Aircraft Local Wind Estimation From Radar Tracker Data

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Agenda

- Why wind estimation is needed for air traffic management ?
- Kalman Filtering
- Mode-S radar
- Classical radar

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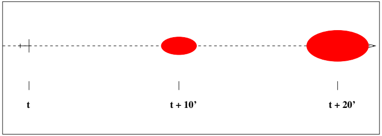
Trajectory Prediction Applications

An accurate trajectory prediction is useful for :

- conflict prediction
- landing sequencing
- airspace sector overload detection
- traffic organization
- etc ...

Trajectory Prediction Limitations



Limitation factors :

- wind (kinematic)
- temperature, pressure (engine preformance)
- aircraft weight (dynamic models)

How it works today ?

Today, a wind map is produced every 3 hours for several altitude levels which is not enough for accurate trajectory prediction.

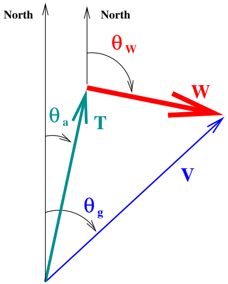
What do we propose ?

The key idea of this work is to use aircraft as sensors in order to produce and update wind maps every minute.

Remarks :

- Any aircraft when turning may be considered as a wind sensor.
- Aircraft with Mode_S transponder may downlink useful data for wind estimation.

Speed Vectors Relations



Relations

$$\vec{V} = \vec{T} + \vec{W}$$

Notations : $T = \|\vec{T}\|, V = \|\vec{V}\|, W = \|\vec{W}\|$

Turning Rates

Air Turning Rate

$$\omega_a = \frac{d\theta_a(t)}{dt}$$

Ground Turning Rate

$$\omega_g = \frac{d\theta_g(t)}{dt}$$

$$\boxed{\omega_g = \frac{\gamma_x . v_y - v_x . \gamma_y}{V^2}}$$

Relation

$$\boxed{\omega_g = \left(\frac{T^2 + T * W * \cos(\theta_a(t) - \theta_g(t))}{T^2 + W^2 + 2.T.W \cos(\theta_a(t) - \theta_g(t))} \right) \omega_a}$$

Radar used for air traffic management

Primary radar

Position measures (x,y)

Secondary radar

Position measures (x,y), mode A, mode C

Mode S radar

For this radar some on-board information may be downlinked to the ground :

- Position measures (x,y)
- Air Speed measures $(\|\vec{T}\|, \theta_a)$
- Air Turning Rate measures (ω_a)

All those measures are disturbed by noise. (additive in models)

Hypothesis

1. En route traffic is considered
2. Aircraft are supposed to fly at constant true air speed ($\|\vec{T}\|$)
3. Aircraft are supposed to turn at constant air turning rate (ω_a)

Based on those hypothesis, our objective is to find a method for extracting wind from radar measures

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Kalman Filtering : Linear Form

Linear Model

$$\begin{cases} \vec{X}(k+1) = F(k).\vec{X}(k) + G(k).(\underline{\vec{U}}(k)) + \vec{v}(k) \\ \vec{Z}(k) = H(k).\vec{X}(k) + \vec{w}(k) \end{cases}$$

Noise properties

$$\underline{\vec{U}}(k) = \vec{U}(k) + \vec{n}(k)$$

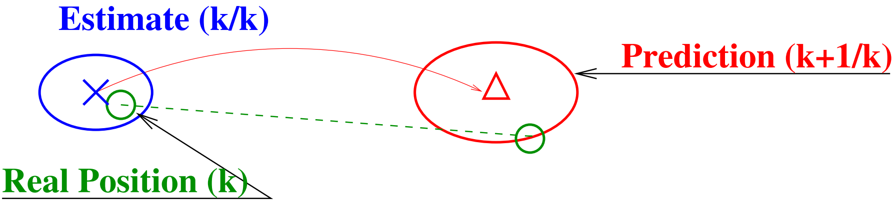
$$\begin{aligned} p(\vec{v}(k)) &\backsim \mathcal{N}(\vec{0}, Q(k)) \\ p(\vec{w}(k)) &\backsim \mathcal{N}(\vec{0}, R(k)) \\ p(\vec{n}(k)) &\backsim \mathcal{N}(\vec{0}, N(k)) \end{aligned}$$

