Selected topics in statistics
Spatial Statistics
Homework 6

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Suggested programming language: R. Suggestion: re-use the code of HW5 whenever it is convenient. You do not have to hand back this homework which is not marked.

We model the deterministic function $f : [0, 1] \to \mathbf{R}$, with $f(x) = \cos(4x) + x + x^2 - \exp(x)$, as the realization of a Gaussian process Y, with zero mean function, and Matérn $\frac{3}{2}$ covariance function, with variance parameter σ^2 and correlation-length parameter ℓ.

1)

Create a R function with

- Inputs
  - A vector x_{obs} of size n
  - A vector y_{obs} of size n
  - $\ell > 0$
- Output
  - The value of the profile likelihood function for the Matérn $\frac{3}{2}$ covariance function, evaluated at ℓ, when x_{obs} is the vector gathering the n observation points and y_{obs} is the corresponding vector of observed values.

Suggestion: same structure as for the functions implemented in HW5.

2)

Let $x_{obs} = (0, 0.3, 0.6, 1)$ and $y_{obs} = (f(0), f(0.3), f(0.6), f(1))$. Plot the 1000 values of the profile likelihood function, where ℓ takes 1000 values that are equally spaced between 0.1 and 2.

Suggestion: make a loop to compute these 1000 values. Use the same plot command as in Hw5 4). Re-use your function f from HW5.

3)

Find the correlation length l_0 that minimizes the profile likelihood function. Suggested R commands: *which.min*.

4)

Create a R function with

- Inputs
  - A vector x_{obs} of size n
  - A vector y_{obs} of size n
  - $\ell > 0$
- Output

- The estimation of the variance parameter σ^2, given the correlation length ℓ, when x_{obs} is the vector gathering the n observation points and y_{obs} is the corresponding vector of observed values.

Suggestion: same structure as for the functions implemented in HW5.

5)

Evaluate the function of 4) with $x_{obs} = (0, 0.3, 0.6, 1)$, $y_{obs} = (f(0), f(0.3), f(0.6), f(1))$ and $\ell = \ell_0$ where ℓ_0 is the result of 3). We call σ_0^2 this result

5)

Redo the questions 4), 5), 8), 9), 12) of HW5, with $x_{obs} = (0, 0.3, 0.6, 1)$, $y_{obs} = (f(0), f(0.3), f(0.6), f(1))$, $\ell = \ell_0$ and $\sigma^2 = \sigma_0^2$.