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Multipoint Taylor interpolation

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Abstract. We construct new multivariate polynomial interpolation schemes of Hermite type. The interpolant of a function is obtained by specifying suitable discrete differential conditions on the restrictions of the function to algebraic hypersurfaces. The least space of a finite-dimensional space of analytic functions plays an essential role in the definition of these differential conditions.

1 Introduction

An n-dimensional Hermite (or Birkhoff) interpolation scheme of degree d is a collection $H = \{\mu_s : s \in S\}$ of discrete (differential) functionals μ_s such that for every suitably defined function f there exists a unique polynomial p of n variables and degree at most d satisfying $\mu_s(p) = \mu_s(f)$, $s \in S$. The polynomial p is then called the H-interpolation polynomial of f. Classical Lagrange-Hermite interpolation furnishes the most important general example of a specific univariate Hermite scheme. In the multivariate case it is generally difficult to check whether a given set of functionals H is a Hermite scheme, even when every $\mu_s \in H$ is a point-evaluation functional, $\mu_s(f) = f(u_s)$, which corresponds to ordinary Lagrange interpolation. Actually, the sole multivariate case for which the verification that H is a Hermite scheme is absolutely straightforward is obtained by taking $H = \{f \to D^\alpha(f)(a),\ |\alpha| \le d\}$. In that

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