L'inversion du tourbillon potentiel

Algorithme et applications

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Ertel (1942) Potential Vorticity I

$$DP/Dt = 0$$

where

$$P = \frac{\zeta_3 \nabla_3 \theta}{\rho^*}$$

$$\rho^* P = -\frac{\partial v}{\partial z^*}\frac{\partial \theta}{\partial x} + \frac{\partial u}{\partial z^*}\frac{\partial \theta}{\partial y} + (f + \zeta)\frac{\partial \theta}{\partial z^*} \qquad (1)$$

If the wind field $\overrightarrow{v_g} = \overrightarrow{v} - \overrightarrow{v_a}$ where $\overrightarrow{v_g}$ is the geostrophic wind and $\overrightarrow{v_a}$ the ageostrophic one.
Thermal-wind balance :

$$f\frac{\partial v_g}{\partial z^*} = \frac{g}{\theta_0}\frac{\partial \theta}{\partial x} \qquad \text{and} \qquad f\frac{\partial u_g}{\partial z^*} = -\frac{g}{\theta_0}\frac{\partial \theta}{\partial y}$$

Ertel (1942) Potential Vorticity II

$$\theta(x, y, z^*, t) = \overline{\theta}(z^*) + \tilde{\theta}(x, y, z^*, t)$$

Therefore :

$$\rho^* P = (f + \zeta)\left(\frac{\partial\overline{\theta}}{\partial z^*} + \frac{\partial\tilde{\theta}}{\partial z^*}\right) - \frac{f\theta_0}{g}\left[\left(\frac{\partial u}{\partial z^*}\right)^2 + \left(\frac{\partial v}{\partial z^*}\right)^2\right] + \frac{f\theta_0}{g}\left[\frac{\partial u}{\partial z^*}\frac{\partial u_a}{\partial z^*} + \frac{\partial v}{\partial z^*}\frac{\partial v_a}{\partial z^*}\right] \quad (2)$$

Potential Vorticity scaling I

$\mathcal{M} = [\mathcal{M}]\mathcal{M}'$
$[\mathcal{M}]$ is a scaling of the corresponding variable and $\mathcal{M}'$ the variable without dimension. The scaling for the subset of independent variables is :

- $[U] = U$
- $[x] = [\delta x] = [y] = [\delta y] = L$,
- $[z^*] = [\delta z^*] = H$,
- $N^2 = g/\theta_o \, [\partial\theta/\partial z^*]$ where N^2 is the brunt-vaissala frequency of the reference flow.

Then,

- $[\zeta] = U/L$
- $[\partial U/\partial z^*] = U/H$
- $\left[\partial\tilde{\theta}/\partial z^*\right] = f\theta_0 LU/gH^2$.

Potential Vorticity scaling II

Using the Rossby $R = U/fL$ and Froude $F = U/NH$ numbers the non-dimensional form of the Potential Vorticity equation is :

$$\begin{aligned}
\frac{g}{f\theta_0 N^2}{\rho^*}' P' &= \frac{\partial\overline{\theta'}}{\partial {z^*}'} + F^2 R^{-1}\frac{\partial\tilde{\theta}'}{\partial {z^*}'} + R\zeta'\frac{\partial\overline{\theta'}}{\partial {z^*}'} + F^2\zeta'\frac{\partial\tilde{\theta}'}{\partial {z^*}'} \\
&- F^2\left(\frac{\partial u'}{\partial {z^*}'}\right)^2 + F^2 R\frac{\partial u'}{\partial {z^*}'}\frac{\partial u'_a}{\partial {z^*}'} \\
&- F^2\left(\frac{\partial v'}{\partial {z^*}'}\right)^2 + F^2 R\frac{\partial v'}{\partial {z^*}'}\frac{\partial v'_a}{\partial {z^*}'}. \qquad (3)
\end{aligned}$$

small Rossby and $F \simeq R^{1/2}$ I

$$\frac{g}{f\theta_0 N^2}\rho^{*\prime}P' = \frac{\partial\overline{\theta'}}{\partial z^{*\prime}} + \frac{\partial\tilde{\theta}'}{\partial z^{*\prime}} + R^1\zeta'\frac{\partial\overline{\theta'}}{\partial z^{*\prime}} + R^1\zeta'\frac{\partial\tilde{\theta}'}{\partial z^{*\prime}} - R^1\left(\frac{\partial u'}{\partial z^{*\prime}}\right)^2 - R^1\left(\frac{\partial v'}{\partial z^{*\prime}}\right)^2. \qquad (4)$$

