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Interpolation in Fréchet spaces with an application to complex function theory

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ABSTRACT

We give a necessary and sufficient condition on a sequence of linear functionals on a Fréchet space to be interpolating. A practical criterion is given and applied to several natural interpolation problems in complex function theory.

1. INTRODUCTION

Let E be a Fréchet space, E^* the topological dual of E; $L_n \in E^*$, $n=1,2,\ldots$ is said to be an interpolating sequence if for any scalar sequence A_n, $n=1,2,\ldots$ there exists $f\in E$ such that $L_n(f)=A_n$, $n=1,2,\ldots$. The problem is to find conditions on the sequence L_n to ensure that it is an interpolating sequence. Let us note that obviously the answer does not depend on the ordering of the sequence (L_n).

This problem has very natural applications when $E=H(\Omega)$, the space of holomorphic functions in an open set Ω in $\mathbb{C}^n$, endowed with the topology of uniform convergence on compact subsets of Ω.

P. Gauthier and L.A. Rubel have given a necessary and sufficient condition on (L_n) to be an interpolating sequence when E is separable.

DEFINITION 1.1. *For each n, denote by V_n the space span$(L_1,\ldots,L_n)$. The sequence (L_n) is said to be totally linearly independent if*

(1) *(L_n) is linearly independent in E^*;*