stabilization of some nonlinear parabolic systems with a finite number of controllers. We can apply such a criterion to many systems such as the classical Navier–Stokes system.
We also use this abstract method to some fluid-structure interaction systems: a 1d simplified model composed by a particle and where the fluid motion is modeled by the viscous Burgers equation and the 2d/3d model corresponding to the coupling between Navier–Stokes equations and the motion of a rigid body. In that case, one of the difficulty consists in handling the moving domain of the fluid equations by using an appropriate change of variables.

Time optimal control of infinite dimensional linear systems

Marius Tucsnak
Institut Élie Cartan, Henri Poincaré University (Nancy, France)

We consider the time optimal control problem, with a point target, for infinite dimensional time invariant linear systems. We first consider systems with a dynamics governed by an abstract Schrödinger type equation, with bounded control operator. The main results establish a Pontryagyn type maximum principle and give sufficient conditions for the bang-bang property of optimal controls. The results are then applied to some systems governed by partial differential equations. We discuss possible extensions and we state some open problems concerning time reversible systems.
We next consider systems which are not time reversible and in particular the heat equations. The fact that the time optimal controls for parabolic equations have the bang-bang property has been recently proved for controls distributed inside the considered domain (interior control). The main result in this article asserts that the *boundary controls* for the heat equation have the same property, at least in rectangular domains. This result is proved by combining methods from traditionally distinct fields: the Lebeau-Robbiano strategy for null controllability and estimates of the controllability cost in small time for parabolic systems, on one side, and a Remez-type inequality for Müntz spaces and a generalization of Turán's inequality, on the other side.

Keywords: Heat and Schrödinger equations, time optimal control, maximum principle, bang-bang.

Uniqueness results by partial Cauchy data on arbitrary subboundary for 2-dimensional elliptic systems

Masahiro Yamamoto
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I will present our recent results on the uniqueness in determining coefficients in various 2-dimensional elliptic systems by all the set of Cauchy data with Dirichlet data supported on arbitrary subboundary Γ and Neumann data on Γ. The classical Dirichlet-to-Neumann map corresponds to a special case where Γ is the whole boundary, and our results are the best possible uniqueness results in two dimensions within some smoothness assumptions.

Invited talks

Reduced number of controls for N-coupled systems of PDE's

Fatiha Alabau-Boussouira
LMAM, Inria Equipe-projet CORIDA, University of Lorraine (Metz, France)

Systems of PDE's describe in general complex interactions between several unknowns which characterize the state of the physical devices under study. In a first approximation some of these interactions may be neglected so that the system reduces to decoupled scalar equations. Here we focus on "fully" coupled systems of PDE's. One of the main challenging issue since more than a decade is the question of their controllability/observability by a reduced number of controls/observations. This means that the number of controls is strictly less than the number of unknowns/equations which is a more difficult situation in control questions. We shall present a general approach based on a two-level energy method and its generalization for handling this issue.

We consider two classes of coupled systems, namely: 2-coupled symmetric systems (coupling two symmetric evolution PDE's by symmetric lower order coupling terms) and N-coupled cascade systems (coupling N evolution PDE's in a lower or upper matrix form). We present several positive boundary or localized controllability/observability results by a reduced number of controls for these two classes of systems in a multi-dimensional framework. Moreover we shall focus on geometrical situations for which the regions of localization of the controls/observations do not meet the regions of localization of the couplings terms. This situation is richer and more complex to study than the one for which these regions meet.

We shall also give several examples of applications to coupled wave, heat or Schrödinger equations. The striking property is that the lower order coupling terms allow to transfer information from the controlled/observed PDE to the uncontrolled/unobserved ones even if the corresponding control/observation and coupling regions are away from each other. This shows that the interactions between the different state components may contain very useful properties and have important geometrical characteristics in the context of control theory so that they should not be neglected.

Keywords: Control theory. Coupled systems of evolution PDE's. Reduced number of controls. Hyperbolic equations. Diffusive equations. Heat equations.

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Optimal time for the null controllability of parabolic systems: the effect of the condensation index of complex sequences

Assia Benabdallah
Aix-Marseille University (France)
(joint work with F. Ammar Khodja, M. González-Burgos and L. de Teresa)

This talk studies the relation between the condensation index of a sequence of complex numbers, with positive real part, and the null controllability of parabolic systems. In particular we show that a minimal time is required for controllability. We present illustrative examples of the null controllability problem associated with coupled parabolic equations.

Positive and negative results on the control of fluids with memory

Enrique Fernández-Cara
Dpto. EDAN, University of Sevilla (Spain)

Abstract: The goal of this contribution is to present some results concerning the controllability of fluids where memory effects are important. We consider linear visco-elastic models of the Maxwell and Jeffreys kinds and some other similar systems.

Keywords: Null and approximate controllability, visco-elastic fluids, memory effects.

Positive results for Maxwell fluids

First, we analyze the controllability properties of systems which provide a description, at first approximation, of a kind of viscoelastic fluids. We consider linear Maxwell fluids. We establish the large time approximate-finite dimensional controllability of the system, with distributed or boundary controls supported by arbitrary small sets. Then, we prove the large time exact controllability of fluids of the same kind, with controls supported by suitable large sets. The proofs of these results rely on classical arguments. In particular, the approximate controllability result is implied by appropriate unique continuation properties, while exact controllability is a consequence of observability (inverse) inequalities. We also discuss some other questions concerning to the controllability of viscoelastic fluids and some related open problems. These results have been taken from [2].

Positive results for Jeffreys fluids

We analyze the control of vicoelastic fluids of the Jeffreys kind, also known as Oldroyd models. We present the interesting problems, with special emphasis in the difficulties that they involve. Then, we will consider appropriate linear approximations and we will establish some partial approximate-finite dimensional controllability results in arbitrarily small time, with distributed or boundary controls supported by arbitrarily small sets. The proofs rely on some specific unique continuation properties which are implied by the structure of the solutions; see [1,3].

Negative results for fluids where viscosity and memory effects co-exist

Here, we deal with the boundary null controllability problem for the Stokes system with a memory term. For any positive final time T, with controls acting on the whole boundary, we show that there exist initial conditions such that the null controllability property fails. This is closely related to the results and arguments in [4], where the authors consider heat equations with memory terms.

Other similar problems and results

We also present other similar results obtained by other authors, not necessarily concerning memory effects. In particular, we take a look to the role of compressibility in the context of controllability problems.

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Exact controllability for a system of coupled wave equations on a compact manifold

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(joint work with Matthieu Léautaud and Jérôme Le Rousseau)

Let (M, g) be a compact connected n-dimensional Riemannian manifold without boundary. We take two smooth functions b_ω and b on M and we consider the controllability problem for the system of coupled wave equations

$$\begin{cases} (\partial_t^2 - \Delta)u_1 + b(x)\, u_2 = 0 & \text{in } (0,T) \times M, \\ (\partial_t^2 - \Delta)u_2 = b_\omega(x)\, f & \text{in } (0,T) \times M. \\ \text{Initial Data in } (H^2 \times H^1) \times (H^1 \times L^2) \end{cases} \tag{S}$$

Here, the state of the system is $(u_1, u_2, \partial_t u_1, \partial_t u_2)$ and f is our control function, localized on $\{b_\omega \neq 0\}$.
For most results proved in this paper, we shall assume that the function b is non-negative on M, and denote by $\omega = \{b_\omega \neq 0\}$ the control set and by $O = \{b \neq 0\}$ the coupling set (which is the indirect control set for the first equation).
A natural necessary and sufficient condition to have controllability for wave equations is to suppose that the control set satisfies the Geometric Control Condition (GCC) defined in Bardos-Lebeau-Rauch. For $\omega \subset M$ and $T > 0$, we shall say that (ω, T) satisfies GCC if every geodesic traveling at speed one in M meets ω in a time $t < T$. We say that ω satisfies GCC if there exists $T > 0$ such that (ω, T) satisfies GCC. Note that in the situation of system (S), a necessary condition is that both sets ω and O satisfy GCC.