$$\begin{array}{cc} E\left[\vec{v}(k)\right] = 0 & E\left[\vec{v}(k)\vec{v}(j)^T\right] = Q(k).\delta_{kj} \\ E\left[\vec{w}(k)\right] = 0 & E\left[\vec{w}(k)\vec{w}(j)^T\right] = R(k).\delta_{kj} \\ E\left[\vec{n}(k)\right] = 0 & E\left[\vec{n}(k)\vec{n}(j)^T\right] = N(k).\delta_{kj} \end{array}$$

Prediction Phase

$$\begin{cases} \vec{X}(k+1/k) & = & F(k).\vec{X}(k/k) + G(k).\underline{\vec{U}}(k) \\ P(k+1/k) & = & F(k).P(k/k).F(k)^T + G(k).N(k).G(k)^T + Q(k) \end{cases}$$

Prediction



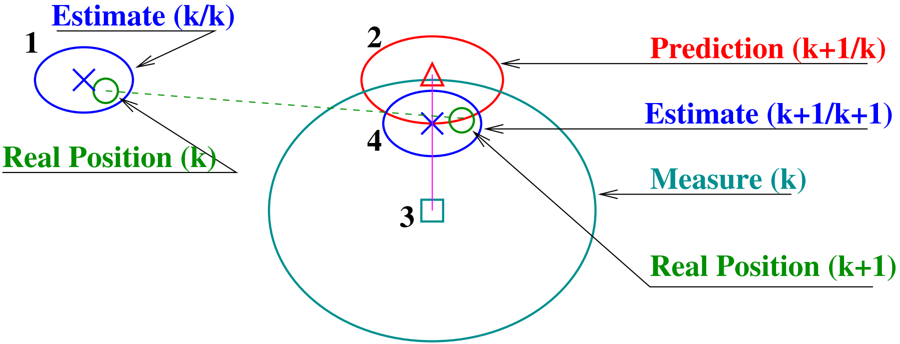
Kalman Filtering : Linear Form

Updating Phase

$$K(k+1) =$$
$$P(k+1/k).H(k+1)^T.\left[H(k+1).P(k+1/k).H(k+1)^T + R(k+1)\right]^{-1}$$

$$\begin{cases} \vec{X}(k+1/k+1) &= \vec{X}(k+1/k) + \\ & \quad K(k+1).\left[\vec{Z}(k+1) - H(k+1).\vec{X}(k+1/k)\right] \\ P(k+1/k+1) &= \left[I - K(k+1).H(k+1)\right].P(k+1/k) \end{cases}$$

Update



Estimate (k/k)
1
2
Prediction (k+1/k)
Estimate (k+1/k+1)
4
Real Position (k)
Measure (k)
3
Real Position (k+1)

Kalman Filtering : Non Linear Form (EKF)

$$
\begin{cases}
\vec{X}(k+1) = \mathcal{F}\left[k, \vec{X}(k), \underline{\vec{U}}(k)\right] + \vec{v}(k) \\
\vec{Z}(k+1) = \mathcal{H}\left[k, \vec{X}(k)\right] + \vec{w}(k)
\end{cases}
$$

Kalman Filtering : Non Linear Form

Prediction Phase

$$\begin{cases} \vec{X}(k+1/k) &= \mathcal{F}\left[k, \vec{X}(k/k), \underline{U}(k)\right] \\ P(k+1/k) &= \mathcal{F}_{\vec{X}}(k).P(k/k).\mathcal{F}_{\vec{X}}(k)^T + \\ & \quad \mathcal{F}_{\vec{U}}(k).N(k).\mathcal{F}_{\vec{U}}(k)^T + Q(k) \end{cases}$$

where $\mathcal{F}_{\vec{X}}(k)$ is the Jacobian matrix of partial derivatives of $\mathcal{F}$ with respect to the vector $\vec{X}$:

$$\mathcal{F}_{\vec{X}}(k) \hat{=} \left[\nabla_{\vec{X}}\left(\mathcal{F}\left[k, \vec{X}, \vec{U}\right]^T\right)\right]^T_{\vec{X}=\vec{X}(k/k), \vec{U}=\underline{U}(k)}$$

$$\nabla_{\vec{S}} \hat{=} \left[\frac{\partial}{\partial S_1}, \frac{\partial}{\partial S_2}, ..., \frac{\partial}{\partial S_{n_S}}\right]^T$$

