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ABSTRACTS

Eric BEDFORD : Dynamics of rational surface automorphisms of positive entropy.

A complex surface is said to be rational if it is birationally equivalent to the projective plane. An automorphism will have positive entropy exactly when the induced mapping on cohomology has an eigenvalue greater than 1. It is known that (essentially speaking) the basic possibilities for positive entropy occur for tori, $K3$, and rational surfaces. The positive entropy maps are obtained by blowing up the projective plane, and we will discuss some constructions of this. Some of these examples involve simple blowups, and sometimes we must take iterated blowups. We will also touch on the dynamical behavior of the maps that are obtained this way.

Taib BELGHITI : Approximation polynomiale sur des compacts vérifiant la condition SL.

Le but de ce papier est de déterminer le degré d'approximation ainsi que la construction de meilleurs polynômes approximants pour des fonctions appartenant à des classes de Denjoy-Carleman holomorphes sur certains compacts de $\mathbb{C}^N$, étendant, ainsi, le fameux théorème de Bernstein-Walsh-Siciak, ainsi que les résultats de Baouendi et Goulaouic pour la classe de Gevrey sur la boule unité de $\mathbb{R}^N$. Un critère géométrique du type inégalité de Lojasiewicz pour la fonction de Green associée au compact est introduit (condition SL).

Bo BERNDTSSON : Bergman kernels and Fekete points (After Berman, Boucksom and Witt)

The Bergman kernel asymptotics of Bouche-Tian-Catlin-Zelditch deals with the behaviour of Bergman kernels for the space of sections to a high power of a positive line bundle. Here the Bergman kernel depends on the choice of a metric on the line bundle and of some measure with strictly *positive density on the base manifold. Berman, Boucksom and Witt have recently found the (first order) asymptotics when the measure only needs to satisfy a Bernstein-Markov property. For the case when the support of the measure is compact in $\mathbb{C}^n$, regarded as a subset of $n-$dimensional projective space, this gives strong information on orthogonal polynomials, and it also turns out to imply the equidistribution of Fekete points towards the equilibrium measure on the compact.*

Léa BLANC-CENTI : STATIONARY PSEUDOHOLOMORPHIC DISCS

Stationary discs are natural invariants of manifolds with boundary with respect to biholomorphisms. The aim of this talk is to present an explicit parametrization of the stationary discs glued to small almost complex deformations of a non-degenerate hyperquadric. This gives a result of uniqueness for pseudo-biholomorphisms with given $1-$jet at some convenient point.

Thomas BLOOM : RANDOM MATRICES AND POTENTIAL THEORY.

The initial impetus for the study of random matrices stems from the work of the physicist E. Wigner, done about 50 years ago. Today, the theory of random matrices has connections with many different areas of mathematics and physics. I will discuss the interaction between the theory of random matrices and potential theory.

Urban CEGRELL : THE MONGE-AMPÈRE EQUATION

Let $\Omega \subset \mathbb{C}^n$ be a hyperconvex domain and denote by $\mathcal{E}(\Omega) = \mathcal{E}$ the negative plurisubharmonic functions on Ω with well-defined Monge-Ampre measure.

We will discuss the following theorem.

Assume that $0 \leq \mu \leq (dd^c v)^n$, $v \in \mathcal{E}$ and $H \in \mathcal{E}$, $(dd^c H)^n = 0$. Then there is a function $u \in \mathcal{E}, H \geq u \geq v + H$ such that $\mu = (dd^c u)^n$, (Joint work with P. åhag, R. Czyz and H.H. Pham).

Jean-Pierre DEMAILLY : UNIVERSAL CANONICAL METRICS ON PSEUDOEFFECTIVE KAWAMATA LOG TERMINAL PAIRS (X, D)

By adapting ideas of Narasimhan-Simha, also recently investigated by H. Tsuji, we will propose a very simple method for constructing universal canonical singular metrics on every Kawamata log terminal pair (X, D). This metric is strongly tied to important questions such as the finiteness of the canonical ring and the abundance conjectures, and should be in a weak sense a solution of a Kähler-Einstein equation. Under the assumption that the line bundles involved are big, suitable regularity properties can also be proved.