Definition: Assume that the sets O and ω satisfy GCC. We define $T_{\omega \to O \to \omega}$ to be the infimum of times $T > 0$ for which the following assertion is satisfied:
every geodesic traveling at speed one in M meets ω in a time $t_0 < T$, meets O in a time $t_1 \in (t_0, T)$ and meets again ω in a time $t_2 \in (t_1, T)$.

Theorem 1: Assume that ω and O both satisfy GCC. Then system (S) is controllable if $T > T_{\omega \to O \to \omega}$ and is not controllable if $T < T_{\omega \to O \to \omega}$.

On the other hand, by a change of functional spaces and a well adapted splitting, one can work in the space $H = L^2_+(M) \times \mathbb{C}^4$, where L^2_+ is the subspace of L^2functions with non zero frequencies.

Theorem 2: Under GCC and in the splitting above, the HUM control operator is, up to a smoothing operator, an elliptic pseudodifferential operator with zero order.

We also consider the case of two different speeds, i.e we study exact controllability of the following system, with $\gamma \neq 0, 1$:

$$\begin{cases} (\partial_t^2 - \Delta)u_1 + b(x)\, u_2 = 0 \\ (\partial_t^2 - \gamma^2\Delta)u_2 = b_\omega(x)\, f \end{cases} \tag{S$_\gamma$}$$

Theorem 3: Assume that $\omega \cap O$ satisfies GCC. Then (S_γ) is controllable in the space $(H^3 \times H^2) \times (H^1 \times L^2)$, in any time $T > \max(T^1_{\omega\cap O}, T^\gamma_{\omega\cap O})$.

The tools used in the proofs are essentially of microlocal nature. For Th.1, we use propagation properties of microlocal defect measures attached to bounded sequences of solutions of the adjoint system. Th.2 is based on Egorov Theorem, and finally Th.3 uses in a crucial way smoothing properties of system S_γ for $\gamma \neq 1$.

Lagrangian controllability of some fluid models

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I will present the notion of Lagrangian controllability. Specific results concerning mainly the Euler equations will be given describing how to settle the main definition of this notion will be given as well as a comparison with the controllability in Eulerian descriptions. Works in progress on numerics and other fluid models will be discussed according to their state at the present time of the conference. All results presented are in common with Olivier Glass. More precisely I will describe the results proven in [1] and [2]. I will also describe difficulties arising in numerical issues as shown by G. Legendre in a work in progress ([3]). Ideally, it would be certainly a great progress to transpose the already proven results to the case of multifluids. In order to motivate a possible scheme, I will give some ideas on quasistatic motions of fluids.

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Inverse problems for elliptic PDEs and Hardy classes of generalized analytic functions

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(joint work with Laurent Baratchart and Yannick Fischer)

Abstract: Issues concerning solutions to boundary inverse problems from partial overdetermined Dirichlet-Neumann data for Laplace, conductivity or Schrödinger equations in smooth plane domains are approached in normed Hardy classes of generalized holomorphic complex valued functions.

Keywords: Inverse problems, Cauchy-type problems, Hardy classes, Holomorphic functions, Best constrained approximation issues.

Whenever Ω is a smooth domain in $\mathbb{R}^2$, solutions to families of elliptic partial differential equations –like Laplace, conductivity or Schrödinger PDEs– can be described as (real or imaginary parts) of holomorphic or generalized holomorphic functions of the complex variable [1] (solutions to $\bar{\partial}$ equations, also called pseudo-holomorphic [7]).
Weak assumptions on available Dirichlet or Neumann boundary data can then be formulated as boundedness conditions in Hardy norm. This leads to the introduction of associated Hardy classes of generalized analytic functions [2, 3].
Both direct and boundary inverse recovery problems can be stated and solved as best (constrained) approximation issues in these classes. This leads to well-posed approximate formulations of Cauchy-type inverse problems from partial overdetermined Dirichlet-Neumann data [5]. On the way, unique continuation and Runge density properties are established. Further, constructive recovery schemes and algorithms are available in a number of situations, where complete families of solutions are available. This namely holds for Laplace equations [6] and for particular conductivity coefficients that arise in physical applications related to plasma confinement [4], stemming from Maxwell equations.

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On the controllability of driftless swimmers

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CORIDA Project, Numerics, SOLEIL project
(Joint work with Thomas Chambrion, Marc Fuentes, Jérôme Lohéac and Bruno Pinçon)

The modeling of swimming usually leads to a complex system of equations coupling PDEs (governing the fluid flow) and ODEs (governing the motion of the swimmer). In this talk, I will address two particular cases referred to as "driftless models". The first one (called "resistive model") is relevant for microswimmers (like microorganisms) and consists in neglecting the inertial effects in the modeling. The second one (called "reactive model") is obtained by neglecting instead the viscous forces and is supposed to be relevant for swimmers with elongated bodies (like eels). Surprisingly, the dynamics are very similar in both cases.

I will state some (generic) controllability results for these models and give the taste of the proofs, which relies deeply on the analyticity of the dynamics. The Orbit Theorem (of Nagano-Sussman) and its Corollary (referred to as Rashevsky-Chow theorem) play a crucial role as well.

Time permitting, I will show some numerical simulations from the SOLEIL project (Solveur d'Équations Intégrales pour la Locomotion). SOLEIL is a set of Matlab functions to study fish locomotion. More details are available on the web page http://soleil.gforge.inria.fr/.

Carleman estimate for Zaremba boundary condition

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Le problème de Zaremba est un problème elliptique avec une condition mixte au bord. Plus précisément, d'un côté d'une hypersurface du bord on impose la condition de Dirichlet et de l'autre la condition de Neumann. Pour ce problème nous démontrons une inégalité de Carleman près du bord. Dans l'exposé nous essaierons de donner les grandes lignes de la preuve. Celle-ci consiste à se restreindre au bord. Cela donne une équation pseudo-différentielle sur le bord qui permet d'estimer les traces de la solution en fonction des données. L'application qui a motivé notre travail est un problème de stabilisation. Cette inégalité peut aussi s'appliquer pour le contrôle de l'équation de la chaleur.

Control and mixing for 2D Navier-Stokes equations with space-time localised force

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We consider 2D Navier-Stokes equations in a bounded domain with smooth boundary and discuss the interconnection between controllability for the deterministic problem and mixing properties of the associated random dynamics. Namely, we first consider the problem of stabilisation of a given non-stationary solution, assuming that the control is localised in space and time and is finite-dimensional as a function of both variables. We next replace the control by a random force and prove that the resulting random dynamical system is exponentially mixing in the Kantorovich-Wasserstein distance.
Some of the results of this talk are obtained in collaboration with V. Barbu and S. Rodrigues.

