Selected topics in statistics
Spatial Statistics
Mid-term exam

Lecturer: François Bachoc, PhD

Specifications

- No documents, no calculators.
- You can use the results of the lectures without reproving them.
- Tentative notation: 7/5/8.

Exercise 1

Let $(X_i)_{i \in \mathbb{Z}}$ be a sequence of *iid* random variables, with mean μ and variance σ^2. For $i \in \mathbb{Z}$, let

$$Z(i) = \frac{1}{i^2 + 1}(X_{i-1} + i^2 X_i).$$

a) Show that Z is a stochastic process on $\mathcal{D} = \mathbb{Z}$.
b) Calculate the mean and covariance function of Z.
c) Is Z stationary? Prove your answer.

Exercise 2

Let Y be a stationary stochastic process on $\mathcal{D} = \mathbb{R}$, so that the sequence of random variables $(Y(n))_{n \in \mathbb{N}}$ goes to zero in probability when $n \to +\infty$. Show that, for any $x \in \mathbb{R}$, $Y(x) = 0$, almost-surely.

Exercise 3

Let Y be a stochastic process on $\mathcal{D} = [0, 1]$, with mean function 0 and covariance function $K(x, y) = xy$. Let, for $n \in \mathbb{N}^*$ and $x \in [0, 1]$,

$$I_n(x) = \frac{1}{n} \sum_{i=1,\dots,n; \frac{i}{n} \leq x} Y(\frac{i}{n}).$$

We admit that there exists a stochastic process I on $[0, 1]$ so that, for any $x \in [0, 1]$, $I_n(x)$ converges to $I(x)$ in the mean square sense. We also admit that, for any $x, y \in [0, 1]$, $\mathbb{E}(I(x)) = lim_{n \to +\infty} \mathbb{E}(I_n(x))$ and $cov(I(x), I(y)) = lim_{n \to +\infty} cov(I_n(x), I_n(y))$.

a) Calculate the covariance function of I.

b) Prove that I is mean square differentiable on $[0, 1]$. Calculate the covariance function of the stochastic process $\frac{\partial}{\partial x} I(x)$, on $\mathcal{D} = [0, 1]$.