The R^1 expansion of the left-hand side of this equation is now non-linear.

$$\phi' = f(\theta') \text{ hydrostatic relation}$$
$$\phi' = \psi' \text{ geostrophic relation}$$
$$(5)$$

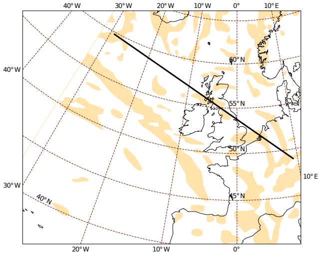
small Rossby and $F \simeq R^{1/2}$ II

Ertel PV is now proportional to the quantity Q where :

$$Q = (f^2 + \nabla^2\phi)\phi_{zz} - \phi_{xz}^2 - \phi_{yz}^2$$

which belongs to the Monde-Ampere family.

ellipticity criterion I



The shaded area corresponds to atmosphere columns with potential vorticity values less than -0.01 pvu

Linear PV inversion without specifying any balance conditions between velocity and mass I

at small Rossby and Froude/$O(1)$ expansion :

$$\frac{g}{f\theta_0 N^2}\rho^{*\prime}P' = \frac{\partial\overline{\theta'}}{\partial z^{*\prime}} + R^1\frac{\partial\tilde{\theta}'}{\partial z^{*\prime}} + R^1\zeta'\frac{\partial\overline{\theta'}}{\partial z^{*\prime}}. \quad (6)$$

With physical variables :

$$P = -g\left((f_0+\zeta)\frac{\partial\bar{\theta}}{\partial p} + f_0\frac{\partial\theta}{\partial p}\right)$$

For a single layer defined by two isentropic surfaces with uniform density ($\Delta\theta = \theta_U - \theta_b$) whose upper boundary height ϕ_U or pressure $_U$ is allowed to vary whereas the bottom $\phi_B/_B$ is fixed.

Linear PV inversion without specifying any balance conditions between velocity and mass II

Δp is defined as the mean depth of the fluid and δp a small amplitude deviation :

$$P = -g\left(\zeta\frac{\Delta\theta}{\Delta p} + f_0\frac{\Delta\theta}{\Delta p + \delta p}\right)$$

At the first order :

$$P = -g\frac{\Delta\theta}{\Delta p}\left(f_0 + \zeta - f_0\frac{\delta p}{\Delta p}\right)$$

$$P = -g\frac{\Delta\theta}{\Delta p}\left(f_0 + \frac{\Delta\psi}{f_0} - f_0\frac{\phi}{\Delta\phi}\right)$$

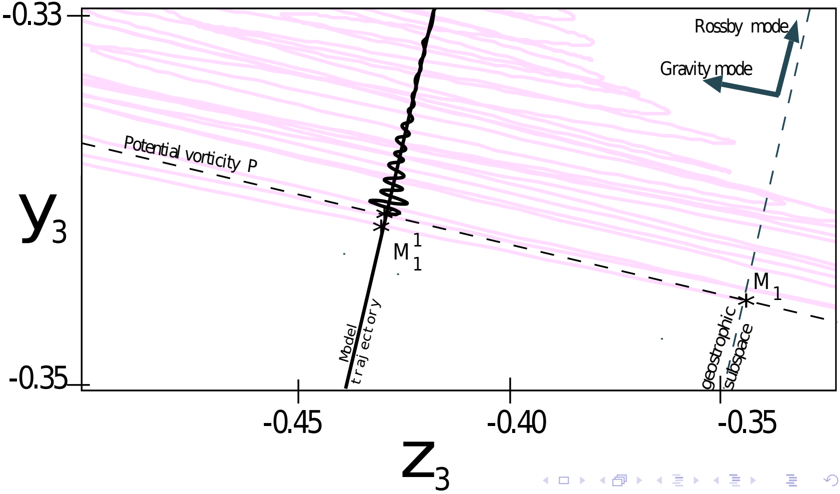
Linear PV inversion without specifying any balance conditions between velocity and mass III

The variable χ is expanded as a sum of $\chi_k e^{ikx}$ functions,the k^{th} component of the expansion of P following the same way writes :

$$P_k = -g\frac{\Delta\theta}{\Delta p}\left(f_0 - \frac{-k^2\psi_k}{f_0} - f_0\frac{\phi_k}{\Delta\phi}\right)$$

Lorenz (1980) model which is based on 3 wave numbers.