Posters

New finite dimensional observer for the boundary control of fluid flows

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(Joint work with Laleh Ravanbod and Jean-Pierre Raymond)

We present a new finite dimension estimator for infinite dimensional systems with discrete spectrum and finitely many eigenvalues in $Re(s) > -\delta$ for all $\delta > 0$ [1].
The existence of a finite dimensional compensator for stabilizing such systems was first proved by Schumacher [2, 3]. The considered class included parabolic systems with bounded control and observation operators. Subsequently, Curtain in [4, 5, 6] proposed an alternative scheme applicable to a general class of parabolic or hyperbolic systems with Neumann or Dirichlet boundary conditions and bounded or unbounded control and observation operators. It was named integral dynamic output feedback control and it was based on the relocation of the eigenvalues. All these approaches share the common disadvantage that the dynamics corresponding to an infinite number of eigenvalues in $Re(s) < -\delta$ are neglected in the estimator equations.
The approach proposed by Fujii in [7] seems to overcome this problem. There, a functional observer of Luenberger type was presented. The solution of linear parabolic initial boundary value problem was decomposed into the solution when the control input was zero and the solution when the initial condition was zero. The observation law, then, included the convolution integral of these decomposed solutions with the solution of the state feedback control law. However, the resulting observer was of infinite dimension.
Here, we propose and analyse a new observer of finite dimension, coupled with a finite dimensional feedback controller. As in [2, 3, 4, 5, 6], we still express the dynamics in terms of a stable and an unstable part. In our new approach, we take advantage of the fact that the stable part can be decomposed as in [8]. This is the new trick which enables us to determine our observer.

Our numerical test concerns the stabilization of two-dimensional linearized Navier-Stokes equations by a boundary control and using boundary observations of velocity and of pressure in the case of a flow around a circular cylinder. We show also that the new estimator remains efficient above an acceptable Signal to Noise Ratio for the nonlinear Navier-Stokes equations.

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Stabilization of a fluid-solid system, by the deformation of the self-propelled solid

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Abstract: We consider a deformable solid immersed in a viscous incompressible fluid filling a bounded domain $\mathcal{O}$ of $\mathbb{R}^2$ or $\mathbb{R}^3$. The solid's position, the fluid's velocity and the fluid's pressure are assumed to satisfy a given coupled system. We explain how this fluid-solid system can be stabilized to zero, in acting on the proper solid's deformation.

Keywords: Fluid-structure interactions, Navier-Stokes equations, Stabilization.

Presentation

The solid occupies the domain $\mathcal{S}(t)$, and the fluid occupies $\mathcal{F}(t) = \mathcal{O} \setminus \overline{\mathcal{S}(t)}$. The fluid's velocity u, its pressure p, and the solid's position given by h and ω are assumed to satisfy the following coupled system

$$\begin{aligned} \frac{\partial u}{\partial t} - \nu\Delta u + (u\cdot\nabla)u + \nabla p &= 0, \quad x \in \mathcal{F}(t), \quad t \in [0,T], && (1)\\ \mathrm{div}\, u &= 0, \quad x \in \mathcal{F}(t), \quad t \in [0,T], && (2) \end{aligned}$$

$$\begin{aligned} u = 0, &\qquad x \in \partial\mathcal{O}, \quad t \in [0,T], && (3)\\ u = h'(t) + \omega(t) \wedge (x - h(t)) + w(x,t), &\qquad x \in \partial\mathcal{S}(t), \quad t \in [0,T], && (4) \end{aligned}$$

$$Mh''(t) = -\int_{\partial\mathcal{S}(t)} \sigma(u,p)n\mathrm{d}\Gamma, \quad t \in [0,T], \tag{5}$$

$$(I\omega)'\,\mathbf{1}.(t) = -\int_{\partial\mathcal{S}(t)} (x - h(t)) \wedge \sigma(u,p)n\mathrm{d}\Gamma, \quad t \in [0,T], \tag{6}$$

$$u(y,0) = u_0(y), \quad y \in \mathcal{F}(0), h(0) = h_0 \in \mathbb{R}^d, \quad h'(0) = h_1 \in \mathbb{R}^d, \quad \omega(0) = \omega_0 \in \mathbb{R}^3. \tag{7}$$

The angular velocity ω is associated with a rotation $\mathbf{R}$. The control is seen through the velocity w, defined as

$$w(x,t) \;=\; \mathbf{R}(t)\, w^*\left(\mathbf{R}(t)^T(x - h(t)), t\right), \quad x \in \mathcal{S}(t). \tag{8}$$

The Eulerian velocity w^* can be chosen as the control, but also the Lagrangian flow X^* associated with w^* and defined as

$$\frac{\partial X^*}{\partial t}(y,t) = w^*(X^*(y,t),t), \quad X^*(y,0) = y - h_0, \quad y \in \mathcal{S}(0). \tag{9}$$

The mapping X^* must be a C^1-diffeomorphism. It defines $\mathcal{S}(t)$, as

$$\mathcal{S}(t) \quad = \quad h(t) + \mathbf{R}(t) X^*\left(\mathcal{S}(0), t\right),$$

and must satisfies some constraints that guarantee the *self-propelled* character of the solid.

The main result

System (1)-(7) is linearized, and the corresponding linear system is the following

$$\frac{\partial U}{\partial t} - \nu \Delta U + \nabla P \quad = \quad 0, \quad \text{in } (0,T) \times \mathcal{F}, \tag{10}$$

$$\operatorname{div} U \quad = \quad 0, \quad \text{in } (0,T) \times \mathcal{F}, \tag{11}$$

$$U = 0, \qquad \text{in } (0,T) \times \partial\mathcal{O}, \tag{12}$$

$$U = H'(t) + \Omega(t) \wedge y + \frac{\partial X^*}{\partial t}(y,t), \qquad y \in \partial\mathcal{S}, \quad t \in [0,T], \tag{13}$$

$$MH''(t) = -\int_{\partial\mathcal{S}} \sigma(U,P) n \mathrm{d}\Gamma, \quad t \in [0,T], \tag{14}$$

$$I_0 \Omega'(t) = -\int_{\partial\mathcal{S}} y \wedge \sigma(U,P) n \mathrm{d}\Gamma, \quad t \in [0,T], \tag{15}$$

$$U(y,0) = u_0(y), \quad y \in \mathcal{F}, \quad H'(0) = h_1 \in \mathbb{R}^d, \quad \Omega(0) = \omega_0 \in \mathbb{R}^3. \tag{16}$$

Theorem. *For all $\lambda > 0$, and all $(u_0, h_1, \omega_0) \in \mathbf{H}_{cc}$, there exists $X^* \in \mathcal{W}_{0,m}(0,T;\mathcal{S})$, such that the solution to system (10)-(16) obeys*

$$\|(U, H', \Omega)\|_{\mathrm{L}^2(0,\infty;\mathbf{H}_{cc})} \quad < \quad C \exp(-\lambda t).$$

Ideas of the proof

1. We show that system (10)-(16) can be written through an operator, which acts on (U, H', Ω), and which defines an analytic semigroup. Thus the unstable modes of this operator are in finite number, and they define a finite-dimensional system for which approximate controllability implies stabilizability.

2. Introducing an adjoint system (associated with system (10)-(16)), whose unknowns are (Φ, ψ, k', r), the approximate controllability of system (10)-(16) is reduced to a unique continuation problem for which we have

$$\int_0^T \int_{\partial \mathcal{S}} \frac{\partial X^*}{\partial t} \cdot \sigma(\Phi, \psi) n \mathrm{d}\Gamma = 0.$$

3. We choose X^* as the solution of a modified Lamé system, with a nonhomogeneous Dirichlet condition, such that X^* satisfies constraints which make the solid *self-propelled*, and such that the equality (17) leads us to

$$\sigma(\Phi, \psi) n \quad = \quad 0.$$

This boundary condition (combined to the a Dirichlet one) enables us to get $(\Phi, \psi) = 0$.

Remark.
1. A similar result is obtained for the nonlinear system (1)-(7).
2. One of the main difficulty lies in the fact that the regularity of the control X^* is limited.

