Workshop on Topology and Languages

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Institut de Mathématiques de Toulouse

LIST OF ABSTRACTS

Abstracts

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The (poly)topologies in provability logic

The Gödel-Löb logic GL is a modal logic which is sound and complete for its arithmetical interpretation, where the modal [ ] represents Gödel's provability predicate. It also enjoys both relational and topological semantics, the latter based on Cantor's scattered spaces. There are various completeness results for this interpretation due to Esakia, Abashidze and Blass.

Meanwhile, Japaridze's polymodal logic GLP is an extension of GL which has one modality [n] for each natural number. These operators also enjoy a natural proof-theoretic interpretation using a notion of n-provability, but in contrast to GL, GLP is incomplete for its relational semantics. Fortunately, it is complete for its topological interpretation, using polytopological spaces with an increasing chain of scattered topologies, as was shown by Beklemishev and Gabelaia. The proof of this is non-constructive and required novel techniques in the study of scattered spaces. Moreover, as the speaker has shown, this result also holds for Beklemishev's transfinite extension of GLP.

In this presentation, we will introduce the logics GL and GLP along with their intended proof-theoretic interpretation. We will discuss their relational and topological semantics, and sketch the main ideas behind the completeness proofs for the latter.

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Infinite computations, logic, and topological complexity

Infinite words and infinite trees are standard models of executions for reactive systems. Therefore, it is natural to seek for decidable formalisms allowing to express properties of such objects. The classical results of Buchi and Rabin from the 60's provide the landmark of this field - Monadic Second-Order logic (MSO) is decidable over both classes of structures. Since then, the study of decidable logics over such infinite structures has been extensively developed. An important type of problems in the area is to effectively characterise which properties expressible in a stronger formalism can also be specified in a weaker one. This line of research turned out to be very fruitful, just to mention the celebrated result of Schutzenberger, McNaughton, and Papert that effectively characterises the expressive power of First-Order logic (FO) among the MSO-definable sets of infinite words. Nonetheless, many other questions about characterisations remain open.

During my talk I will argue that the classical notions of topological complexity are natural tools to provide an insight into effective characterisations for the formalisms described above. The entry point to such applications is the fact that spaces of infinite words and infinite trees are topologically equivalent to the Cantor set. Therefore, all the methods of descriptive set theory apply. In particular, for each set of words or trees, we can ask which level of the Borel or the projective hierarchy (jointly known as the boldface hierarchies) it inhabits. It turns out that the topological complexity of a set is strongly related to the strength of a formalism needed to define it. Therefore, the boldface hierarchies provide an easy way to stratify the formalisms with respect to their expressive power. Additionally, it is possible to obtain effective results that invoke, in the proof of correctness, purely topological arguments.

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The word problem in $\mathbb{Z}^2$ and formal language theory

This talk, will present a proof that the word problem for the group $\mathbb{Z}^2$ can be solved by a natural extension of context-free languages : Multiple Context-Free Languages. This problem is connected with the MIX conjecture coming from computational linguistics. After I have given an overview of these problems, I will present a proof of the result that crucially relies on arguments from algebraic topology. It is now conjectured that this result should extend for the groups $\mathbb{Z}^k$, but there is no clue in the proof I will present about how to proceed.

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A gentle introduction to the profinite theory of rational languages

Profinite theory is a topological approach in the study of rational languages. In this talk, I will introduce the profinite monoid over a finite alphabet using a distance on words. I will explain why this object is suitable to obtain characterisations of some classes of rational languages, leading sometimes to the decidability of the membership of these classes. To this aim, I will introduce the notion of equations of profinite words and illustrate it by a recent work with Charles Paperman, which gives a characterisation of four classes of rational languages (lattice, Boolean algebra, closure under quotient) generated by the languages of the form u*, where u is a word.

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Distributive law and Koszulity

Koszulity is a topological notion defined on quadratic algebras and operads. To such an algebra or operad, one can associate a complex called its Koszul complex. An algebra is then Koszul if and only if the associated Koszul complex is acyclic. This notion implies nice topological properties on the associated algebra/operad and can be proven under some assumptions thanks to the confluence of some rewriting systems on the algebra or on the operad. The notion of distributive laws for operads was introduced by Beck in 1969 : it encoded the relations between operations given by different operads. A simple example of such a law is the one giving the distributivity of multiplication over addition in the field of complex numbers. Mixed distributive law can also be reformulated in terms of rewriting theories. After recalling the different notions needed, we will explain the connection between distributive laws and koszulity, shown by Markl in 1994, and open this problem to new subjects of research.

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The Garside groups and some of their properties

Garside groups have been first introduced by P.Dehornoy and L.Paris in 1990. In many aspects, Garside groups extend braid groups and more generally finite-type Artin groups. These are torsion-free groups with a word and conjugacy problems solvable, and they are groups of fractions of monoids with a structure of lattice with respect to left and right divisibilities. It is natural to ask if there are additional properties Garside groups share in common with the intensively investigated braid groups and finite-type Artin groups. In this talk, I will introduce the Garside groups in general, and a particular class of Garside groups, that arise from certain solutions of the Quantum Yang-Baxter equation. I will describe the connection between these theories arising from different domains of research, present some of the questions raised for the Garside groups and give some partial answers to these questions.

