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CONSTRUCTIVE APPROXIMATION


# Kergin Interpolants of Holomorphic Functions

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**Abstract.** Let $D$ be a $\mathbf{C}$-convex domain in $\mathbf{C}^n$. Let $\{A_{dj}\}$, $j = 0, \ldots, d$, and $d = 0, 1, 2, \ldots$, be an array of points in a compact set $K \subset D$. Let $f$ be holomorphic on $\overline{D}$ and let $K_d(f)$ denote the Kergin interpolating polynomial to $f$ at $A_{d0}, \ldots, A_{dd}$. We give conditions on the array and $D$ such that $\lim_{d\to\infty} \|K_d(f) - f\|_K = 0$. The conditions are, in an appropriate sense, optimal.

This result generalizes classical one variable results on the convergence of Lagrange–Hermite interpolants of analytic functions.

## 1. Introduction

The procedure known as Kergin interpolation (see below) is a multivariate counterpart of the classical Lagrange–Hermite interpolation. It was introduced in 1978 by Kergin [Ke] for sufficiently differentiable functions on a convex open set in $\mathbf{R}^n$. A little later, a constructive approach was given by Micchelli [M], see also [MM]. In the complex case the work of Andersson and Passare, see [AP1] and [AP2], showed the crucial role played by $\mathbf{C}$-convexity.

The main properties of convergence are known for the case of entire functions, see [B1] and [AP1]. In this paper, we are concerned with convergence problems for nonentire functions. More precisely, we will study the

**Problem 1.1.** Let $A = (A_{dj})$ be a triangular array of (not necessarily distinct) points in a compact set $K$ in $\mathbf{C}^n$, find a domain $D$ in $\mathbf{C}^n$ as small as possible (or a compact set $K_1$) such that for every function $f$ holomorphic on $D$ (or in a neighborhood of $K_1$) the Kergin interpolation polynomial of $f$ at the points $A_{d0}, \ldots, A_{dd}$ exists and converges to $f$ uniformly on $K$ as $d$ approaches $\infty$.

Note that the existence requirement is not superfluous or straightforward for the definition of the Kergin operator for functions holomorphic on a domain $D$ needs a strong geometric condition, namely, $D$ must be $\mathbf{C}$-convex (see below again).

As it turns out, the problem is closely related to the distribution of the points $(A_{dj})$,

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