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Lipschitz stability estimate for the Stokes system with mixed boundary conditions

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Abstract: We consider the Stokes system and we assume that mixed Dirichlet, Neumann and Robin boundary conditions are prescribed. We are interested here in identifiability and stability properties for the inverse problem of identifying a Robin coefficient on some non accessible part of the boundary from available measurements on another part of the boundary. We provide a Lipschitz stability estimate under the further a priori assumption that the Robin coefficient is piecewise constant.

Keywords: Inverse problem, Stokes system, Lipschitz stability estimate.

Introduction

Let $d \in \mathbb{N}^*$ and $\Omega \subset \mathbb{R}^d$ be a Lipschitz connected bounded open set such that $\partial\Omega = \Gamma_0 \cup \Gamma_e \cup \Gamma_l$. Let $\Gamma \subset \Gamma_e$. We assume that Γ and Γ_0 are of class $\mathcal{C}^\infty$. We are interested in the following Stokes system:

$$\left\{\begin{array}{ccll} -\Delta u + \nabla p & = & 0, & \text{in } \Omega, \\ \nabla \cdot u & = & 0, & \text{in } \Omega, \\ u & = & 0, & \text{in } \Gamma_l, \\ \nabla u \cdot n - pn & = & g, & \text{on } \Gamma_e, \\ \nabla u \cdot n - pn + qu & = & 0, & \text{on } \Gamma_0. \end{array}\right. \tag{1}$$

We want to identify the Robin coefficient q defined on Γ_0 from measurements available on Γ. Such problems can be viewed as a generalization of some inverse problems which appear naturally in the modeling of some biological problems, like for instance blood flow in the cardiovascular system (see [1] or the airflow in the lungs (see [2]). Analogous inverse problems for the Stokes system have already been studied in [3, 4]. In both papers, logarithm stability estimates are obtained. Note that E. Sincich obtained in [6] a Lipschitz stability estimate for a similar inverse problem but concerning the Laplace equation.

Main results

Uniqueness result concerning this inverse problem can be obtained as a corollary of C. Fabre and G. Lebeau unique continuation result for the Stokes equations (see [5]). It states that, under some regularity assumption on the data g, if the velocities are equal on Γ, then the Robin coefficients are equal on Γ_0.

Henceforth, we assume that q is piecewise constant on Γ_0. As regards the stability estimate, we are going to prove a Lipschitz stability estimate. Let us be more precise: we consider, for $i = 1, 2$, (u_i, p_i) solution of (1) associated with q_i. We obtain that there exists $C > 0$ such that:

$$\begin{aligned} &\|q_1 - q_2\|_{L^\infty(\Gamma_0)} \\ &\leq C \left(\|u_1 - u_2\|_{L^2(\Gamma)} + \left\| \frac{\partial u_1}{\partial n} - \frac{\partial u_2}{\partial n} \right\|_{L^2(\Gamma)} + \|p_1 - p_2\|_{L^2(\Gamma)} + \left\| \frac{\partial p_1}{\partial n} - \frac{\partial p_2}{\partial n} \right\|_{L^2(\Gamma)} \right). \end{aligned} \tag{2}$$

It is interesting to compare identifiability and stability results. In the identifiability result, we need to have equality of the velocities together with equality of the normal component of the constraints on Γ. In the stability estimate, the constraint is divided into two terms: $\left\| \frac{\partial u_1}{\partial n} - \frac{\partial u_2}{\partial n} \right\|_{L^2(\Gamma)}$ in one hand and $\|p_1 - p_2\|_{L^2(\Gamma)}$ in the other hand. Moreover, in inequality (2), there is also an additional term: $\left\| \frac{\partial p_1}{\partial n} - \frac{\partial p_2}{\partial n} \right\|_{L^2(\Gamma)}$. Therefore, it might be interesting to know if it is possible to obtain a stability inequality with less measurement terms and in particular, if it is possible to get rid of the gradient term $\frac{\partial p_1}{\partial n} - \frac{\partial p_2}{\partial n}$.

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Influence of boundary on the motility of micro-swimmers

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(Joint work with François Alouges)

Abstract: Swimming, i.e., being able to advance in the absence of external forces by performing cyclic shape changes, is particularly demanding at low Reynolds numbers which is the regime of interest for micro-organisms and micro-robots. We focus on self-propelled stokesian robots composed of assemblies of balls and we prove that the presence of a wall has an effect on their motility. To rest on what has been done in [1] for such system swimming on $\mathbb{R}^3$, we demonstrate that a controllable swimmer remains controllable in a half space whereas the reachable set of a non fully controllable one is affected by the presence of a wall.

Keywords: Control theory, Biological and artificial micro-swimmers, Self propulsion, Movement and locomotion, Low-Reynolds-number, Stokes equation, Boundary effect.

Self-propulsion at low Reynolds number is a problem of considerable biological and biomedical relevance which has also great appeal from the point of view of fundamental science. Many applications are concerned as for example the creation of micro devices be able to swim in a narrow channel. Swimming in a geometrically confined environment is a subject of growing interest. Since the sixties, experiments proved that in confined geometries, microorganisms are attracted by the boundaries, as for example the study of Rothschild in [4] on bull spermatozoa. On a more theoretical side, R. Zagar, A. Najafi and M. Miri proved in [2] that the dynamic of the Three-sphere swimmer, a model introduced by A. Najafi and R.Golestanian in [3], is affected by the plane wall. We attack the same problem (the influence of a plane wall in the motion of this swimmer) by means of control theory. The question that we want to address is whether the presence of the plane wall modifies the controllability of specific swimmers as such studied in the whole space by F. Alouges, A. Desimone, L. Heltai, A. Lefebvre-Lepot and B. Merlet in [1]. The main results are applied on two specific swimmers which consist of N (here $N = 3, 4$) spheres connected by thin jacks which are able to elongate or shrink.

- $N = 3$ represents the case of the Three-sphere swimmer.
- $N = 4$ corresponds to the Four-sphere swimmer depicted below.

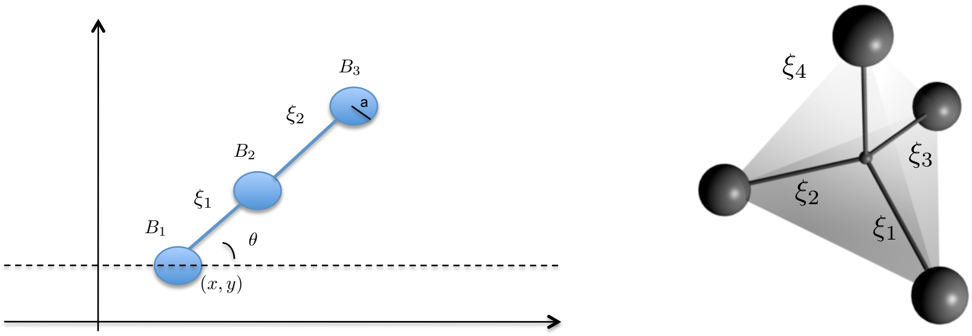

Figure 1: The Three-sphere swimmer and the Four-sphere swimmer

First, we prove that the Four-sphere swimmer remains controllable in the half space. This result is based on the fact that when the swimmer is sufficiently far from the wall, its dynamic is close to the one without boundary.
Then, concerning the Three-sphere swimmer, we describe its reachable set. In the whole space, it can only reach one direction (see [1]), whereas, we prove that if the initial position of the swimmer is not perpendicular to the plane wall, it can move in one more direction. In others words, the system related to the motion of the Three-sphere swimmer becomes controllable for almost all initial positions. The proof is based on the description of the orbits of the vector fields related to the swimmer's motion by using the Nagano's theorem.