Kalman Filtering : Non Linear Form

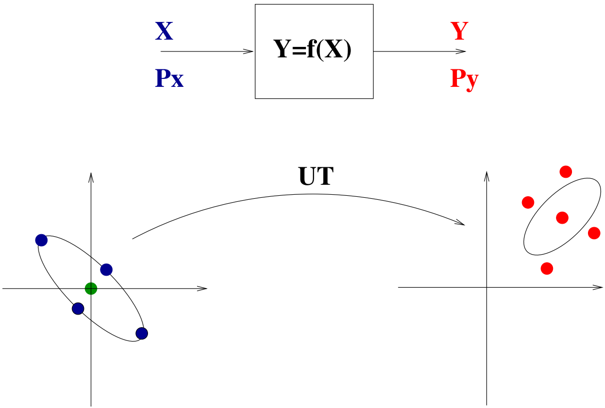
Updating Phase

$$\begin{cases} K(k+1) = P(k+1/k).\mathcal{H}_{\vec{X}}(k+1)^T. \\ \quad \left[\mathcal{H}_{\vec{X}}(k+1).P(k+1/k)\mathcal{H}_{\vec{X}}(k+1)^T + R(k+1)\right]^{-1} \\ \vec{X}(k+1/k+1) = \vec{X}(k+1/k) + K(k+1). \\ \quad \left[Z(k+1) - \mathcal{H}\left(k, \vec{X}(k+1/k)\right)\right] \\ P(k+1/k+1) = \left[I - K(k+1).\mathcal{H}_{\vec{X}}(k+1)\right].P(k+1/k) \end{cases}$$

Uncented Kalman Filter

- EKF is difficult to tune, the Jacobian can be hard to derive, and it can only handle limited amount of nonlinearity.
- PF can handle arbitrary distributions and non-linearities but is computationally very complex
- UKF gives a nice tradeoff between PF and EKF.

Uncented Transform



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Observability conditions : Mode S radar

$$\begin{cases} v_x = T\sin(\theta_a) + w_x \\ v_y = T\cos(\theta_a) + w_y \end{cases}$$

(2 equations and to unknowns)
For Mode-S radars a Kalman filter will be used in order to produce wind estimate in real time.

Hypothesis

Air Speed $(\|\vec{T}\|, \theta_a)$ and Air Turning Rate (ω_a) are both available

State vector

$$\vec{X}(k) = [x(k),\ y(k),\ t_x(k),\ t_y(k),\ w_x(k),\ w_y(k)]^T$$

Measure

$$\vec{Z}(k) = [x_m(k),\ y_m(k),\ t_{x_m}(k),\ t_{y_m}(k)]^T$$

Wind Estimation from Mode S Radar : Model 1

Having access to the air turning rate (ω_a), the system is fully linear :

$$F(k) = \begin{bmatrix} 1 & 0 & C_1(\omega_a) & C_2(\omega_a) & \Delta_t & 0 \\ 0 & 1 & -C_2(\omega_a) & C_1(\omega_a) & 0 & \Delta_t \\ 0 & 0 & C_3(\omega_a) & C_4(\omega_a) & 0 & 0 \\ 0 & 0 & -C_4(\omega_a) & C_3(\omega_a) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} C_1(\omega_a(k)) = \frac{\sin(\omega_a(k)\Delta_t)}{\omega_a(k)} \\ C_2(\omega_a(k)) = \frac{1-\cos(\omega_a(k)\Delta_t)}{\omega_a(k)} \\ C_3(\omega_a(k)) = \cos(\omega_a(k)\Delta_t) \\ C_4(\omega_a(k)) = \sin(\omega_a(k)\Delta_t) \end{cases}$$