Henri De THELIN : DYNAMICS OF MEROMORPHIC MAPS

We study the dynamics of a meromorphic map on a compact Kähler manifold. We give a criterion that allows us to produce a measure of maximal entropy and we apply this criterion for a family of generic birational maps of $\mathbb{P}^k$*. This is a joint work with Gabriel Vigny.*

Tien-Cuong DINH : EXPONENTIAL ESTIMATES FOR MONGE-AMPÈRE MEASURES AND COMPLEX DYNAMICS

We prove exponential estimates for plurisubharmonic functions with respect to Monge-Ampère measures with Hölder continuous potential. As an application, we obtain several stochastic properties for the equilibrium measures associated to holomorphic maps on projective spaces. More precisely, we prove the exponential decay of correlations, the central limit theorem for general d.s.h. observables, and the large deviations theorem for bounded d.s.h. observables and Hölder continuous observables. This is joint work with Viêt-Anh Nguyên and Nessim Sibony.

Christophe DUPONT: ON THE DIMENSION OF THE EQUILIBRIUM MEASURE

Let f *be an holomorphic endomorphism of* $\mathbb{CP}(2)$ *and* μ *be its equilibrium measure. We prove a lower bound for the Hausdorff dimension of* μ*, involving the degree of* f *and the Lyapounov exponents of* μ*.*

Julien DUVAL : SUR LE DEUXIÈME THÉORÈME DE NEVANLINNA

Le théorème de Nevanlinna mesure l'impact d'une courbe entière (une application holomorphe du plan complexe dans la sphère de Riemann) sur des cibles fixes. L'exposé parlera de la généralisation (due à Yamanoi) de ce théorème aux cibles lentes.

Vincent GUEDJ : A BIG YAU THEOREM

A celebrated theorem of Yau asserts that a Kähler class α *always contains Kähler forms with prescribed Ricci curvature. We establish a similar result when the class* α *is merely big. This is a joint work with S.Boucksom, P.Eyssidieux and A.Zeriahi*

Friedrich HASLINGER : ASPECTS OF COMPACTNESS FOR THE $\bar{\partial}$-NEUMANN PROBLEM.

Let Ω *be a bounded pseudoconvex domain in* $\mathbb{C}^n$*. We show that compactness of the canonical solution operator to* $\overline{\partial}$ *restricted to* $(0,1)$*-forms with holomorphic coefficients is equivalent to compactness of the commuta-*

tor $[\mathcal{P}, \overline{M}]$ defined on the whole $L^2_{(0,1)}(\Omega)$, where $\overline{M}$ is the multiplication by $\bar{z}$ and $\mathcal{P}$ is the orthogonal projection of $L^2_{(0,1)}(\Omega)$ to the subspace of $(0,1)$ forms with holomorphic coefficients.

We also discuss the $\overline{\partial}$-Neumann problem in the setting of weighted L^2-spaces on $\mathbb{C}^n$. For this purpose we apply ideas which were used for the Witten Laplacian in the real case and various methods of spectral theory of these operators, and a version of the Rellich lemma for weighted Sobolev spaces.

Burglind JÖRICKE : Pluripolar hulls and fine analytic continuation.

At present stage fine analytic continuation seems the most adequate tool to understand the pluripolar hull of graphs of analytic functions over the unit disc. We will address recent results of several authors.

Ha Hui KHOAI : Nevanlinna theory, unique range sets and decomposition of meromorphic functions.

R. Nevanlinna showed that a nonconstant meromorphic function on $\mathbb{C}$ is uniquely determined by the inverse images, not counting multiplicities of 5 distinct values. The theorem is either referred to as a "uniqueness" or "unicity" theorem, or as a "sharing values" theorem.

Two meromorphic functions f, g defined on $\mathbf{C}$ are said to share a value $a \in \mathbf{C} \cup \{\infty\}$, ignoring multiplicities if $f^{-1}(a) = g^{-1}(a)$ as a point set, and to share a value $a \in \mathbf{C} \cup \{\infty\}$, counting multiplicities, if $E_f(a) = E_g(a)$, where

$$E_f(a) := \{(m, z) | f(z) = a \text{ with multiplicity } m\}.$$

Now let S be a non-empty subset of $\mathbb{C} \cup \{\infty\}$, and f be a meromorphic function. Define

$$E_f(S) = \bigcup_{a \in S} \{(m, z) | f(z) = a \text{ with multiplicity } m\}.$$

A set S is called a unique range set (URS), not counting multiplicities (resp. counting multiplicities), for meromorphic functions if for two meromorphic functions f, g the condition $f^{-1}(S) = g^{-1}(S)$ (resp. $E_f(S) = E_g(S)$) implies $f \equiv g$.