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Applications of defects localization for the reconstruction of an acoustic refraction index

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Abstract: We are investigating numerical methods to retrieve information about an acoustic scatterer's refraction index from far field measurements. Problems of this kind are commonly non linear and ill-posed.
We have developed a sampling method to localize differences, which we call defects, in the actual index when compared to a known reference index. So, we use this information to propose two separate strategies to reduce the number of parameters needed in the reconstruction of the actual refraction index. Moreover, we investigate the minimization of defects as a new approach for the complete index reconstruction. Our results are illustrated by numerical experiments.

Keywords: Inverse problems, Acoustic scattering, Iterative methods.

Introduction

In inverse acoustic scattering, one tries to recover information about a scatterer from measurements. Penetrable scatterers are frequently referred to as inhomogeneous media and characterized by a refraction index [1, 3]. We are interested in the reconstruction of the refraction index from far-field measurements. This generally leads to iterative methods involving numerous heavy computations.
We have extended the Factorization method [2] to localize the differences between the actual index and a known reference index. These differences will be called defects. Thus, differences between some computed index and the actual index can be seen as defects and we use this information in the reconstruction of the actual index.

Enhancement of iterative strategies for index reconstruction by selective focusing

First, we propose a strategy to reconstruct a perturbed version of some reference index. These perturbations are treated as defects and thus, can be localized. So, only the parameters

corresponding to these defects need to be reconstructed. This naturally provides a substantial reduction in computational costs.
Secondly, we propose an iterative refinement strategy to compute a more precise reconstruction of a refraction index with few parameters. Each step of this strategy has two stages. First, we refine the zone containing to the most contrasting defect. Then, the reconstruction is computed with this new set of parameters. This leads to an approximation of the actual index with a constrained number of parameters positioned to fit as much as possible the geometry of this index.

Index reconstruction by defects minimisation

Lastly, another interpretation for our defects localization result, is that the absence of defects means that the actual index is equal to the reference index. Therefore, we propose a new way to reconstruct an index of refraction by looking for the reference index such that the actual index presents no more defects. We compare this approach to the classical reconstruction which consists in matching directly the simulation with the measurements. Also, we show some numerical results obtained with a Gauss-Newton method coupled with a total variation regularization [4].

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Indirect stabilization of weakly coupled systems

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Abstract: We investigate stability properties of indirectly damped systems of evolution equations in Hilbert spaces, under new compatibility assumptions. We prove polynomial decay for the energy of solutions and optimize our results by interpolation techniques, obtaining a full range of power-like decay rates. In particular, we give explicit estimates with respect to the initial data. We discuss several applications to hyperbolic systems with *hybrid* boundary conditions and globally distributed coupling, including the system of two wave equations subject to Dirichlet and Robin type boundary conditions, respectively.
Moreover, we present some new results concerning the stabilization properties of systems of weakly coupled hyperbolic equations with both damping and coupling acting only on a subset of the boundary.

Keywords: Indirect stabilization, energy estimates, interpolation spaces, evolution equations, hyperbolic systems.

Introduction

In recent years, the interest of the scientific community in the stabilization and control of systems of partial differential equations has remarkably increased, due to the fact that such systems arise in several applied mathematical models, such as those used for studying the vibrations of flexible structures and networks, or fluids and fluid-structure interactions. Moreover, it becomes essential to study whether controlling only a reduced number of state variables suffices to ensure the stability of the full system.
As an example, we are concerned with the stabilization properties of systems like

$$\begin{cases} \partial_t^2 u - \Delta u + \partial_t u + \alpha v = 0 & \text{in} \quad \Omega \times \mathbb{R} \\ \partial_t^2 v - \Delta v + \alpha u = 0 & \text{in} \quad \Omega \times \mathbb{R} \\ u + \frac{\partial u}{\partial \nu} = 0 = v & \text{on} \quad \partial\Omega \times \mathbb{R}\,, \end{cases} \tag{1}$$

where Ω is a bounded open domain of $\mathbb{R}^N$, and the 'frictional' term $\partial_t u$ acts as a stabilizer.

Main results

In a real Hilbert space H, with scalar product $\langle \cdot, \cdot \rangle$ and norm $|\cdot|$, we study the system of evolution equations

$$\begin{cases} u''(t) + A_1 u(t) + Bu'(t) + \alpha P_1 v(t) = 0 \\ v''(t) + A_2 v(t) + \alpha P_2 u(t) = 0 \end{cases} \tag{2}$$

with appropriate assumptions on operators B, A_i, P_i $(i = 1, 2)$ and the coefficient $\alpha \in \mathbb{R}$.
When P_i are bounded and coercive operators (that is, when the coupling between the two components of the system is globally distributed), we present results assuring the polynomial stabilization property of system (2) under suitable compatibility conditions on A_i [4] [3].
Moreover, in the case of operators P_i unbounded (when, for example, the coupling acts on the boundary of the domain or on a proper subset of it), we present new stabilization results, showing a polynomial decay rate for the total energy associated to system (2), related to the regularity of the initial condition of the system.

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Optimal Dirichlet boundary control for the Navier–Stokes equations

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(joint work with Olaf Steinbach)

Abstract: We consider an optimal Dirichlet boundary control problem for the Navier–Stokes equations. The control is considered in the energy space where the related norm is realized by the so called Steklov–Poincaré operator. We introduce a stabilized finite element method for the optimal control problem, for elements of lowest order. Further we present some numerical results which demonstrate the differences of a control in $L_2(\Gamma)$ and in the energy space $H^{1/2}(\Gamma)$, with an application to arterial blood flow.

Keywords: Optimal Dirichlet boundary control, energy space, Navier–Stokes equations, stabilized finite elements, arterial blood flow.

The Optimal control problem

Let $\Omega \subset \mathbb{R}^n$ $(n = 2, 3)$ be a bounded Lipschitz domain with boundary $\Gamma = \partial\Omega$. We consider the following optimal Dirichlet boundary control problem for the Navier–Stokes equations: Minimize the cost functional

$$\mathcal{J}(\underline{u}, \underline{z}) := \frac{1}{2} \|\underline{u} - \overline{\underline{u}}\|^2_{L_2(\Omega)} + \frac{1}{2} \varrho \, |\underline{z}|^2_{H^{1/2}(\Gamma_c)}$$

under the constraint

$$\begin{aligned} -\nu\Delta\underline{u} + (\underline{u} \cdot \nabla)\underline{u} + \nabla p &= \underline{f} && \text{in } \Omega, \\ \nabla \cdot \underline{u} &= 0 && \text{in } \Omega, \\ \underline{u} &= \underline{g} && \text{on } \Gamma_\mathrm{D}, \\ \nu(\nabla\underline{u})\underline{n} - p\underline{n} &= \underline{0} && \text{on } \Gamma_\mathrm{N}, \\ \underline{u} &= \underline{z} && \text{on } \Gamma_\mathrm{c}, \end{aligned}$$

where $\underline{u}$ and p are denoting velocity and pressure, respectively. The Dirichlet boundary control $\underline{z}$ is considered in the energy space $H^{1/2}(\Gamma)$, which is motivated by the trace theorem, applied to the standard weak formulation in $H^1(\Omega)$.
This approach was at first used for the Dirichlet control of the Poisson equation, see [4]. There it was shown that the control in the classical framework of $L_2(\Gamma)$ has several disadvantages in comparison to the framework of the energy space. The following extension for the Navier–Stokes equations was done in [2].

Control in $H^{1/2}(\Gamma)$

The application of the standard optimal control theory requires a suitable representation of the $H^{1/2}(\Gamma)$ semi–norm. This can be done by the so called Steklov–Poincaré operator S (Dirichlet to Neumann map), which is a bounded and semi–elliptic mapping

$$S : H^{1/2}(\Gamma) \to H^{-1/2}(\Gamma).$$

It is realized by a homogeneous Poisson problem, the details can be found in [2, 5].