Wind Estimation from Mode S Radar : Model 2

Hypothesis

Air Speed only available $(\|\vec{T}\|, \theta_a)$

State vector

$$\vec{X}(k) = [x(k),\ y(k),\ t_x(k),\ t_y(k),\ w_x(k),\ w_y(k)]^T$$

Measure

$$\vec{Z}(k) = [x_m(k),\ y_m(k),\ t_{x_m}(k),\ t_{y_m}(k)]^T$$

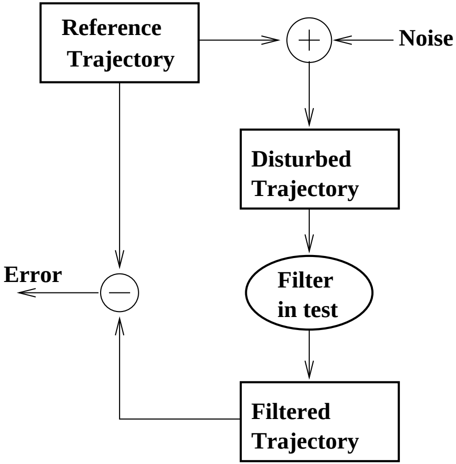
Wind Estimation from Mode S Radar : Model 2

$$F(k) = \begin{bmatrix} 1 & 0 & \Delta_t & 0 & \Delta_t & 0 \\ 0 & 1 & 0 & \Delta_t & 0 & \Delta_t \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This model is linear but is false, then the following model noise covariance matrix is included in the filter :

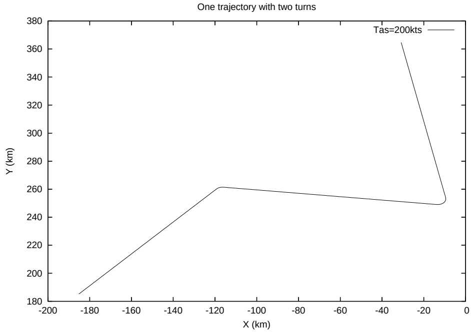
$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Simulation Framework



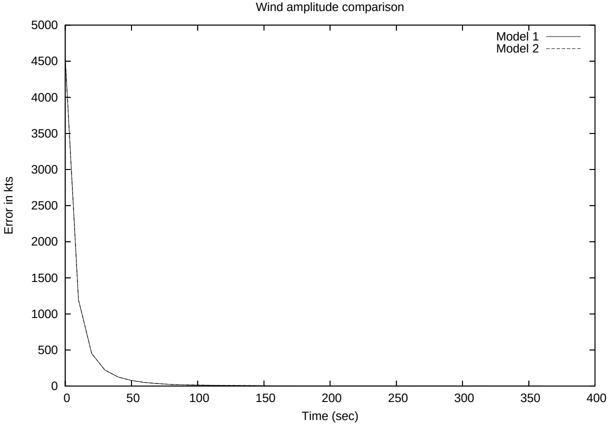
Trajectory Used for Experiments

Trajectory with 3 straight lines (20 minutes for each, $|\omega_a| = 1$ deg/sec).
Aircraft speed : 400kts

