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Polynomial Interpolation with Prescribed Analytic Functionals

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We study the reconstruction of an analytic function of several complex variables by means of interpolating polynomials obtained from pieces of information given by functionals of derivatives of the function. Several classical interpolation methods are examples of our general problem. © 1993 Academic Press, Inc.

INTRODUCTION

Let us suppose that, by some process, you know m numerical pieces of information on a function f; let us suppose also that you can construct a polynomial (of degree less than or equal to $m-1$) which, by the same process, gives the same m informations: you have found an interpolating polynomial (for the process in question) of the function f.

When the m pieces of information are the values of f at m distinct points, the interpolating polynomial is only the Lagrange polynomial; when these are the values of the m first derivatives of f at the point x, then the polynomial is the Talyor's expansion of f at order m and at the point x.

The usual problem is: if the number of pieces of information grows larger and larger, does the interpolating polynomial converge (uniformly) to the function f?

In general no, but sometimes yes (as is well known for the above examples) when f has appropriate analytic properties.

We study such a problem for functions of several complex variables and a quite general process: the information is given by analytic functionals of the derivatives of the function f; see Problem 1.1.

This work finds its origin essentially in the study by Gelfond [11] of the general divided differences interpolation which already generalized previous work of Gontcharoff; see [12]. In the multivariate context this procedure has been studied by Cavaretta *et al.* for the definition, see [8], and by Goodman and Sharma for the convergence; see [13]. The methods of

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