Discretization and numerical results

We discretize the optimality system by a finite element method with Dohrmann–Bochev stabilization, see [1]. For such formulation we can use elements of lowest order, i.e. linear shape functions, which is suitable for large systems of otherwise high number of degrees of freedom.
The corresponding Galerkin matrix of the Steklov–Poincaré is given by the Schur complement system of the standard stiffness matrix, i.e.

$$S_h = A_{CC} - A_{CI} A_{II}^{-1} A_{IC},$$

and thus no further implementation of boundary integrals is required, the details can be found in [2, 4].
We compare the two control approaches, the classical $L_2(\Gamma)$ and the $H^{1/2}(\Gamma)$ formulation. For the former case we are able to prove, independent of the given data, that the control is always zero in each corner of the domain, for example when considering a rectangular domain. For the $H^{1/2}(\Gamma)$ control this behavior does not occur. Some numerical examples shall be presented, which confirm these theoretical results.
Additionally, we present numerical results for the errors of the control for both approaches. Here we observe better accuracy and order of convergence for the $H^{1/2}(\Gamma)$ case.

Applications to arterial blood flow

Here, in the last part, we consider an application to arterial blood flow. More precisely, the optimal control of the inflow to a bypass. The blood flow is described by the steady Navier–Stokes equations for Reynolds number $Re \approx 100$. For this application several problems like optimal vortex reduction and the minimization of wall shear stresses are discussed. Moreover, for these numerical results we present the differences of the control in $L_2(\Gamma)$ and $H^{1/2}(\Gamma)$.

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Inverse problem for a parabolic equation with periodic conditions

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Abstract: In this paper, we study the existence and uniqueness of the solution of a nonlinear parabolic equation with periodic conditions. Afterward, using an adapted Carleman inequality, we look at the stable reconstruction of the potential μ by partial measurements of the solution.

keywords: Inverse problem, Parabolic operator, Carleman inequality, periodic coefficients.

Introduction

In this work, we deal with the following nonlinear parabolic system $(E_{\mu,\nu,\gamma})$:

$$\begin{gathered} \partial_t u = \Delta u - \mu(x)u + \nu(x)u^2, \quad 0 < t < T, x \in \mathbb{R}^n, \\ u(0,x) = \gamma(x) \\ u\big|_{\Gamma_j^0} = u\big|_{\Gamma_j^1}, \quad \frac{\partial u}{\partial x_j}\Big|_{\Gamma_j^0} = \frac{\partial u}{\partial x_j}\Big|_{\Gamma_j^1}, \quad 1 \leq j \leq n \end{gathered} \tag{1}$$

We note the cell $\Omega = \Pi_{i=1}^n (0, L_i)$,
L_i is the period with regard to variable x_i.
$Q = (0,T) \times \Omega$, $\Sigma = (0,T) \times \partial\Omega$,
$\Gamma_j^0 = \partial\Omega \cap \{x_j = 0\}$, $\Gamma_j^1 = \partial\Omega \cap \{x_j = L_j\}$.
$\mu(x), \nu(x)$, and $\gamma(x)$ are periodic functions.
This problem intervenes in biology for example, it can model the population growth in a heterogeneous medium. The coefficient $\mu(x)$ corresponds to the intrinsic growth rate and usually it can not be directly measured. So its reconstruction via the density $u(t,x)$ is of great interest. This work generalizes in the nonlinear case the result obtained by Choi [1] in the linear case.

Theorem 1

In this part in addition, we suppose that $\mu, \nu \in L^\infty(\mathbb{R}^n)$, γ is bounded and continuous, $\gamma > 0$, $\nu \geq 0$ continuous. Then the previous problem admits a unique periodic solution.
For the proof, we adapt a result of [2],

Theorem 2

In this part we improved the result obtained by Choi by choosing an arbitrary open set ω, $\omega \subset \overline{\Omega}$, $\omega \neq \Omega$, $\theta \in (0,T)$ fixed. Then there exists a constant $C > 0$ such that:

$$||\mu - \tilde{\mu}||_{L^2(\Omega)} \leq C(||u - \tilde{u}||_{H^1(0,T;L^2(\omega))} +$$
$$||(u - \tilde{u})(\theta, .)||_{H^2(\Omega)})$$

where u (resp. $\tilde{u}$) is a solution of $(E_{\mu,\nu,\gamma})$ (resp. $(E_{\tilde{\mu},\nu,\gamma})$).

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Existence of a weak solution for a moving boundary fluid-structure interaction problem in blood flow

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Abstract: We study a moving boundary fluid-structure interaction problem arising in modeling blood flow through viscoelastic arteries and prove existence of a weak solution by using a novel approach based on a semi-discrete, operator splitting Lie scheme. The motivation for this proof comes from a numerical scheme, first introduced in 2009 ([3]), where a stable, loosely coupled scheme was proposed to solve the problem under consideration. We effectively prove convergence of that numerical scheme to a solution of the corresponding fluid-structure interaction problem.

Keywords: Fluid-structure interaction, Hemodynamics, Existence of a weak solution, Operator splitting, Semi-discretization

Introduction and statement of the result

This study is motivated by a fluid-structure interaction (FSI) problem in hemodynamics. We consider the flow of an incompressible, viscous flow through a two-dimensional axially symmetric pipe with deformable, thin walls. Let η denote the vertical displacement of the deformable boundary. Then the fluid domain at time t is given by

$$\Omega_\eta(t) = \{(x,z) : 0 < x < 1, 0 < z < 1 + \eta(x,t)\}$$

with deformable boundary $\Gamma(t) = \{(x, 1+\eta(x,t)) : 0 < x < 1\}, t \in [0,T)$. The fluid flow is governed by the incompressible Navier-Stokes equations:

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \sigma,\ \nabla \cdot \mathbf{u} = 0,\ \Omega_\eta(t),\ t \in (0,T),$$

where σ is the fluid stress tensor. The flow is driven by a prescribed dynamic pressure drop at the inlet and outlet boundaries: $p + \frac{1}{2}|u|^2 = P_{in/out}(t),\ \mathbf{u} \times \mathbf{n} = 0, \quad \text{on } \Gamma_{in/out}$. At the bottom boundary we prescribe the symmetry boundary conditions: $u_2(t,x,0) = \partial_z u_1(t,x,0) = 0, \quad x \in (0,1),\ t \in (0,T)$. The structure is modeled by a cylindrical linearly viscoelastic Koiter shell model (see [1]):

$$\varrho_s h \partial_{tt}^2 \eta + C_0 \eta - C_1 \partial_{xx}^2 \eta + C_2 \partial_{xxxx}^4 \eta + D_0 \partial_t \eta - D_1 \partial_{txx}^3 \eta + D_2 \partial_{txxxx}^5 \eta = f,$$

where f is the force applied to the structure, and constants $C_i,\ D_i \geq 0,\ \varrho_s, h > 0$. The fluid and structure are coupled through the kinematic and dynamic coupling conditions, respectively:

$$\mathbf{u}_{|\Gamma(t)}(t,.) = \partial_t \eta(t,.)\mathbf{e}_z, \quad f = -\sqrt{1+(\partial_x \eta)^2}\sigma \mathbf{n} \cdot \mathbf{e}_z, \text{ on } \Gamma(t),\ t \in (0,T).$$

The system is supplemented with initial conditions $\mathbf{u}(0,.) = \mathbf{u}_0,\ \eta(0,.) = \eta_0,\ \partial_x \eta(0,.) = v_0$.
The main result of this work is the proof of the existence of a weak solution (in a sense defined in [2]) of the problem under consideration. In contrast with the related works that already exist in literature, our work utilizes a physiologically reasonable structure model [1], and the pressure inlet and outlet boundary conditions which introduce some technical difficulties. However, the main novelty of this work is the approach used in proving the existence theorem. Our proof is based on a semi-discrete, operator splitting Lie scheme, which was used in [3] for a design of a stable, loosely coupled numerical scheme, called the kinematically coupled scheme. The main steps in the proof include the ALE weak formulation and Lie operator splitting, which splits the FSI problems into certain fluid and structure sub-problems. Then we discretize the sub-problems in time, and apply careful analysis of each substep to obtain suitable semi-discrete energy estimates. By using a compactness argument we show convergence to a weak solution. Thus, we effectively prove convergence of the kinematically coupled scheme to a weak solution of the FSI problem.

