Polynomial projectors preserving homogeneous partial differential equations

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Abstract

A polynomial projector Π of degree d on $H(\mathbb{C}^n)$ is said to *preserve homogeneous partial differential equations* (HPDE) of degree k if for every $f \in H(\mathbb{C}^n)$ and every homogeneous polynomial of degree k, $q(z) = \sum_{|\alpha|=k} a_\alpha z^\alpha$, there holds the implication: $q(D)f = 0 \Rightarrow q(D)\Pi(f) = 0$. We prove that a polynomial projector Π preserves HPDE of degree k, $1 \leqslant k \leqslant d$, if and only if there are analytic functionals $\mu_k, \mu_{k+1}, \ldots, \mu_d \in H'(\mathbb{C}^n)$ with $\mu_i(1) \neq 0$, $i = k, \ldots, d$, such that Π is represented in the following form

$$\Pi(f) = \sum_{|\alpha|<k} a_\alpha(f)u_\alpha + \sum_{k \leqslant |\alpha| \leqslant d} D^\alpha \mu_{|\alpha|} u_\alpha$$

with some a_α's $\in H'(\mathbb{C}^n)$, $|\alpha| < k$, where $u_\alpha(z) := z^\alpha/\alpha!$. Moreover, we give an example of polynomial projectors preserving HPDE of degree k ($k \geqslant 1$) without preserving HPDE of smaller degree. We also give a characterization of Abel–Gontcharoff projectors as the only Birkhoff polynomial projectors that preserve all HPDE.
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