Linear PV inversion without specifying any balance conditions between velocity and mass IV



Linear PV inversion using a minimum energy constraint ; A variational approach of PV inversion I

$$P = -f_0 g \frac{\partial \theta}{\partial p} - g \frac{\partial \bar{\Theta}}{\partial p} \zeta \qquad (7)$$

The heart of the inversion is based on the minimization of :

$$J = \int_\Omega \left\{ \frac{1}{2} \zeta \Delta^{-1} \zeta - \sigma \frac{1}{2} \frac{\partial \bar{\Theta}}{\partial p}^{-1} \theta^2 \right\} d\Omega - \int_\Omega \Lambda \left(-f_0 g \frac{\partial \theta}{\partial p} - g \frac{\partial \bar{\Theta}}{\partial p} \zeta - P \right) d$$

$$\sigma = -\frac{R}{p} \left(\frac{p}{p_0} \right)^{R/C_p}$$

Linear PV inversion using a minimum energy constraint ; A variational approach of PV inversion II

The saddle point of J obeys to :

$$\frac{\partial J}{\partial \Lambda} = 0 \ , \ \frac{\partial J}{\partial \theta} = 0 \ , \ \frac{\partial J}{\partial \zeta} = 0 \text{ and } \frac{\partial J}{\partial D} = 0$$

leading to the following Euler-Lagrange equations :

$$P = -f_0 g \frac{\partial \theta}{\partial p} - g \frac{\partial \bar{\Theta}}{\partial p} \zeta \tag{9}$$

$$\zeta = g \frac{\partial \bar{\Theta}}{\partial p} \Delta \Lambda \tag{10}$$

$$\theta = g f_0 \sigma^{-1} \frac{\partial \bar{\Theta}}{\partial p} \frac{\partial \Lambda}{\partial p} \tag{11}$$

Linear PV inversion using a minimum energy constraint ; A variational approach of PV inversion III

Eqs.(9)-(11) can be rewritten as follows :

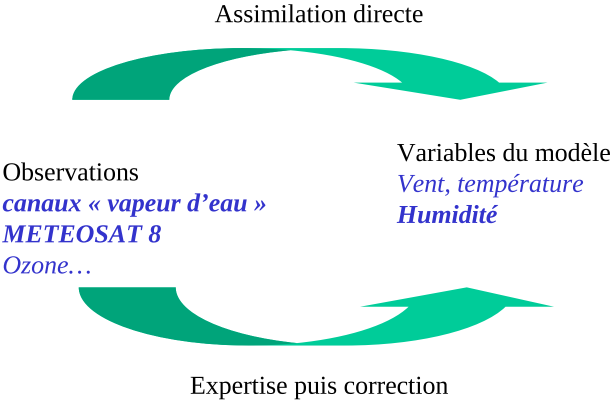
$$\frac{\Delta\Lambda}{f_0} + f_0 \frac{\partial\bar{\Theta}}{\partial p}^{-2} \frac{\partial}{\partial p}\left(\sigma^{-1}\frac{\partial\bar{\Theta}}{\partial p}\frac{\partial\Lambda}{\partial p}\right) = -\frac{1}{f_0 g^2}\frac{\partial\bar{\Theta}}{\partial p}^{-2} P \tag{12}$$

A variational approach of non-linear PV inversion I

$$P = -f_0 g \frac{\partial \theta}{\partial p} - g \frac{\partial \bar{\Theta}}{\partial p} \zeta + \text{non-linear terms} \tag{13}$$