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Hautus test for the approximate controllability of linear parabolic systems

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Abstract: We introduce some generalization of the Hautus test to linear parabolic systems and give some applications to the distributed and boundary approximate controllability of such systems.

Keywords: Hautus test, parabolic systems, distributed controllability, boundary controllability.

In finite dimension we have a well-known condition to check whether or not a system is controllable, this is the so-called Hautus test:

Theorem. *Let* $A \in \mathcal{M}_n(\mathbb{R})$*,* $C \in \mathcal{M}_{n\times m}(\mathbb{R})$*,* $f \in L^2(0,T)^m$ *and* $T > 0$*.*
The O.D.E.

$$\left\{\begin{array}{rcl} \dfrac{d}{dt}y(t) & = & Ay(t) + Cf(t), \quad t \in (0,T). \\ y(0) & = & x \in \mathbb{R}^n. \end{array}\right.$$

is controllable if and only if

$$\operatorname{Ker}(sI - A^*) \cap \operatorname{Ker}(C^*) = \{0\}, \quad \forall s \in \mathbb{C}.$$

In this work we generalize this theorem to a class of linear parabolic systems in view of approximate controllability.
Through the Hautus test we are able to give a necessary and sufficient condition for several kind of parabolic systems, namely:

Example 1. [Boundary controllability in dimension $N \geq 1$]

$$\left\{\begin{array}{rcll} \partial_t y & = & (\Delta + A)y & \text{in } (0,T)\times\Omega, \\ y(t,\sigma) & = & 1_\gamma(\sigma)Bf(t,\sigma) & \text{on } (0,T)\times\partial\Omega, \end{array}\right.$$

where $A \in \mathcal{M}_n(\mathbb{R})$, $B \in \mathcal{M}_{n\times m}(\mathbb{R})$, $f \in L^2\left(0,T;L^2\left(\partial\Omega\right)^m\right)$ and $\gamma \subset \partial\Omega$.

Example 2. [Distributed controllability with a first order coupling term]

$$\left\{\begin{array}{rcll} \partial_t y & = & \begin{pmatrix} \Delta & 0 \\ A & \Delta \end{pmatrix} y + 1_\omega(x)Bf(t,x) & \text{in } (0,T)\times\Omega, \\ y(t,\sigma) & = & 0 & \text{on } (0,T)\times\partial\Omega, \end{array}\right.$$

where $A = G(x) \cdot \nabla + a(x)$ with $a \in L^\infty(\Omega)$, $G \in W^{1,\infty}(\Omega)^N$, $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $f \in L^2\left(0,T;L^2\left(\Omega\right)\right)$, $\omega \subset \Omega$.

Example 3. [Controllability with different diffusion coefficients]

$$\left\{\begin{array}{rcll} \partial_t y &=& \begin{pmatrix} \Delta & 0 \\ a & \nu\Delta \end{pmatrix} y & \text{in } (0,T)\times\Omega, \\ y(t,\sigma) &=& 1_\gamma(\sigma) B f(t,\sigma) & \text{on } (0,T)\times\partial\Omega, \end{array}\right.$$

where $\nu > 0$, $a \in \mathbb{R}$, $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $f \in L^2\left(0,T;L^2\left(\partial\Omega\right)\right)$ and $\gamma \subset \partial\Omega$.

Performance/Robustness trade off for stabilizing a flow around a cylinder

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We study the performance/robustness trade off in some finite dimensional observer-based structure controllers used to stabilize the two-dimensional linearized Navier-Stokes equations around an unstable stationary solution by a boundary control. For this type of systems, we have already defined finite dimensional observers coupled with finite dimensional control laws.
In this work, taking advantage of the finite dimensional character of our observer, we adapt tools from finite dimensional systems to do the best performance/robustness trade off for the stabilization of a flow around a cylinder.
As in [1], Kalman filter gain and state gain are determined by solving two finite dimension Algebraic Riccati Equations involving the projection of the linearized equations onto the unstable subspace of the linearized operator.
This Linear Quadratic Gaussian controller is the first observer-based structure controller; It is of high-performance (in $\mathcal{H}_2$ norm sense) but not necessarily robust (in $\mathcal{H}_\infty$ norm sense) against the perturbation in the actuator dynamics [2].
To increase the robustness, we perform a generalization of Loop Transfer Recovery procedure [3, 4] via a nonlinear and nonconvex optimization program, which results in the second observer-based structure controller. However, more this second controller is robust, lower is its performance.
Finally, an observer-based structure [4, 5] $\mathcal{H}_2/\mathcal{H}_\infty$ controller is proposed and computed once again via nonlinear and nonconvex optimization. It is of high-performance and at the same time it is robust.

The approach is tested numerically for stabilization of two-dimensional linearized Navier-Stokes equations by a boundary control and using boundary observations of velocity in the case of a flow around a circular cylinder.

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On the identifiability of a rigid body moving in a stationary viscous fluid

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Abstract: I present a work that I carried out with the collaboration of Carlos Conca (Universidad de Chile) and Takéo Takahashi (Université de Lorraine) during my PhD. This work is devoted to a geometrical inverse problem associated to a fluid–structure system. More precisely, we consider the interaction between a moving rigid body and a viscous and incompressible fluid. Assuming a low Reynolds regime, the inertial forces can be neglected and therefore, the fluid motion is modelled by the Stokes system. We first prove the well-posedness of the corresponding system. Then we show an identifiability result: with one measure of the Cauchy forces of the fluid on one given part of the boundary and at some positive time, the shape of a convex body and its initial position is identified.

Keywords: Fluid–structure interaction, Navier-Stokes equations, geometric inverse problems, rigid body dynamics.

In this work, we are interested in identifying an inaccessible solid structure, denoted by $\mathcal{S}(t)$, which is moving in a viscous incompressible fluid occupying a region denoted by $\mathcal{F}(t)$. We assume that both the fluid and the structure are contained in a bounded fixed domain (i.e. connected and open set) Ω of $\mathbb{R}^3$ so that $\mathcal{F}(t) = \Omega \setminus \overline{\mathcal{S}(t)}$.
We assume that the structure is a rigid body so that it can be described by its center of mass $\boldsymbol{a}(t) \in \mathbb{R}^3$ and by its orientation $\boldsymbol{Q}(t) \in SO_3(\mathbb{R})$ as follows:

$$\mathcal{S}(t) := \mathcal{S}(\boldsymbol{a}(t), \boldsymbol{Q}(t)),$$

with

$$\mathcal{S}(\boldsymbol{a}, \boldsymbol{Q}) := \boldsymbol{Q}\mathcal{S}_0 + \boldsymbol{a}, \quad (\boldsymbol{a}, \boldsymbol{Q}) \in \mathbb{R}^3 \times SO_3(\mathbb{R}).$$

where $\mathcal{S}_0$ is a smooth non empty domain which is given.
Now, let us take two smooth non empty domains $\mathcal{S}_0^{(1)}, \mathcal{S}_0^{(2)}$. Let us also consider $\left(\boldsymbol{a}_0^{(1)}, \boldsymbol{Q}_0^{(1)}\right)$, $\left(\boldsymbol{a}_0^{(2)}, \boldsymbol{Q}_0^{(2)}\right) \in \mathbb{R}^3 \times SO_3(\mathbb{R})$ such that

$$\overline{\mathcal{S}^{(1)}\left(\boldsymbol{a}_0^{(1)}, \boldsymbol{Q}_0^{(1)}\right)} \subset \Omega \quad \text{and} \quad \overline{\mathcal{S}^{(2)}\left(\boldsymbol{a}_0^{(2)}, \boldsymbol{Q}_0^{(2)}\right)} \subset \Omega.$$

Thanks to the well-posedness of the corresponding systems, we know there exist $T_*^{(1)} > 0$ $\left(\text{respectively } T_*^{(2)} > 0\right)$ for which there exists a unique solution of the corresponding system.