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Dismantling in graphs and a relation with evasiveness conjecture for simplicial complexes

A boolean function is evasive if its decision tree complexity is maximal ; this notion has been investigated for studying monotone graph properties. In 1984, Kahn, Saks and Sturtevant, by converting this question in terms of simplicial complexes, have obtained some results following a topological approach. In this talk, we will recall these origins and we will introduce the notions of k-dismantling in graphs in order to study the evasiveness conjecture in the case of flag simplicial complexes (which claims that "if K is a non evasive vertex-homogeneous simplicial complex, then K is a simplex").

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Handsome Proof-Nets for Cyclic Linear Logic
(Joint work with Christian Retoré)

Proof-nets appeared together with linear logic. They identify proofs whose behaviors are the same, typically up to rule permutations. But it is also possible to consider proofs up to the algebraic properties of the logical connectives : associativity and commutativity.

Lambek calculus, that was proposed to model the combinatory possibilities of the syntax of natural languages (Lambek 1958), can be seen as a fragment of non-commutative (cyclic) linear logic.

For commutative linear logic, Retoré (2003) introduced proof-nets that use the bijective correspondence between formulas, up to associativity and commutativity, and a well structured class of plain graphs, namely cographs. The latter replace labeled trees and have colored edges. Axioms are represented by non-adjacent edges of a different color. Among these bicolored graphs, proofs are characterized by the absence of alternate elementary cycles without chords and by the connectivity with elementary paths.

We propose a similar characterization of proof-nets for non-commutative linear logic (cyclic linear logic and Lambek calculus). The criterion consists in the commutative condition plus a bracketing condition along an Hamiltonian cycle which is provided by the structure of the graph. We also discuss extension of this criterion to cuts.

Ivano Ciardelli and Christian Retoré

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A DIRECT PROOF OF THAT INTUITIONISTIC PREDICATE CALCULUS IS COMPLETE WRT PRESHEAVES OF CLASSICAL MODELS

Presheaf models [Osius, 1975, Fourman and Scott, 1979, Moerdijk and Mac Lane, 1992] are a simple generalization of Kripke models and provide one of the most natural semantics for intuitionistic predicate logic. Although a completeness result for this semantics is known, this is usually obtained either non-constructively, by defining a canonical Kripke model, or indirectly, e.g. via categorical equivalences with omega-models as in [Troelstra and Van Dalen, 1988]. This result (due to the first author and presented à Topology, Algebra and Categories in Logic 2011) describes an elementary canonical (separated) pre sheaf model construction by means of which completeness is established in a perspicuous, direct and constructive way. Next we shall discuss wether the same construction is possible with sheaves.
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OPERADS AND REGULAR LANGUAGES

The notion of operad appeared in the 70's to describe loop spaces. Actually, each operad encodes a certain type of algebraic structure and this notions is nowadays used in several fields beyond topology and algebra. In this talk, we give several definitions related to operads and we propose motivations and applications in topology and in the study of regular languages.

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THE GENUS OF REGULAR LANGUAGES AND OTHER IDEAS FROM LOW-DIMENSIONAL TOPOLOGY
(joint work with Guillaume Bonfante)

A regular language is the simplest class of languages in Chomksy's classification : it is the language computed by some finite deterministic automaton (DFA). Forgetting (part of the) structure, a DFA is a digraph. The genus of a regular language L is the minimum orientable genus of all DFAs computing L. We shall make the case why this number is useful by comparing genus and size, showing that it provides a hierarchy of languages and relating it to graph theory via directed emulators. A key question is that of the calculability of the genus. We shall prove the conjecture for a fairly generic class of languages and speculate about the remaining cases.

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Directed minors for minimal automata

I will present a graph-theoretic approach of the following decidability problem called C-recognizability : given a regular language L and a class of (directed or undirected) graphs C, is there a deterministic automaton for L whose underlying graph is in C ? I will present a notion of directed minor that is adapted to treat this problem, by allowing to look solely at the minimal automaton of L. It is open whether this directed minor relation is well-ordered, leaving unresolved the decidability of C-recognisability, even if C is a particular class such as planar graphs. In this particular case, I will show a connection between this problem and undirected graph-theoretic conjectures related to planar emulation.

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Diagrammatic Quantum Reasoning : Completeness and Incompleteness

The ZX-calculus introduced by Coecke and Duncan is a graphical formal language for quantum reasoning based on the complementarity of observables. I ?ll introduce this category-based diagrammatic calculus, give some examples, and focus on the question of completeness of the language for quantum mechanics. The language is complete for a fragment of quantum mechanics, the so-called stabiliser quantum mechanics, a fragment with is not universal for quantum computing. The completeness for the universal "Clifford+T" fragment has been conjectured. We show that supplementarity, a property of interacting observables pointed out by Coecke and Edwards, cannot be derived in the ZX-calculus, implying in particular that the ZX-calculus is not complete for the "Clifford+T" fragment.