Shape optimization problems for elliptic operators with drift

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In this talk we will present some very recent advances in the field of shape optimization for operators with drift. We consider the model problem

$$\min\Big\{\lambda_1(\Omega,V)\ :\ \Omega\subset D,\ V:D\to\mathbb{R}^d,\ \|V\|_{L^\infty}\le\tau,\ |\Omega|\le m\Big\},$$

where $m,\tau>0$ and the bounded open domain D are given and $\lambda_1(\Omega,V)$ denotes the first eigenvalue of the operator $-\Delta+V\cdot\nabla$ with Dirichlet Boundary conditions in Ω. We show that an optimal domain Ω and optimal drift V do exist in the class of *quasi-open* sets. Moreover, if we restrict our attention to the class of vector fields V such that $V=\nabla\Phi$, for some τ-Lipschitz continuous function $\Phi:D\to\mathbb{R}$, we also prove that the optimal sets have $C^{1,\alpha}$ smooth free boundary.
We notice that in the case $D=\mathbb{R}^d$, it was proved by Hamel, Nadirashvili and Russ [3] that the optimal domain is a ball and the optimal vector field is $V(x)=\tau x/|x|$.
The operator $-\Delta+V\cdot\nabla$ is not a self-adjoint operator, so the definition itself of the first eigenvalue requires special attention. For an open set Ω, it was proved by Beresticky, Nirenberg and Varadhan [1] that there exists a real eigenvalue $\lambda_1(\Omega,V)$ of $-\Delta+V\cdot\nabla$ such that $\lambda_1(\Omega,V)\le Re\,\lambda$ for every other eigenvalue λ of the same operator. In order to prove our existence result, we extend this theorem to the case of quasi-open sets and, we use the theory of Buttazzo and Dal Maso to prove an existence of an optimal quasi-open set.

References

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