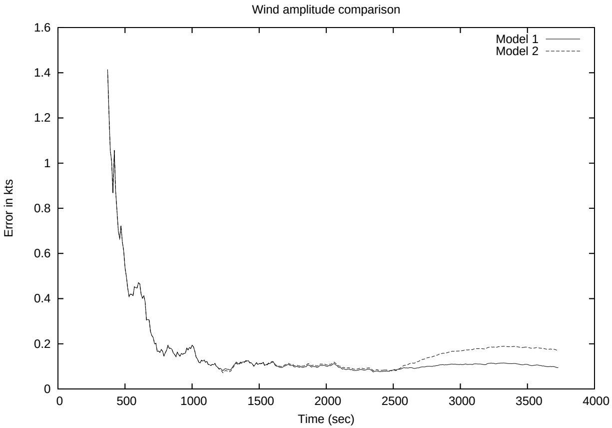


For all experiments, a wind of 40 kts has been used with $\theta_w = 240^o$

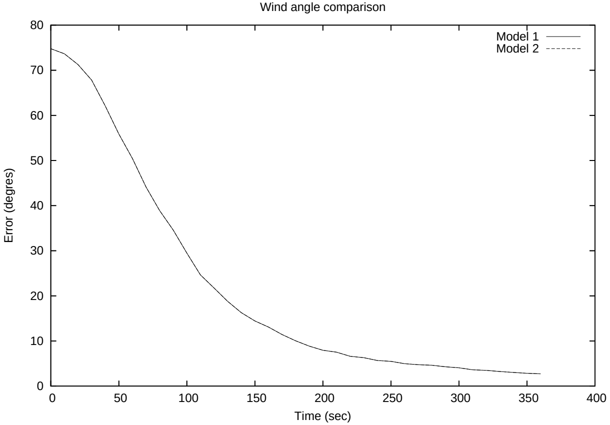
Wind strength error (models 1 and 2).
Convergence phase



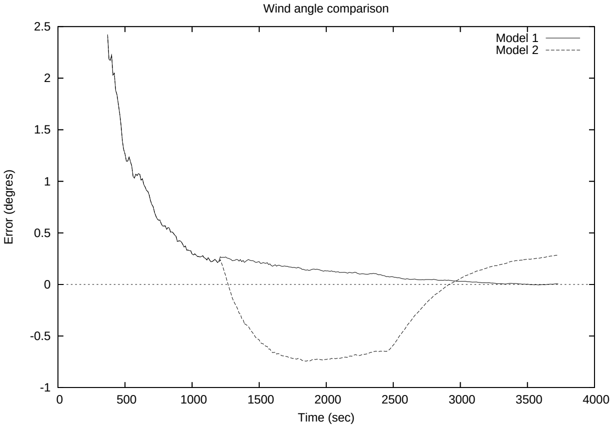
Wind strength error (models 1 and 2)



Wind angle error (model 1 and 2)
Convergence phase



Wind angle error (model 1 and 2)



Hypothesis

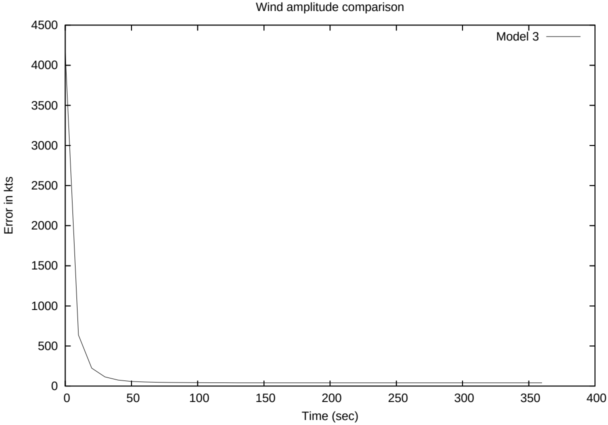
Turning rate available only (ω_a)

The same as model 1 unless for the measure equation :

$$\vec{Z}(k) = [x_m(k),\ y_m(k)]^T$$

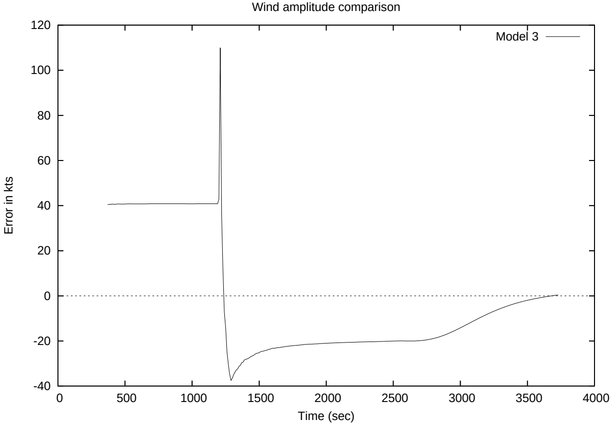
Results for Mode-S radar (model 3)

