Aircraft Local Wind Estimation From Radar Tracker Data

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Why wind estimation is needed for air traffic management?
Kalman Filtering
Mode-S radar
Classical radar
Agenda

- Why wind estimation is needed for air traffic management?
- Kalman Filtering
- Mode-S radar
- Classical radar
An accurate trajectory prediction is useful for:

- conflict prediction
- landing sequencing
- airspace sector overload detection
- traffic organization
- etc ...

Trajectory Prediction Applications
Limitation factors:

- wind (kinematic)
- temperature, pressure (engine performance)
- aircraft weight (dynamic models)
How it works today?

Today, a wind map is produced every 3 hours for several altitude levels which is not enough for accurate trajectory prediction.

What do we propose?

The key idea of this work is to use aircraft as sensors in order to produce and update wind maps every minute.

Remarks:

- Any aircraft when turning may be considered as a wind sensor.
- Aircraft with Mode_S transponder may downlink useful data for wind estimation.
\[ \vec{V} = \vec{T} + \vec{W} \]

Notations: \( T = \|\vec{T}\|, \ V = \|\vec{V}\|, \ W = \|\vec{W}\| \)
Turning Rates

Air Turning Rate

\[ \omega_a = \frac{d\theta_a(t)}{dt} \]

Ground Turning Rate

\[ \omega_g = \frac{d\theta_g(t)}{dt} \]

\[ \omega_g = \frac{\gamma_x \cdot v_y - v_x \cdot \gamma_y}{V^2} \]

Relation

\[ \omega_g = \left( \frac{T^2 + T \cdot W \cdot \cos(\theta_a(t) - \theta_g(t))}{T^2 + W^2 + 2 \cdot T \cdot W \cdot \cos(\theta_a(t) - \theta_g(t))} \right) \omega_a \]
### Primary radar

- Position measures \((x, y)\)

### Secondary radar

- Position measures \((x, y)\), mode A, mode C

### Mode S radar

For this radar some on-board information may be downlinked to the ground:

- Position measures \((x, y)\)
- Air Speed measures \((\|\vec{T}\|, \theta_a)\)
- Air Turning Rate measures \((\omega_a)\)

All those measures are disturbed by noise. (additive in models)
Hypothesis

1. En route traffic is considered.
2. Aircraft are supposed to fly at constant true air speed ($\| \vec{T} \|$).
3. Aircraft are supposed to turn at constant air turning rate ($\omega_a$).

Based on those hypothesis, our objective is to find a method for extracting wind from radar measures.
Agenda

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Kalman Filtering: Linear Form

Linear Model

\[
\begin{align*}
\tilde{X}(k + 1) &= F(k).\tilde{X}(k) + G(k).\tilde{U}(k) + \tilde{v}(k) \\
\tilde{Z}(k) &= H(k).\tilde{X}(k) + \tilde{w}(k)
\end{align*}
\]

Noise properties

\[
\tilde{U}(k) = \tilde{U}(k) + \tilde{n}(k)
\]

\[
p(\tilde{v}(k)) \sim \mathcal{N}(\tilde{0}, Q(k))
\]

\[
p(\tilde{w}(k)) \sim \mathcal{N}(\tilde{0}, R(k))
\]

\[
p(\tilde{n}(k)) \sim \mathcal{N}(\tilde{0}, N(k))
\]

\[
E[\tilde{v}(k)] = 0 \quad E[\tilde{v}(k)\tilde{v}(j)^T] = Q(k).\delta_{kj}
\]

\[
E[\tilde{w}(k)] = 0 \quad E[\tilde{w}(k)\tilde{w}(j)^T] = R(k).\delta_{kj}
\]

\[
E[\tilde{n}(k)] = 0 \quad E[\tilde{n}(k)\tilde{n}(j)^T] = N(k).\delta_{kj}
\]
Kalman Filtering: Linear Form

Prediction Phase

\[
\begin{align*}
\tilde{X}(k + 1/k) & = F(k).\tilde{X}(k/k) + G(k).\tilde{U}(k) \\
P(k + 1/k) & = F(k).P(k/k).F(k)^T + G(k).N(k).G(k)^T + Q(k)
\end{align*}
\]
Prediction

Estimate (k/k)

Real Position (k)

Prediction (k+1/k)

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Updating Phase

\[
K(k + 1) = P(k + 1/k).H(k + 1)^T \cdot [H(k + 1).P(k + 1/k).H(k + 1)^T + R(k + 1)]^{-1}
\]

\[
\begin{align*}
\dot{X}(k + 1/k + 1) &= \dot{X}(k + 1/k) + K(k + 1).\left[\ddot{Z}(k + 1) - H(k + 1).\dot{X}(k + 1/k)\right] \\
P(k + 1/k + 1) &= [I - K(k + 1).H(k + 1)] . P(k + 1/k)
\end{align*}
\]
Update

