Selected topics in statistics
Spatial Statistics
Homework 4

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Exercice 1

Are the following functions, defined on $\mathcal{D}$, symmetric non-negative definite (SNND)? Prove your answer.

a) $\mathcal{D} = \mathbb{R}$ and $K(x, y) = |x - y|$.

b) $\mathcal{D} = (0, 1)$ and $K(x, y) = \frac{1}{1-xy}$. Hint: $\frac{1}{1-xy} = \sum_{k=0}^{+\infty} (xy)^k$.

b) $\mathcal{D} = \mathbb{R}^+$ and $K(x, y) = min(x, y)$. Hint: you can associate, to each $x \in \mathbb{R}^+$, the function $f_x : \mathbb{R}^+ \to \mathbb{R}$, defined by $f_x(t) = \mathbf{1}_{t \leq x}$. Then, you can use the fact that $(f|g) = \int_{\mathbb{R}^+} f(t)g(t)dt$ is a scalar product on the space of functions from $\mathbb{R}^+$ to $\mathbb{R}$ (where a zero function is, by convention a function that is almost surely zero on $\mathbb{R}^+$). Finally, you can use the fact that $min(x, y) = (f_x|f_y)$.

Exercice 2

Let n, $\sigma^2 > 0$, $y_1, ..., y_n \in \mathbb{R}$ and $x_1, ..., x_n, x_{new} \in \mathbb{R}$ be fixed and assume that $x_1, ..., x_n, x_{new}$ are two by two distinct. Consider a Gaussian process Y on $\mathbb{R}$, with mean function 0 and with Matérn $\frac{3}{2}$ covariance function $K_{\sigma^2,\ell}$, with σ^2 fixed as above and varying $\ell > 0$. Consider $n + 1$ different observation points $x_1, ..., x_n, x_{new}$, fixed as above. For each ℓ, we have by the prediction formula of the lecture that, conditionally to $Y(x_1) = y_1, ..., Y(x_n) = y_n$, $Y(x_{new})$ is Gaussian with mean m_ℓ and variance σ_ℓ^2. Show that

$$m_\ell \to_{\substack{\ell \to 0 \\ \ell > 0}} 0$$

and

$$\sigma_\ell^2 \to_{\substack{\ell \to 0 \\ \ell > 0}} \sigma^2.$$