Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5)
Vol. VII (2008), 545-577

Taylorian points of an algebraic curve and bivariate Hermite interpolation

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Abstract. We introduce and study the notion of Taylorian points of algebraic curves in $\mathbb{C}^2$, which enables us to define intrinsic Taylor interpolation polynomials on curves. These polynomials in turn lead to the construction of a well-behaved Hermitian scheme on curves, of which we give several examples. We show that such Hermitian schemes can be collected to obtain Hermitian bivariate polynomial interpolation schemes.

Mathematics Subject Classification (2000): 41A05 (primary); 41A63, 46A32, 14Q05 (secondary).

1. Introduction

In classical univariate Lagrange interpolation theory, we know the values of a function at finitely many points and we construct the polynomial of smallest degree which takes the same value at these points. But we may know more than the mere values of the function, we may know its *local behavior* around the points, that is, its Taylor polynomial at each of the points, each one to a certain order. When we collect these pieces of (local) information and look for the polynomial of smallest degree that matches the same local behavior we find exactly the classical Hermite interpolation polynomial. When working with multivariate functions, apart from the (fundamental) fact that we must now choose the location of the points more carefully, we can follow the same processes. However, knowing the Taylor polynomial, say of degree d, of a function of n complex variables f requires the knowledge of all the partial derivatives of order $\leq d$, or, equivalently, the total Fréchet derivatives $f^{(j)}$, $j = 0, \ldots, d$, of f which is a symmetric j-linear form on $(\mathbb{C}^n)^j$. In many cases, it seems more realistic to know $f^{(j)}$ on a subspace of $(\mathbb{C}^n)^j$, for example on the product of j copies of a hyperplane in $\mathbb{C}^n$, which amounts to knowing the local behavior of the *restriction* of the function to that hyperplane. Likewise, in the bivariate Hermite interpolation theory that we introduce and study in this paper,

Received November 21, 2007; accepted in revised form May 26, 2008.