Estimate \((k/k)\)

Prediction \((k+1/k)\)

Measure \((k)\)

Real Position \((k)\)

Real Position \((k+1)\)

Estimate \((k+1/k+1)\)
\[
\begin{align*}
\ddot{X}(k+1) &= \mathcal{F}[k, \dot{X}(k), \bar{U}(k)] + \tilde{v}(k) \\
\ddot{Z}(k+1) &= \mathcal{H}[k, \dot{X}(k)] + \tilde{w}(k)
\end{align*}
\]
Kalman Filtering: Non Linear Form

Prediction Phase

\[
\begin{align*}
\tilde{X}(k+1/k) &= \mathcal{F}[k, \tilde{X}(k/k), U(k)] \\
P(k+1/k) &= \mathcal{F}_{\tilde{X}}(k).P(k/k).\mathcal{F}_{\tilde{X}}(k)^T + \\
&\quad \mathcal{F}_{\tilde{U}}(k).N(k).\mathcal{F}_{\tilde{U}}(k)^T + Q(k)
\end{align*}
\]

where \( \mathcal{F}_{\tilde{X}}(k) \) is the Jacobian matrix of partial derivatives of \( \mathcal{F} \) with respect to the vector \( \tilde{X} \):

\[
\mathcal{F}_{\tilde{X}}(k) \triangleq \left[ \nabla_{\tilde{X}} \left( \mathcal{F} \left[ k, \tilde{X}, \tilde{U} \right]^T \right) \right]^T \quad \tilde{X} = \tilde{X}(k/k), \tilde{U} = U(k)
\]

\[
\nabla \hat{s} \triangleq \left[ \frac{\partial}{\partial S_1}, \frac{\partial}{\partial S_2}, \ldots, \frac{\partial}{\partial S_{ns}} \right]^T
\]
### Kalman Filtering: Non Linear Form

#### Updating Phase

\[
\begin{align*}
K(k+1) &= P(k+1/k) \cdot H \hat{X}(k+1)^T. \\
[H \hat{X}(k+1) \cdot P(k+1/k) H \hat{X}(k+1)^T + R(k+1)]^{-1} \\
\hat{X}(k+1/k+1) &= \hat{X}(k+1/k) + K(k+1) . \\
[Z(k+1) - H (k, \hat{X}(k+1/k))] \\
P(k+1/k+1) &= [I - K(k+1) H \hat{X}(k+1)] \cdot P(k+1/k)
\end{align*}
\]

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- EKF is difficult to tune, the Jacobian can be hard to derive, and it can only handle limited amount of nonlinearity.
- PF can handle arbitrary distributions and non-linearities but is computationally very complex
- UKF gives a nice tradeoff between PF and EKF.
Uncented Transform

\[ X \rightarrow Y = f(X) \rightarrow Y \]

\[ Px \rightarrow Py \]

UT

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Observability conditions: Mode S radar

\[
\begin{align*}
v_x &= T \sin(\theta_a) + w_x \\
v_y &= T \cos(\theta_a) + w_y
\end{align*}
\]

(2 equations and to unknowns)

For Mode-S radars a Kalman filter will be used in order to produce wind estimate in real time.
Hypothesis

Air Speed ($\|\mathbf{T}\|, \theta_a$) and Air Turning Rate ($\omega_a$) are both available

State vector

$$\mathbf{X}(k) = [x(k), y(k), t_x(k), t_y(k), w_x(k), w_y(k)]^T$$

Measure

$$\mathbf{Z}(k) = [x_m(k), y_m(k), t_{x_m}(k), t_{y_m}(k)]^T$$
Having access to the air turning rate ($\omega_a$), the system is fully linear:

$$F(k) = \begin{bmatrix}
1 & 0 & C_1(\omega_a) & C_2(\omega_a) & \Delta_t & 0 \\
0 & 1 & -C_2(\omega_a) & C_1(\omega_a) & 0 & \Delta_t \\
0 & 0 & C_3(\omega_a) & C_4(\omega_a) & 0 & 0 \\
0 & 0 & -C_4(\omega_a) & C_3(\omega_a) & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}$$

$$
\begin{align*}
C_1(\omega_a(k)) &= \frac{\sin(\omega_a(k)\Delta t)}{\omega_a(k)} \\
C_2(\omega_a(k)) &= \frac{1-\cos(\omega_a(k)\Delta t)}{\omega_a(k)} \\
C_3(\omega_a(k)) &= \cos(\omega_a(k)\Delta t) \\
C_4(\omega_a(k)) &= \sin(\omega_a(k)\Delta t)
\end{align*}
$$
Hypothesis
Air Speed only available \((\| \vec{T} \|, \theta_a)\)

