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Short Communications

On Polynomial Projectors That Preserve Homogeneous Partial Differential Equations

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1. Let $H(\mathbb{C}^n)$ be the space of entire functions on $\mathbb{C}^n$ equipped with its usual compact convergence topology, and $\mathcal{P}_d(\mathbb{C}^n)$ the space of polynomials on $\mathbb{C}^n$ of total degree at most d. A *polynomial projector* of degree d is a continuous linear map Π from $H(\mathbb{C}^n)$ to $\mathcal{P}_d(\mathbb{C}^n)$ for which $\Pi(p) = p$ for every $p \in \mathcal{P}_d(\mathbb{C}^n)$. Such a projector Π is said to *preserve homogeneous partial differential equations (HPDE)* of degree k if for every $f \in H(\mathbb{C}^n)$ and every homogeneous polynomial of degree k,

$$q(z) = \sum_{|\alpha|=k} a_\alpha z^\alpha,$$

we have

$$q(D)f = 0 \Rightarrow q(D)\Pi(f) = 0, \tag{1}$$

where as usual

$$q(D) := \sum_{|\alpha|=k} a_\alpha D^\alpha,$$

$D^\alpha = \partial^{|\alpha|}/\partial z_1^{\alpha_1} \dots \partial z_n^{\alpha_n}$, and $|\alpha| = \sum_{j=1}^n \alpha_j$ denotes the length of the multi-index $\alpha = (\alpha_1, \dots, \alpha_n)$.

In [3] Calvi and Filipsson gave a precise description of the polynomial projectors preserving all HPDE. In particular they show that a polynomial projector preserves all HPDE as soon as it preserves HPDE of degree 1. Then naturally arises the question of the existence of polynomial projectors preserving HPDE