A CONVERGENCE PROBLEM FOR KERGIN INTERPOLATION

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Let \( E, F, G \) be three compact sets in \( \mathbb{C}^n \). We say that \((E,F,G)\) holds if for any choice of an interpolating array in \( F \) and of an analytic function \( f \) on \( G \), the Kergin interpolation polynomial of \( f \) exists and converges to \( f \) on \( E \). Given two of the three sets, we study how to construct the third in order that \((E,F,G)\) holds.


1. Formulating the problem

Let us first recall some basic facts for Kergin interpolation. Let \( \Omega \) be a \( \mathbb{C} \)-convex domain in \( \mathbb{C}^n \), i.e. for each complex line \( l \subset \mathbb{C}^n \), \( l \cap \Omega \) is empty or simply connected. Denote by \( H(\Omega) \) the space of holomorphic functions on \( \Omega \) and \( P_d(\mathbb{C}^n) \) the space of polynomials whose degree does not exceed \( d \).

Let \( A=\{a_0,a_1,\ldots,a_d\} \) be a subset of \( d+1 \) (nonnecessarily distinct) points in \( \Omega \), then there exists a unique continuous linear map:

\[
K_A: H(\Omega) \rightarrow P_d(\mathbb{C}^n)
\]

with the following properties.

(K1) For \( i=0,1,\ldots,d \) and \( f \in H(\Omega) \), \( K_A(f)(a_i) = f(a_i) \).

(K2) If \( g \in H(\Omega) \) is of the form \( g = f \circ u \) with \( u \) an affine map from \( \mathbb{C}^n \) to \( \mathbb{C}^m \) and \( f \in H(u(\Omega)) \) then

\[
K_A(g) = K_{u(A)}(f) \circ u
\]

where \( u(A) = \{u(a_0),u(a_1),\ldots,u(a_d)\} \). Thus if \( m=1 \)

\[
K_A(g) = L_{u(A)}(f) \circ u
\]

where \( L_{u(A)}(f) \) is the usual Lagrange Hermite interpolation polynomial of the one variable function \( f \) with respect to the points \( u(a_0),\ldots,u(a_d) \).

(K3) When all the points \( a_0, a_1,\ldots,a_d \) coincide, \( K_A(f) \) is the Taylor expansion of \( f \) at the point \( a(=a_0,a_1,\ldots,a_d) \) and of degree \( d \).