Influence of boundary on the motility of micro-swimmers

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Joint work with François Alouges and David Gérard-Varet

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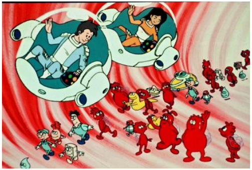
Motivations

- Self-propulsion at micro-scales?
- Many applications are concerned, on fertility, on human diagnosis and therapy...

- Physicians and biologists noticed that the wall attract micro-swimmers: Berke and P. Allison. (2008) - J.R Blake. et al (1971, 2009,2010), H. Winet et al.,(1984), R. Zargar, A. Najafi, and M. Miri. (2009),..
- What is the influence of the presence of the wall on the controllability of such micro-swimmers?



Contents

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- Influence of a plane wall - Joint work with F. Alouges
- Influence of a rough wall - Work in Progress with D. Gérard-Varet

Model swimmer/fluid

The swimmer is described by the vector (ξ, p) such as :

- ξ is a function which defines the shape of the swimmer.
- $p = (c, R) \in \mathbb{R}^3 \times SO(3)$ parametrizes the swimmer's position.

The swimmer changes its shape $\Longrightarrow \xi(t)$ pushes the fluid.
The fluid reacts, under the Stokes Equation

$$\left[\begin{array}{l} -\nu\Delta u + \nabla p = f, \\ \mathrm{div} u = 0. \end{array}\right.$$

Self-propulsion constraints $\Longrightarrow \left\{\begin{array}{ll} \sum \text{Forces} & = 0 \\ \text{Torque} & = 0 \end{array}\right.$

$$\Longleftrightarrow \left\{\begin{array}{l} \int_{\partial\Omega} DN_{p,\xi}\left((\partial_p\Phi)\dot{p} + (\partial_\xi\Phi)\dot{\xi}\right) dx_0 = 0 \\ \int_{\partial\Omega} x_0 \times DN_{p,\xi}\left((\partial_p\Phi)\dot{p} + (\partial_\xi\Phi)\dot{\xi}\right) dx_0 = 0. \end{array}\right.$$

As a result the swimmer moves, under the ODE

$$\dot{p} = V(p, \xi)\dot{\xi}\,.$$

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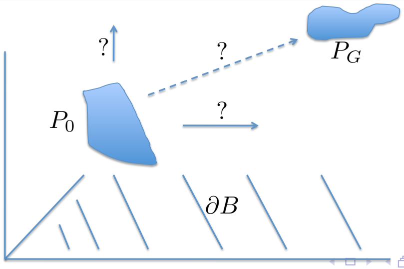
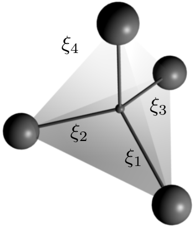
Questions

- Is it possible to control the state of the system (ξ and p) using as controls only the rate of shape changes $\frac{d}{dt}\xi$?
- Does the boundary have an effect on the controllability of the swimmer?

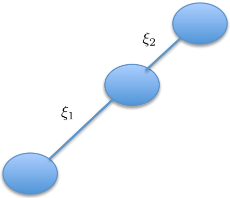


The swimmers

The swimmer that we consider consists of 3 or 4 spheres connected by a thin jacks.
The change of the swimmer's shape consists in changing the length of its arms $(\xi_i)_i$.



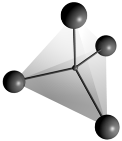
Four sphere swimmer



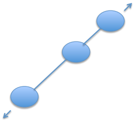
Three sphere swimmer [Golestanian, Najafi 2004]

Example of stroke

Controllability's result in $\mathbb{R}^3$ [Alouges, DeSimone, Heltai, Lefevbre, Merlet (Preprint)]

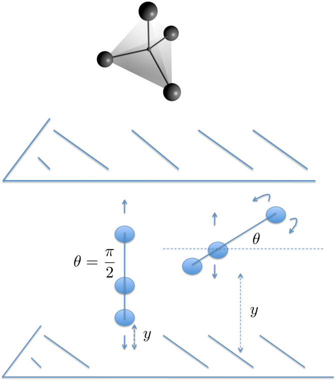
- 

  The 4-sphere swimmer is globally controllable on $\mathbb{R}^3$.

- 

  The 3-sphere swimmer is globally controllable on $\mathbb{R}$.

- Does a confined environnement modify the swimmer's reachable set?



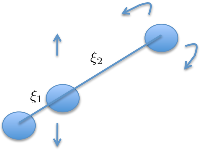
The 4-sphere swimmer is controllable on an dense open set.

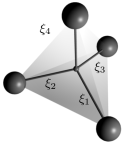
- For almost (x_0, y_0, θ_0), such that $\theta_0 \neq \frac{\pi}{2}$, the 3-sphere swimmer is locally controllable on (x_0, y_0, θ_0).
- If $\theta_0 = \frac{\pi}{2}$ then it moves along a vertical line.

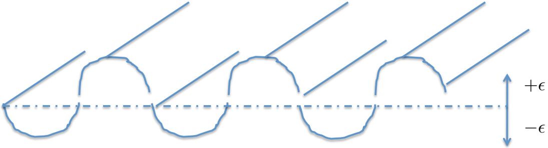
Influence of a rough no slip wall - Work in Progress with D. Gérard-Varet

The rough wall is defined by $z = \epsilon h(x, y)$, $\|h\| = 1$.

- The 4-sphere remain controllable on an dense open set.
- The dimension of the reachable set of the 3-sphere swimmer is greater than or equal to 5.

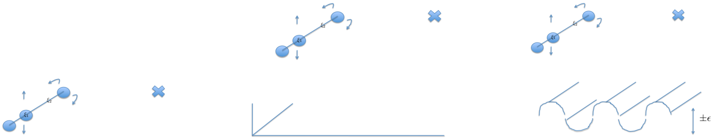






Conclusion and outlook

- Do The 3-sphere be more controllable?
- Influence on the optimal trajectories



Thank you for your attention