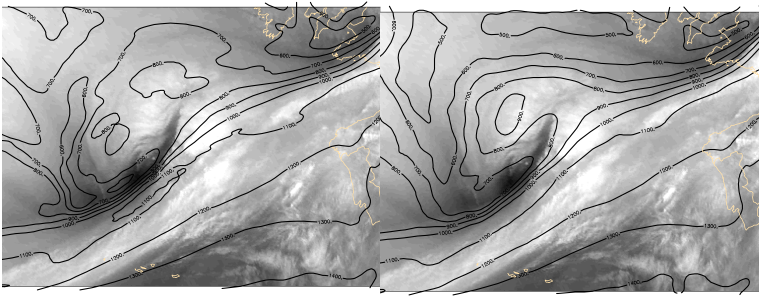
$$\begin{aligned} J = & \int_\Omega \left\{ \frac{1}{2}(\zeta - \zeta_i)\Delta^{-1}(\zeta - \zeta_i) - \sigma \frac{1}{2} \frac{\partial \bar{\Theta}}{\partial p}^{-1} (\theta - \theta_i)^2 \right\} d\Omega \\ & - \int_\Omega \Lambda \left(-f_0 g \frac{\partial \theta}{\partial p} - g \frac{\partial \bar{\Theta}}{\partial p} \zeta - P \right) d\Omega. \end{aligned} \tag{14}$$

Applications à l’expertise humaine

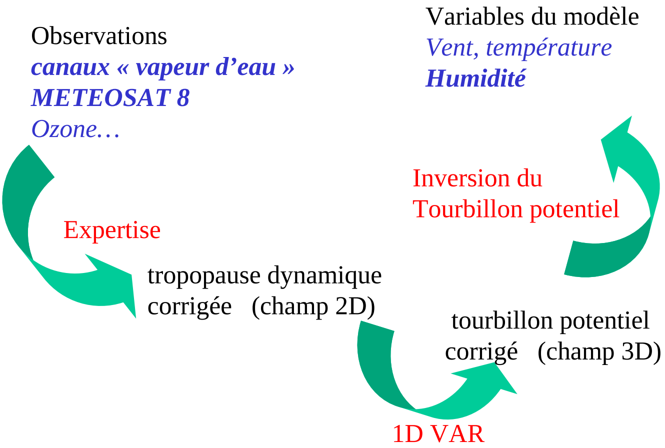


Applications à l'expertise humaine

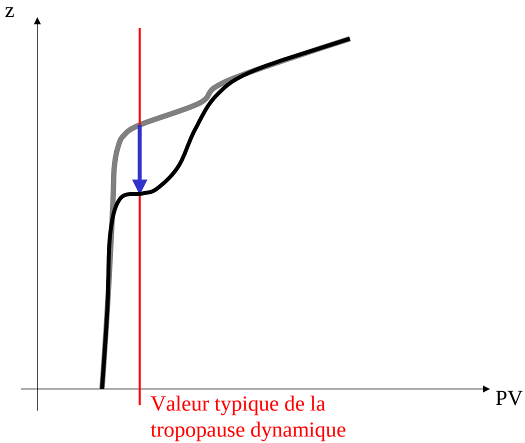
A gauche : avant modifications. A droite : après modifications



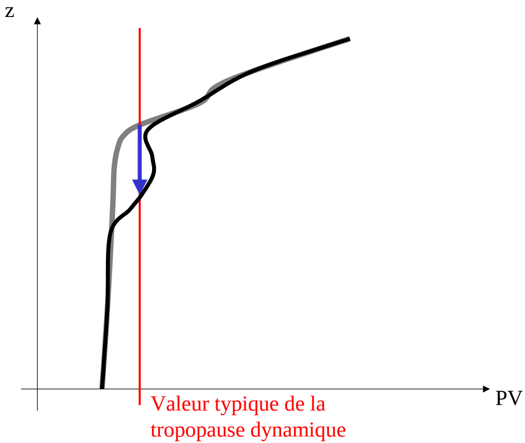
Applications à l'expertise humaine



Reconstruction de profil : ce que l'on veut



Après analyse 1DVAR d'une observation



Applications à l’expertise humaine

A gauche : avant modifications. A droite : après modifications

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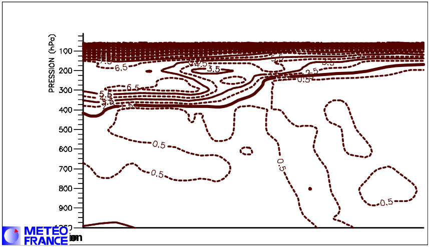