Then we have the main result:

Theorem. *Suppose that the fluid velocity on the $\partial\Omega$ is equal to a known function $\boldsymbol{u}_*$ such that $\boldsymbol{u}_*$ is not the trace of a rigid velocity on $\Gamma \subset \partial\Omega$. Assume also that $\mathcal{S}_0^{(1)}$, $\mathcal{S}_0^{(2)}$ are convex sets. If there exists $0 < t_0 < \min\left(T_*^{(1)}, T_*^{(2)}\right)$ such that*

$$\boldsymbol{\sigma}\left(\boldsymbol{u}^{(1)}(t_0), p^{(1)}(t_0)\right)\ \boldsymbol{n}_{|\Gamma} = \boldsymbol{\sigma}\left(\boldsymbol{u}^{(2)}(t_0), p^{(2)}(t_0)\right)\ \boldsymbol{n}_{|\Gamma}$$

then there exists $\boldsymbol{R} \in SO_3(\mathbb{R})$ such that

$$\boldsymbol{R}\mathcal{S}_0^{(1)} = \mathcal{S}_0^{(2)}$$

and

$$\boldsymbol{a}_0^{(1)} = \boldsymbol{a}_0^{(2)}, \qquad \boldsymbol{Q}_0^{(1)} = \boldsymbol{Q}_0^{(2)}\boldsymbol{R}.$$

In particular, $T_*^{(1)} = T_*^{(2)}$ and

$$\mathcal{S}^{(1)}(t) = \mathcal{S}^{(2)}(t) \qquad \left(t \in \left[0, T_*^{(1)}\right)\right).$$

Here, $\left(\boldsymbol{u}^{(i)}, p^{(i)}\right)$, are the velocity and the pressure of the fluid, whereas $\boldsymbol{\sigma}\left(\boldsymbol{u}^{(i)}, p^{(i)}\right)$ denotes the corresponding Cauchy stress tensor $(i = 1, 2)$.

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Coupling estimation and control for a two dimensional Burgers type equation – Numerical experiments –

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(Joint work with Jean-Marie Buchot and Jean-Pierre Raymond)

Abstract: We consider the problem of stabilizing a two dimensional Burgers equation, when only partial observations are available. We use a Galerkin approximation and we solve two Riccati equations to define the finite dimensional differential system that we should solve. The method defined for the linear system succeeds to stabilize the nonlinear problem when noisy measurements are considered.

Keywords: Feedback control, estimation, finite element method, Burgers equation.

Our aim is to stabilize the solution z of the following problem

$$\begin{cases} \dfrac{\partial z}{\partial t} - \nu\Delta z + (\partial_1 w_s + \partial_2 w_s)z + (\partial_1 z + \partial_2 z)w_s + (\partial_1 z + \partial_2 z)z = 0 \quad \text{in } \Omega \times (0,\infty), \\ \nu\dfrac{\partial z}{\partial n} = 0 \quad \text{on } \Gamma_n \times (0,\infty), \\ z = Mu \quad \text{on } \Gamma_d \times (0,\infty), \\ z(0) = w_0 - w_s = z_0 \quad \text{in } \Omega \end{cases}$$

when we observe only the solution one Γ_n.
For this purpose we consider the corresponding FEM type semi-discretization

$$\begin{aligned} &E\dot{z}^N = Az^N + Bu^N + F(z^N) + E\zeta^N, \quad z^N(0) = z_0^N, \\ &y_{obs}^N = Hz^N, \end{aligned}$$

and look for a feedback law of the type $u^N = Kz_e^N$ where z_e^N is the solution of the estimator system

$$E\dot{z}_e^N = Az_e^N + L(Hz_e^N - y_{obs}^n) + BKz_e^N, \quad z_e^N(0) = z_{e,0}^N.$$

The finite dimensional approximation of the stabilization problem in the frame of linear partial differential equations as been proposed by several authors. Just to mention some, see for instance [3], [4], [1] or [5]. As to the nonlinear Burquers type equation, numerical simulations for the stabilization problem with full measurments were done in [6] within a frame similar to ours. Considering the coupling with the estimator system, the numerical approximation was treated in [2], but for the one dimensional case. We present here numerical simulations for the two dimensional problem, assuming that the observations can be perturbed by some noise with known covariance.

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Best location of actuators for the stabilization of the Navier-Stokes equations

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Abstract: We consider the stabilization by a boundary linear feedback law of the Navier-Stokes equations around an unstable steady solution. We first build a low-dimensional model in two steps for computing stabilizing controls for the linear Navier-Stokes equations. We then improve the approach by establishing optimal control zones that depend on the Reynolds number. The method is tested on the non-linear model for different Reynolds numbers, the flow being destabilized by imposing a perturbation at the inflow. The flow is stabilized at $Re = 80$ while instabilities are considerably reduced for $Re = 150$.

Keywords: Incompressible Navier-Stokes, Feedback control, Best actuator location.

The aim of this work is to determine a boundary linear feedback control which is able to stabilize the incompressible non-stationary Navier-Stokes equations around a steady solution. This problem has received great interest during the last decades due to its practical application in fluid mechanics [2].

Flow setup and main features

We apply our method to the case of flow past a circular cylinder confined in a channel. Visualizations of flow vorticity show that, starting with a steady solution of the Navier-Stokes equations, perturbations to the inflow condition induce vortex shedding in the cylinder wake. Instability is also apparent in the spectrum of the linearized operator which exhibits two unstable eigenvalues.

Formulation of the control problem

Control is performed using blowing/suction actuators placed on the cylinder boundary. The control law is determined for the model linearized around the steady solution and next applied to the nonlinear model. Two difficulties arise: the differential-algebraic nature of the Navier-Stokes equations and the large scale of the discretized problem. We first show that using a projection onto a divergence-free function space as described in [5] we can formulate the problem as a classical linear control problem for which a feedback law can be

determined by solving an Algebraic Riccati Equation. We then introduce the method used in [1, 6] for reducing the problem dimension when there are only a few unstable modes.
Results: The actuation law we calculate suppresses, or at least reduces (depending on the Reynolds number), the effects of the perturbations. The effect of the control is measured by examining the time evolution of the control functional, but also the behavior of the lift and drag coefficients.

Optimal location of actuators

Many authors show the importance of the control zone (see for example [3, 4]). One can hope that improving the actuation law by optimizing this parameter could be a way to improve the domain of validity of the above model. For a given number of actuators of fixed size, we seek the control zone providing a control of minimal norm.
Results: For our example, placing the actuators according to our optimality criterion, brings an improvement in the sense that the control is less costly while still stabilizing the flow, although there is no gain is stabilization time.

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