Wind Stength error:
Convergence phase



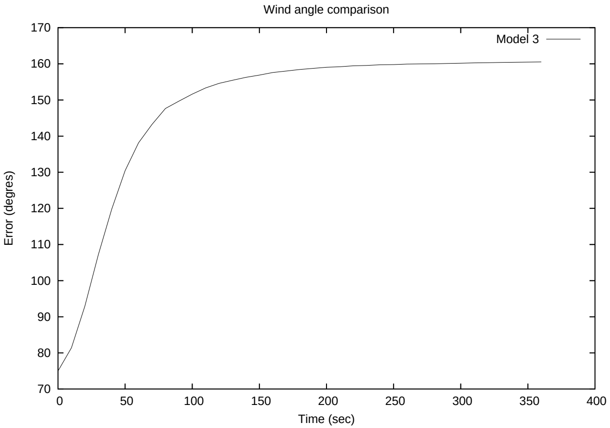
Results for Mode-S radar (model 3)

One has to wait the second turn in order to produce wind estimate.
Wind Stength error:



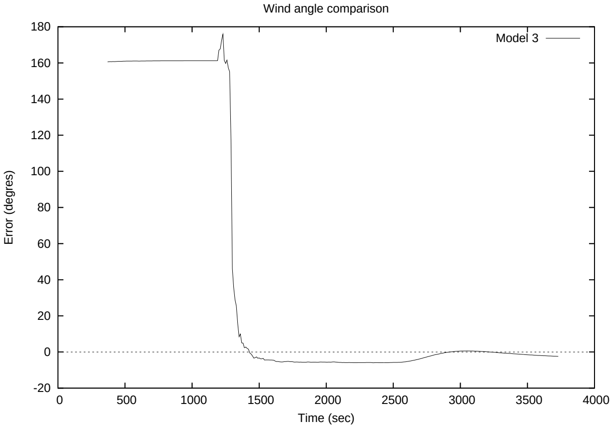
Results for Mode-S radar (model 3)

Wind Angle error:
Convergence phase



Results for Mode-S radar (model 3)

Wind angle error :



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- Why wind estimation is needed for air traffic management ?
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Wind Estimation from Standard Radar : Model 4

Hypothesis

No onboard information

State vector

$$\vec{X}(k) = [x(k),\ y(k),\ t_x(k),\ t_y(k),\ w_x(k),\ w_y(k)]^T$$

Measure

$$\vec{Z}(k) = [x_m(k),\ y_m(k)]^T$$

Having no access to the air turning rate, this parameter may be extracted by a Non Linear Kalman filter (EKF or UKF).

State vector

$$\vec{X}(k) = [x(k),\ y(k),\ t_x(k),\ t_y(k),\ w_x(k),\ w_y(k),\ \omega_a(k)]^T$$

$$\vec{X}(k+1/k) = \mathcal{F}[k, \vec{X}(k/k)] =$$

$$\begin{bmatrix} x + C_1(\omega_a).T_x(k) + C_2(\omega_a).T_y(k) + w_x.\Delta_t \\ y - C_2(\omega_a).T_x(k) + C_1(\omega_a).T_y(k) + w_y.\Delta_t \\ C_3(\omega_a).T_x(k) + C_4(\omega_a).T_y(k) \\ -C_4(\omega_a).T_x(k) + C_3(\omega_a).T_y(k) \\ w_x \\ w_y \\ \omega_a \end{bmatrix}$$

Results for Classical Radar: model 4

- ω_a included in the state vactor
- EKF and UKF has been developed for this approach
- Both filters are not able to estimate the wind correctly.
- Filters are not able to find the part due to the wind and the one due to the error coming from the Taylor expansion.

