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A CONTINUITY PROPERTY OF MULTIVARIATE LAGRANGE INTERPOLATION

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ABSTRACT. Let $\{S_t\}$ be a sequence of interpolation schemes in $\mathbb{R}^n$ of degree d (i.e. for each S_t one has unique interpolation by a polynomial of total degree $\leq d$) and total order $\leq l$. Suppose that the points of S_t tend to $0 \in \mathbb{R}^n$ as $t \to \infty$ and the Lagrange-Hermite interpolants, H_{S_t}, satisfy $\lim_{t\to\infty} H_{S_t}(x^\alpha) = 0$ for all monomials x^α with $|\alpha| = d+1$. **Theorem**: $\lim_{t\to\infty} H_{S_t}(f) = T^d(f)$ for all functions f of class C^{l-1} in a neighborhood of 0. (Here $T^d(f)$ denotes the Taylor series of f at 0 to order d.)

Specific examples are given to show the optimality of this result.

1. INTRODUCTION

Let O be an open neighborhood of the origin in $\mathbb{R}$, $a := (a^0, ..., a^d) \in O^{d+1}$ and f a function of class C^{d+1} on O. As is well known, if $H[a^0, \dots, a^d](f)$ denotes the Lagrange-Hermite interpolation polynomial with respect to the points $a^0, \dots, a^d$ (with the usual convention when some points coincide), then

$$\lim_{a\to 0} H[a^0, \dots, a^d] = T^d f$$

where $T^d f$ denotes the d-th Taylor polynomial of f at the origin. This follows quite easily from the Newton representation formula for the interpolating polynomial, that is

$$H[a^0, \dots, a^d](f, x) = f(a^0) + \sum_{i=1}^{d} f[a^0, \dots, a^i](f, x)(x - a^0)\dots(x - a^{i-1})$$

via the Hermite-Genocchi formula for the divided differences, namely

$$f[a^0, \dots, a^i] = \int_{\Delta^i} f^{(i)}(a^0 + \sum_{j=1}^{d} t_j a^j) dm(t)$$

where dm denotes Lebesgue measure on the simplex

$$\Delta^i = \{(t_j)_{1\leq j\leq i} : t_j \geq 0, \sum_{j=1}^{i} t_j \leq 1\}.$$

More generally, for fixed f of class C^{d+k}, one can prove similarly that the function $a \to H[a^0, \dots, a^d](f)$ is of class C^k on O^{d+1} (see also [N, Th. 2.5]).

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