State vector
\[
\vec{X}(k) = [x(k), y(k), t_x(k), t_y(k), w_x(k), w_y(k)]^T
\]

Measure
\[
\vec{Z}(k) = [x_m(k), y_m(k), t_{x_m}(k), t_{y_m}(k)]^T
\]
Wind Estimation from Mode S Radar : Model 2

\[
F(k) = \begin{bmatrix}
1 & 0 & \Delta_t & 0 & \Delta_t & 0 \\
0 & 1 & 0 & \Delta_t & 0 & \Delta_t \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

This model is linear but is false, then the following model noise covariance matrix is included in the filter:

\[
Q = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
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Simulation Framework

Reference Trajectory

Disturbed Trajectory

Filtered Trajectory

Error

Noise

Filter in test
Trajectory Used for Experiments

Trajectory with 3 straight lines (20 minutes for each, $|\omega_a| = 1$ deg/sec). Aircraft speed : 400kts

For all experiments, a wind of 40 kts has been used with $\theta_w = 240^\circ$.
Results for Mode-S radar (models 1 and 2)

Wind strength error (models 1 and 2).
Convergence phase
Results for Mode-S radar (models 1 and 2)

Wind strength error (models 1 and 2)

![Wind amplitude comparison graph]

- Model 1
- Model 2
Results for Mode-S radar (models 1 and 2)

Wind angle error (model 1 and 2)
Convergence phase

Both models model are able to produce accurate wind estimate all along trajectory, even during turns.
Results for Mode-S radar (models 1 and 2)

Wind angle error (model 1 and 2)

Both models are able to produce accurate wind estimate all along trajectory, even during turns.
Hypothesis

Turning rate available only \((\omega_a)\)

The same as model 1 unless for the measure equation:

\[
\vec{Z}(k) = [x_m(k), y_m(k)]^T
\]
Results for Mode-S radar (model 3)

Wind Stength error: Convergence phase

![Wind amplitude comparison](image)

Model 3
One has to wait the second turn in order to produce wind estimate. Wind Strength error:
Results for Mode-S radar (model 3)

Wind Angle error:
Convergence phase

![Wind angle comparison graph]

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Results for Mode-S radar (model 3)

Wind angle error:

![Wind angle comparison graph](image_url)
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Hypothesis

No onboard information

State vector

\[ \vec{X}(k) = [x(k), y(k), t_x(k), t_y(k), w_x(k), w_y(k)]^T \]

Measure

\[ \vec{Z}(k) = [x_m(k), y_m(k)]^T \]
Having no access to the air turning rate, this parameter may be extracted by a Non Linear Kalman filter (EKF or UKF).

**State vector**

\[ \tilde{X}(k) = [x(k), y(k), t_x(k), t_y(k), w_x(k), w_y(k), \omega_a(k)]^T \]

\[ \tilde{X}(k + 1/k) = F[k, \tilde{X}(k/k)] = \]

\[ \begin{bmatrix} x + C_1(\omega_a).T_x(k) + C_2(\omega_a).T_y(k) + w_x.\Delta t \\ y - C_2(\omega_a).T_x(k) + C_1(\omega_a).T_y(k) + w_y.\Delta t \\ C_3(\omega_a).T_x(k) + C_4(\omega_a).T_y(k) \\ -C_4(\omega_a).T_x(k) + C_3(\omega_a).T_y(k) \\ w_x \\ w_y \\ \omega_a \end{bmatrix} \]
Results for Classical Radar: model 4

- $\omega_a$ included in the state vector
- EKF and UKF has been developed for this approach
- Both filters are not able to estimate the wind correctly.
- Filters are not able to find the part due to the wind and the one due to the error coming from the Taylor expansion.
One aircraft (two turns are needed)

\[
\begin{aligned}
    v_{1x} &= T \cdot \sin(\theta_{a1}) + w_x; & v_{1y} &= T \cdot \cos(\theta_{a1}) + w_y \\
    v_{2x} &= T \cdot \sin(\theta_{a2}) + w_x; & v_{2y} &= T \cdot \cos(\theta_{a2}) + w_y \\
    v_{3x} &= T \cdot \sin(\theta_{a3}) + w_x; & v_{3y} &= T \cdot \cos(\theta_{a3}) + w_y
\end{aligned}
\]
A closed form solution has been exhibited for this system:

Wind closed form expression

\[ w_x = \frac{(v_{3y} - v_{2y}) \cdot V_1^2 + (v_{1y} - v_{3y}) \cdot V_2^2 + (v_{2y} - v_{1y}) \cdot V_3^2}{2 \{ v_{1y} (v_{2x} - v_{3x}) + v_{2y} (v_{3x} - v_{1x}) + v_{3y} (v_{1x} - v_{2x}) \}} \]

\[ w_y = \frac{(v_{2x} - v_{3x}) \cdot V_1^2 + (v_{3x} - v_{1x}) \cdot V_2^2 + (v_{1x} - v_{2x}) \cdot V_3^2}{2 \{ v_{1y} (v_{2x} - v_{3x}) + v_{2y} (v_{3x} - v_{1x}) + v_{3y} (v_{1x} - v_{2x}) \}} \]
System Solution

True Air Speed closed form expression

$$T = \frac{||\Delta \vec{v}_{12}|| \cdot ||\Delta \vec{v}_{13}|| \cdot ||\Delta \vec{v}_{23}||}{2 \cdot (v_{1y} \cdot (v_{2x} - v_{3x}) + v_{2y} \cdot v_{3x} - v_{2x} \cdot v_{3y} + v_{1x} \cdot (v_{3y} - v_{2y}))}$$

Where $\Delta \vec{v}_{ij} = \vec{v}_i - \vec{v}_j$
Two aircraft (one turn for each aircraft is needed)

\[
\begin{align*}
va_{1x} &= T_a \sin(\theta_{a1}) + w_x; \\
va_{1y} &= T_a \cos(\theta_{a1}) + w_y \\
vb_{1x} &= T_b \sin(\theta_{b1}) + w_x; \\
vb_{1y} &= T_b \cos(\theta_{b1}) + w_y \\
va_{2x} &= T_a \sin(\theta_{a2}) + w_x; \\
va_{2y} &= T_a \cos(\theta_{a2}) + w_y \\
vb_{2x} &= T_b \sin(\theta_{b2}) + w_x; \\
vb_{2y} &= T_b \cos(\theta_{b2}) + w_y
\end{align*}
\]
A closed form solution has been exhibited for this system:

Wind extraction

\[
wx = \frac{(vb_1y - vb_2y)(Va_1^2 - Va_2^2) + (va_2y - va_1y)(Vb_1^2 - Vb_2^2)}{2 \{(va_1x - va_2x)(vb_1y - vb_2y) - (va_1y - va_2y)(vb_1x - vb_2x)\}}
\]

\[
w_y = \frac{(vb_2x - vb_1x)(Va_1^2 - Va_2^2) + (va_1x - va_2x)(Vb_1^2 - Vb_2^2)}{2 \{(va_1x - va_2x)(vb_1y - vb_2y) - (va_1y - va_2y)(vb_1x - vb_2x)\}}
\]
Hypothesis
No onboard information

Principle
1. Turns detection
2. Ground speed averaging
3. Computation of the wind expression (closed form)
Results for Classical Radar: Model 5

one aircraft

Ground turning rate estimate for the first trajectory

![Turning rate estimate graph]

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## Wind estimate

<table>
<thead>
<tr>
<th>Wind Est</th>
<th>Strength and Angle</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_x = -17.6798 m/s$</td>
<td>$39.65 kts$</td>
<td>$0.35 kts$</td>
</tr>
<tr>
<td>$w_y = -10.1831 m/s$</td>
<td>$240.053 deg$</td>
<td>$0.053 deg$</td>
</tr>
</tbody>
</table>
Two aircraft trajectories available.

Two trajectories with one turn
First aircraft (Tas=300kts)
Second aircraft (Tas=400kts)
Turning rates estimates

![Graph showing turning rates estimates for two aircraft over time. The x-axis represents time in seconds, ranging from 0 to 2500. The y-axis represents turning rate in degrees per second, ranging from -1 to 1.5. The graph plots two lines: one for the first aircraft and one for the second aircraft. For the first aircraft, the turning rate remains relatively stable with slight fluctuations, while the second aircraft shows a more dramatic change, especially around the middle of the time period.](image-url)
### Wind estimate

<table>
<thead>
<tr>
<th>Wind Est</th>
<th>Strength Angle</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_x = -17.6485 \text{ m/s}$</td>
<td>39.64 \text{kts}</td>
<td>0.36 \text{kts}</td>
</tr>
<tr>
<td>$w_y = -10.2227 \text{ m/s}$</td>
<td>239.918 \text{deg}</td>
<td>0.082 \text{deg}</td>
</tr>
</tbody>
</table>
Conclusion

Trajectory Prediction Limitations

- The trajectory prediction is a key factor for many ATM applications.
- Wind is the most critical parameter that have first to be estimated.

What did we propose?

- To use aircraft as wind sensors.
- A kalman filtering model has been developed in order to estimate the wind in real time (Mode-S radar).
- A close form expression of wind has been exhibited in the classical radar framework (based on turns).