One aircraft (two turns are needed)

$$\begin{cases} v_{1x} = T.sin(\theta_{a1}) + w_x; & v_{1y} = T.cos(\theta_{a1}) + w_y \\ v_{2x} = T.sin(\theta_{a2}) + w_x; & v_{2y} = T.cos(\theta_{a2}) + w_y \\ v_{3x} = T.sin(\theta_{a3}) + w_x; & v_{3y} = T.cos(\theta_{a3}) + w_y \end{cases}$$

System Solution

A closed form solution has been exhibited for this system :

Wind closed form expression

$$w_x = \frac{(v_{3y} - v_{2y}).V_1^2 + (v_{1y} - v_{3y}).V_2^2 + (v_{2y} - v_{1y}).V_3^2}{2\left\{v_{1y}(v_{2x} - v_{3x}) + v_{2y}(v_{3x} - v_{1x}) + v_{3y}(v_{1x} - v_{2x})\right\}}$$

$$w_y = \frac{(v_{2x} - v_{3x}).V_1^2 + (v_{3x} - v_{1x}).V_2^2 + (v_{1x} - v_{2x}).V_3^2}{2\left\{v_{1y}(v_{2x} - v_{3x}) + v_{2y}(v_{3x} - v_{1x}) + v_{3y}(v_{1x} - v_{2x})\right\}}$$

System Solution

True Air Speed closed form expression

$$\boxed{T = \frac{\|\Delta\vec{v}_{12}\|.\|\Delta\vec{v}_{13}\|.\|\Delta\vec{v}_{23}\|}{2.|v_{1y}.(v_{2x}-v_{3x})+v_{2y}.v_{3x}-v_{2x}.v_{3y}+v_{1x}.(v_{3y}-v_{2y})|}}$$

Where $\Delta\vec{v}_{ij} = \vec{v}_i - \vec{v}_j$

Two aircraft (one turn for each aircraft is needed)

$$\begin{cases} va_{1x} = T_a \sin(\theta_{a_{a1}}) + w_x; & va_{1y} = T_a \cos(\theta_{a_{a1}}) + w_y \\ vb_{1x} = T_b \sin(\theta_{a_{b1}}) + w_x; & vb_{1y} = T_b \cos(\theta_{a_{b1}}) + w_y \\ va_{2x} = T_a \sin(\theta_{a_{a2}}) + w_x; & va_{2y} = T_a \cos(\theta_{a_{a2}}) + w_y \\ vb_{2x} = T_b \sin(\theta_{a_{b2}}) + w_x; & vb_{2y} = T_b \cos(\theta_{a_{b2}}) + w_y \end{cases}$$

System Solution

A closed form solution has been exhibited for this system :

Wind extraction

$$w_x = \frac{(vb_{1y} - vb_{2y})(Va_1^2 - Va_2^2) + (va_{2y} - va_{1y})(Vb_1^2 - Vb_2^2)}{2\left\{(va_{1x} - va_{2x})(vb_{1y} - vb_{2y}) - (va_{1y} - va_{2y})(vb_{1x} - vb_{2x})\right\}}$$

$$w_y = \frac{(vb_{2x} - vb_{1x})(Va_1^2 - Va_2^2) + (va_{1x} - va_{2x})(Vb_1^2 - Vb_2\|^2)}{2\left\{(va_{1x} - va_{2x})(vb_{1y} - vb_{2y}) - (va_{1y} - va_{2y})(vb_{1x} - vb_{2x})\right\}}$$