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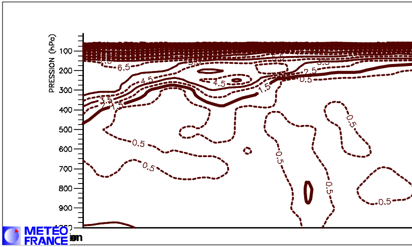
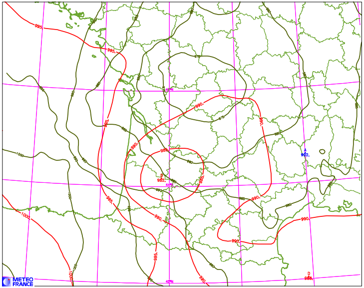


Applications à l'expertise humaine

PMER (à gauche : avant modifications. A droite : après modifications)

Pmer sam 24/01/2009 09:00 Sol SY,RD,R6,Sy,SH,BU,BO,ME,SP,ST,SB,AL,EA,ER,JA,xx



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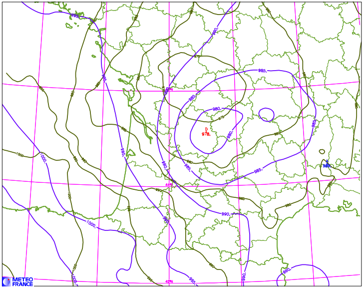
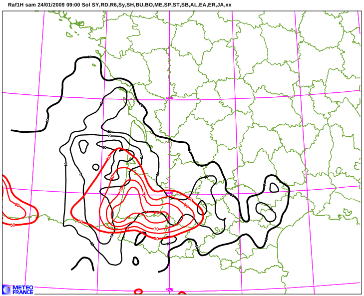
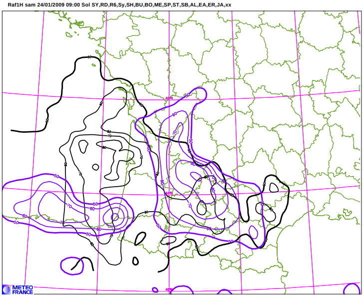


Applications à l'expertise humaine

Rafales (à gauche : avant modifications. A droite : après modifications)

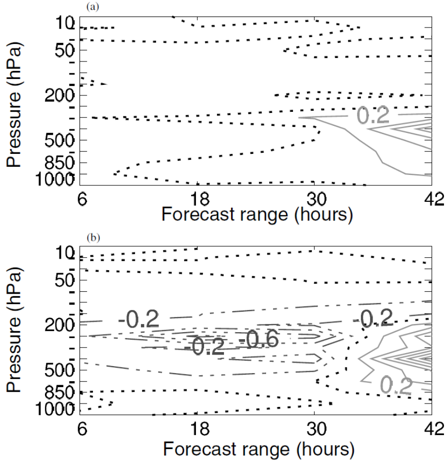




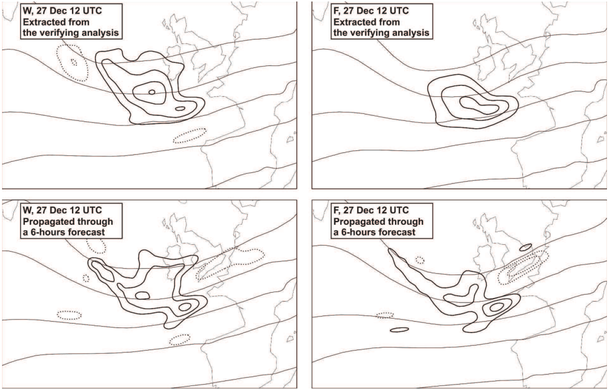
Assimilation of corrections as pseudo-observations within 4DVAR I

PV Correction considered as observations and assimilated (Guérin et al. 2005) Main point : estimation of observation error

Assimilation of corrections as pseudo-observations within 4DVAR II



Coherent structure depiction using wavelets (Plu et al. 2008) I



Perspectives I

- Non-linéarités
- Eventuels problèmes de conditionnement
- Quelles approches pour traiter les forts gradients ?