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On the Siciak extremal function for real compact convex sets

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1. Introduction

Let E be a bounded Borel set in $\mathbf{C}^N$. Define

$$V_E(z) := \sup\{u(z) : u \in L,\ u \le 0 \text{ on } E\}, \tag{1.1}$$

where

$$L := \{u \text{ plurisubharmonic in } \mathbf{C}^N : u(z) \le \log^+|z| + C \text{ for some } C\}$$

is the class of plurisubharmonic functions of logarithmic growth (here we have $|z| = (\sum_{j=1}^N |z_j|^2)^{1/2}$ and $\log^+|z| = \max\{0, \log|z|\}$). Then the upper semicontinuous regularization $V_E^*(z) := \limsup_{\zeta \to z} V_E(\zeta)$ is called the (Siciak) extremal function of E. If K is a compact set in $\mathbf{C}^N$, then the extremal function in (1.1) can be gotten via the formula

(1.2)

$$V_K(z) := \max\left\{0, \sup\left\{\frac{1}{\deg p} \log|p(z)| : p \text{ holomorphic polynomial},\ \|p\|_K \le 1\right\}\right\}$$

(Theorem 5.1.7 in [K1]). Here, $\|p\|_K := \sup_{z \in K} |p(z)|$ denotes the uniform norm on K. We say that K is regular if and only if $V_K^* = V_K$. Note that if we let

$$\widehat{K} := \{z \in \mathbf{C}^N : |p(z_1, \dots, z_N)| \le \|p\|_K \text{ for all polynomials } p\}$$

denote the polynomial hull of K, then

1. $\widehat{K} = \{z \in \mathbf{C}^N : V_K(z) = 0\}$;
2. $V_{\widehat{K}} = V_K$.