In this talk we give a survey of recent results on unique range sets for meromorphic functions and applications in the study of functional equations and decompositions of meromorphic functions.

Christer KISELMAN : Prolongement analytique des solutions fondamentales des équations différentielles à coefficient constants

Pour certains opérateurs différentiels à coefficients constants une formule élémentaire définit une solution fondamentale. À partir de ces opérateurs on peut obtenir par prolongement analytique une solution fondamentale pour d'autres opérateurs pour lesquels il n'existe pas de formule élémentaire. L'étude de ces prolongements, donc la dépendence analytique de la solution E_P du polynôme variable P, est le sujet de cet exposé.

Sławomir KOŁODZIEJ : VOLUME OF SUBLEVEL SETS OF PLURISUBHARMONIC FUNCTIONS AND THE THEOREM OF DEMAILLY
(joint work with P. Ahag, U. Cegrell, H. Pham and A. Zeriahi)

We prove an a priori estimate for volumes of sublevel sets of plurisubharmonic functions with well defined Monge-Ampère measures which can be approximated from above by plurisubharmonic functions with zero boundary values.

Thus for any bounded hyperconvex Ω with diameter d_Ω and any $u \in PSH(\Omega)$ with zero boundary values and satisfying

$$\int_\Omega (dd^c u)^n = t^n$$

the Lebesgue measure (dV) of the set $\{u \leq s\}$ is bounded from above by

$$const(n) d_\Omega^{2n} (1 + s/t)^{n-1} \exp(-2ns/t).$$

For $n = 1$ it follows from the classical Polya inequality. In higher dimension this gives a "pluripotential theoretic" proof of a strengthened version of recent result of Demailly saying that if u is as above with $t < n$ then

$$\int_\Omega \exp(-2u) dV \leq (\pi^n + const(n)t/(n-t)^n) d_\Omega^{2n}.$$

As observed by Demailly such a theorem provides an alternative proof of an inequality from local algebra (due to Corti, de Fernex, Ein, Mustata)

$$lc(I) \geq ne(I)^{-1/n},$$

where I is an ideal of germs of holomorphic functions with isolated singularity at the origin in $\mathbb{C}^n$, $lc(I)$ - denotes the log canonical threshold if I and $e(I)$ - the Hilbert-Samuel multiplicity of the ideal.

Lukasz KOSINSKI : CLASSIFICATION OF PROPER HOLOMORPHIC MAPS BETWEEN NON-HYPERBOLIC REINHARDT DOMAINS IN $\mathbb{C}^2$

In the talk I will present the description of non-elementary proper holomorphic maps between non-hyperbolic Reinhardt domains in $\mathbb{C}^2$ as well as the corresponding pairs of domains. Additionally, I will give some partial *results for proper maps between domains of the form $\mathbb{C}_*^2$, $\mathbb{C}^2$ and $\mathbb{C} \times \mathbb{C}_*$ and I will present some more general results related to proper holomorphic mappings.*

Wiesław PLESNIAK : RECENT PROGRESS IN THE STUDY OF PLURIREGULARITY

The Siciak extremal function establishes an important link between polynomial approximation in several variables and pluripotential theory. This yields its numerous applications in complex and real analysis. The crucial point is to establish continuity of the function Φ_E. In such a case the (compact) set E is said to be L-regular. The aim of my talk is to give a state-of-the-art survey of investigations concerning this property whose satisfactory theory has been developed due to applications of both the Gabrielov-Hironaka-Łojasiewicz subanalytic geometry together with its further generalizations which are o-minimal structures and pluripotential methods based on the complex Monge-Ampère operator.