Wind Estimation from Standard Radar : Model 5

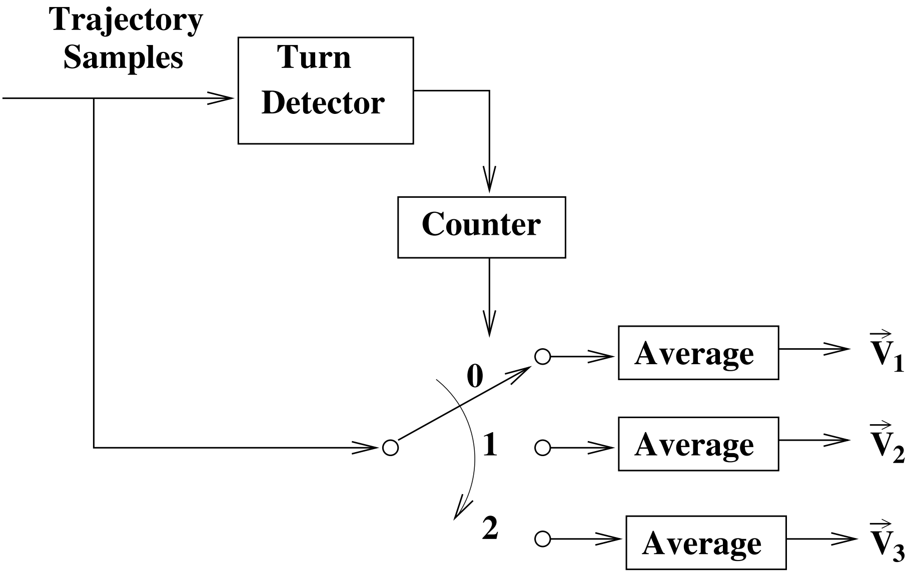
Hypothesis

No onboard information

Principle

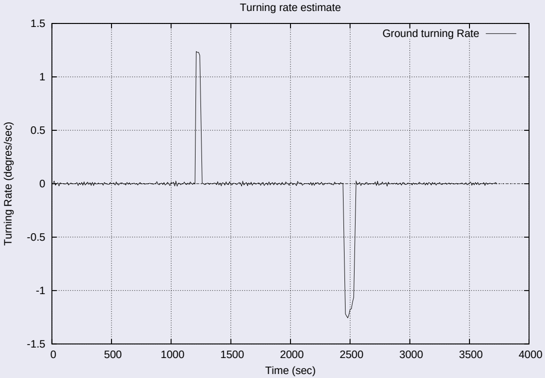
1. Turns detection
2. Ground speed averaging
3. Computation of the wind expression (closed form)

Model 5 : Framework



one aircraft

Ground turning rate estimate for the first trajectory



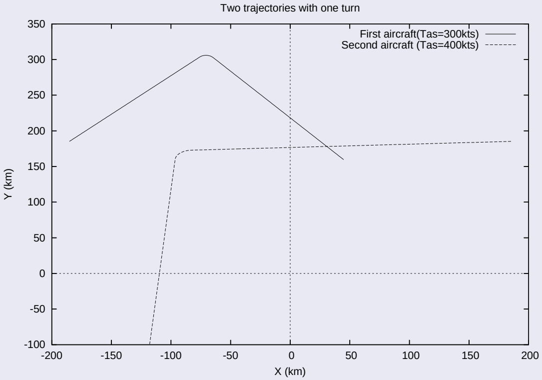
Results for Classical Radar : Model 5

Wind estimate

Wind Est	Strength and Angle	Error
$w_x = -17.6798 m/s$	$39.65 kts$	$0.35 kts$
$w_y = -10.1831 m/s$	$240.053 deg$	$0.053 deg$

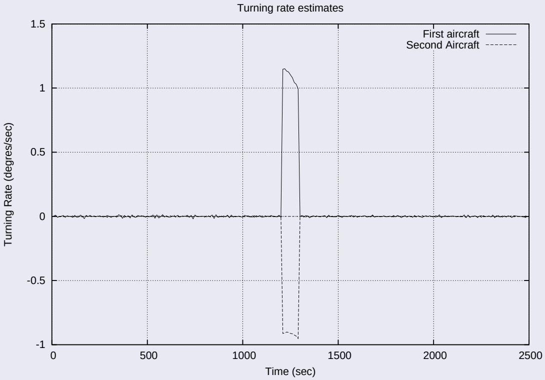
Results for Classical Radar : Model 5

Two aircraft trajectories available.



Results for Classical Radar : Model 5

Turning rates estimates



Results for Classical Radar : Model 5

Wind estimate

Wind Est	Strength Angle	Error
$w_x = -17.6485 m/s$ $w_y = -10.2227 m/s$	$39.64 kts$ $239.918 deg$	$0.36 kts$ $0.082 deg$

Conclusion

Trajectory Prediction Limitations

- The trajectory prediction is a key factor for many ATM applications
- Wind is the most critical parameter that have first to be estimated

What did we propose ?

- To use aircraft as wind sensors.
- A kalman filtering model has been developped in order to estimate the wind in real time (Mode-S radar).
- A close form expression of wind has been exhibited in the classical radar framework (based on turns).