Azim SADULLAEV : : PLURIHARMONIC CONTINUATION IN A FIXED DIRECTION

Complex potential theory plays an important role in solving problems of continuation of analytic functions. Let $f('z, z_n)$ be a holomorphic function in the polydisk $'U \times U_n \subset \mathbb{C}^{n-1}_{'z} \times \mathbb{C}_{z_n}$. For each fixed $'z^0 \in' U$, denote respectively by $R^1('z^0)$, $R^2('z_0)$ and $R^3('z_0)$ the maximal radius of the disc to which the function $z_n \longmapsto f('z_0, z_n)$ extends as a holomorphic function, a meromorphic function, a holomorphic function with a polar set of singularities.

Théorème 1 : *The (well-known) regularizations $-\ln R^1_\star('z), -\ln R^2_\star('z)$ and $-\ln R^3_\star('z)$ are plurisubharmonic in $'U$. Moreover, the function $f('z, z_n)$ extends respectively as a holomorphic function in $\{('z, z_n) \in' U \times U_n; |z_n| < R^1_\star('z)\}$, a meromorphic function in $\{('z, z_n) \in' U \times U_n; |z_n| < R^2_\star('z)\}$, holomorphic function with a pluripolar set of singularities in $\{('z, z_n) \in' U \times U_n; |z_n| < R^3_\star('z)\}$ and the sets $\{R^1_\star('z) < R^1('z)\}$, $\{R^2_\star('z) < R^2('z)\}$ and $\{R^3_\star('z) < R^3('z)\}$ are pluripolar in $'U$.*

Plurisubharmonicity of $-\ln R^j_\star$ $(1 \leq j \leq 3)$ helps us to apply the complex potential theory, in particular, the well-known $P-$measure $\omega^\star('z, \cdot)$, to the description of the maximal domain of holomorphicity, meromorphicity or extension as a holomorphic function with pluripolar singular set (see [1-6]).

Analogous questions for harmonic and pluriharmonic functions are more complicated. The case of harmonic functions was studied by Korevaar J., Meyers J., Zaharuta V.P., Nguyen T. V., Zeriahi A., Hécart J.M. and others (see [3,7-11]).

Let us consider a plurisubharmonic function $u('z, z_n)$ in the polydisk $'U \times$

U_n *and for each fixed* $'z_0 \in' U$ *let* $R('z^0)$ *the maximal radius such that* $u('z, z_n)$ *extends harmonically to the disk* $\{|z_n| < R('z^0)\}$ *.Then :*
a) $-\ln R_\star('z)$ *is plurisubharmonic in* $'U$*,*
b) the function $u('z, z_n)$ *is plurisubharmonic in* $\{(z', z_n) \in' U \times U_n; |z_n| < R_\star('z)\}$*,*
c) the set $\{R_\star('z) < R('z)plur\}$ *is ipolar in* $'U$*.*

When $u('z, z_n)$ *has singularities in the direction of* z_n*, the analogous result is not true.*

Example. *The function* $u(z, w) := \Re[z \log(w-1)]$ *is pluriharmonic in the bidisk* $\{|z| < 1\} \times \{|w| < 1\}$*; for each fixed* $z = x \in (-1, +1)$*, t the function* $u(x, w) = x \ln|w-1|$ *can be extended as a harmonic function of* w *on the entire plane, except* $w = 1$*. However,* $u(z, w)$ *is not single-valued in* $(\{|z| < 1\} \times \mathbb{C}) \setminus \{w = 1\}$ *so that it could not be continued to* $\{|z| < 1\} \times \mathbb{C}$ *as a plurisubharmonic function with an analytic singular set.*

Théorème 2 : (Sadullaev A., Imamkulov S. [11]). *Let* $u('z, z_n)$ *be a pluriharmonic function in the polydisk* $'U \times U_n \subset \mathbb{C}^{n-1}_{'z} \times \mathbb{C}_{z_n}$ *and, for any fixed* $'z_0$ *in a certain nonpluripolar set* $E \subset' U$*, the function* $u('z, z_n)$ *of the variable* z_n *can be extended to a harmonic function on the entire plane except for a finite set of singular points. Then* $u('z, z_n)$ *can be extended pluriharmonically (possibly, in a multivalued manner) to* $('U \times \mathbb{C}) \setminus S$ *, where* S *is an analytic set in* $'U \times \mathbb{C}$*. Moreover, if* E *is not contained in a countable union of local sets of zeros of pluriharmonic functions, then the extension of* $u('z, z_n)$ *is a single ?valued pluriharmonic function.*

Note. *Analogous questions in the boundary case* $E \subset \partial D$ *will also discussed in the talk.*

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Ragnar SIGURDSSON : Siciak-Zahariuta extremal functions, analytic discs and polynomial hulls.

This is a joint work with Finnur Larusson, University of Adelaide, Australia. We prove two disc formulas for the Siciak-Zahariuta extremal function of an arbitrary open subset of complex affine space. We use these formulas to characterize the polynomial hull of an arbitrary compact subset of complex affine space in terms of analytic discs. Similar results in previous work of ours required the subsets to be connected.

B. A. TAYLOR : High order tangents to analytic varieties and Phragmén-Lindelöf conditions.

A pure dimensional variety V *in a neighborhood of the origin in* $\mathbf{C^n}$ *is said to satisfy the local Phragmén-Lindelöf condition if the local plurisubharmonic extremal function,* $U(z)$*, for the set of real points in the variety grows linearly in some neighborhood of the origin; that is,* $U(z) \leq A|Im\ z|$ *if* $z \in V$ *and* $|z| < \delta$*. In joint work with R. Braun and R. Meise, we have shown that this condition is characterized, at least when* $n = 2, 3$*, by the behavior of the limit varieties that are highly tangent to* V *in real directions. We will report on progress in extending this result to* $n \geq 4$*, explain why the methods succeed when* $n = 4$*, and discuss the obstruction that remains in treating higher dimensional cases. At least for hypersurfaces, the characterization of varieties satisfying the Phragmén-Lindelöf condition depends on properties of certain algebraic varieties of the same dimension that are the high order real tangents to* V*. All such varieties* W *are of the form* $W = \lim_{j\to\infty}\{w \in \mathbf{C^n} : a_j + r_j w \in V\}$ *where* $a_j \in \mathbf{R^n}$*,* $r_j > 0$*, and* $|a_j| + r_j \to 0$*.*

Vyacheslav ZAKHARYUTA : Kolmogorov problem on widths asymptotics.

Given a compact set K *in an open set* D *on a Stein manifold* $\Omega, \dim\Omega = n$*, the set* A_K^D *of all restrictions of functions, analytic in* D *with absolute value bounded by* 1*, is considered as a compact subset in* $C(K)$*. The problem about the strict asymptotics for Kolmogorov diameters:*

$$-\ln d_s\left(A_K^D\right) \sim \sigma\ s^{1/n},\ s \to \infty. \tag{1}$$

was stated by Kolmogorov (in an equivalent formulation for ε*-entropy of this set). It was conjectured in [4,5] that for "good" pairs* $K \subset D$ *the asymptotics (1) holds with* $\sigma = 2\pi\left(\frac{n!}{C(K,D)}\right)^{1/n}$*, where* $C(K,D)$ *is the pluricapacity of* *the "pluricondenser" (K, D) (see, e.g., [1]). In the one-dimensional case this hypothesis is equivalent to Kolmogorov's conjecture about asymptotics of ε-entropy of the set A_K^D, which has been confirmed by efforts of many authors (Erokhin, Babenko, Zahariuta, Levin-Tikhomirov, Widom, Nguyen, Skiba - Zahariuta, Fisher - Miccheli, et al).*

In [4,5] the problem (1) was reduced (the proof was only sketched, for a detailed proof see, [6]) to the problem of Pluripotential Theory about approximation of the relative Green pluripotential of the pluricondenser (K, D) by pluripotentials with finite set of logarithmic singularities. The last problem has been solved recently by Nivoche-Poletsky ([2,3]), which results, together with [4,5,6], a final proof of our conjecture about strong asymptotics (1) under some natural restrictions about K, D.

In the talk we are focused on a proper modification of the Kolmogorov problem, which is connected more tightly with certain natural pluripotential properties of a pluricondenser (K, D); the asymptotics (1) is treated in the frame of those more general considerations. On the other hand, a considerable attension is paid to an important device in the proof of the reduction - special extendible bases with good estimates on sublevel domains of a multipolar pluricomplex Green function.

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