Separability criteria
for high dimensional bipartite quantum states

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Outline

1. Introduction
2. Mean-width of the sets of separable and k-extendible states
3. Separability and k-extendibility of random states
4. Summary

Mathematical formalism of quantum physics in a nutshell

- **State of a quantum system :** Positive and trace 1 operator ρ (*density operator*) on a Hilbert space H (*state space*).
  Finite number n of free parameters $\to \mathrm{H} \equiv \mathbf{C}^n$.

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- **Separability vs Entanglement :** A bipartite quantum state is *separable* if it may be written as a convex combination of product states. Otherwise, it is *entangled*.
- **Reduced state :** For ρ_{AB} a state on $\mathrm{A} \otimes \mathrm{B}$, its *reduced state on* A is the partial trace $\rho_A = \mathrm{Tr}_\mathrm{B}\, \rho_{AB}$.

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Solution : Find set of states which are easier to characterize and which contain the set of separable states.
$\rightarrow$ Necessary conditions for separability that have a simple mathematical description and that may be checked efficiently on a computer (e.g. by a semi-definite programme).

The k-extendibility criterion for separability (1)

Definition (k-extendibility)

Let $k \geqslant 2$. A state ρ_{AB} on $\mathrm{A} \otimes \mathrm{B}$ is k-extendible with respect to B if there exists a state ρ_{AB^k} on $\mathrm{A} \otimes \mathrm{B}^{\otimes k}$ which is invariant under any permutation of the B subsystems and such that $\rho_{AB} = \mathrm{Tr}_{\mathrm{B}^{k-1}} \rho_{AB^k}$.

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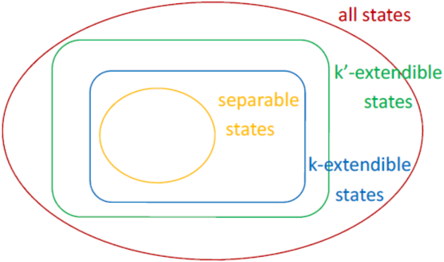
Proof idea :

• "ρ_{AB} separable $\Rightarrow$ ρ_{AB} k-extendible w.r.t. B for all $k \geqslant 2$" is obvious since $\sigma_A \otimes \tau_B = \mathrm{Tr}_{\mathrm{B}^{k-1}} \left[\sigma_A \otimes \tau_B^{\otimes k} \right]$.

• "ρ_{AB} k-extendible w.r.t. B for all $k \geqslant 2 \Rightarrow \rho_{AB}$ separable" relies on the quantum De Finetti theorem (Christandl/König/Mitchison/Renner).

The k-extendibility criterion for separability (2)

Observation : ρ_{AB} k-extendible w.r.t. B $\Rightarrow$ ρ_{AB} k'-extendible w.r.t. B for $k' \leqslant k$.
$\rightarrow$ Hierarchy of NC for separability, which an entangled state is guaranteed to stop passing at some point.



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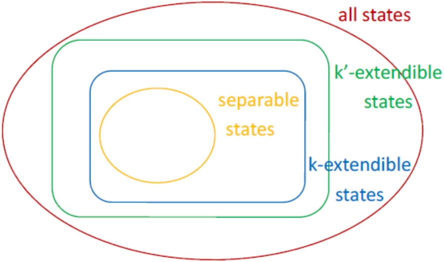

Problem : For a given $k \geqslant 2$, how "close" to the set of separable states is the set of k-extendible states ? how "powerful" is the k-extendibility NC for separability ?

Reminder about Gaussian and Wishart matrices

Definitions (Gaussian Unitary and Wishart ensembles)

• G is a $n \times n$ GUE matrix if $G = (H + H^{\dagger})/2$ with H a $n \times n$ matrix having independent complex normal entries.
• W is a (n, s)-Wishart matrix if $W = HH^{\dagger}$ with H a $n \times s$ matrix having independent complex normal entries.

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$$\mathrm{d}\mu_{SC(\sigma^2)}(x) = \frac{1}{2\pi\sigma^2}\sqrt{4\sigma^2 - x^2}\,\mathbf{1}_{[-2\sigma,2\sigma]}(x)\mathrm{d}x$$

$$\mathrm{d}\mu_{MP(\lambda)}(x) = \left(1 - \frac{1}{\lambda}\right)_+ \delta_0 + \frac{\sqrt{(\lambda^+ - x)(x - \lambda^-)}}{2\pi\lambda x}\mathbf{1}_{[\lambda^-,\lambda^+]}(x)\mathrm{d}x,\ \lambda^\pm = (\sqrt{\lambda} \pm 1)^2$$

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Link with random matrices : When $n \to +\infty$, the spectral distribution of a $n \times n$ GUE matrix (rescaled by $\sqrt{n}$) converges to $\mu_{SC(1)}$, and that of a $(n, \lambda n)$-Wishart matrix (rescaled by λn) converges to $\mu_{MP(\lambda)}$.

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Mean-width of a set of states

Definitions

Let K be a convex set of states on $\mathbf{C}^n$ containing Id/n.

• For a $n \times n$ Hermitian Δ, the *width of K in the direction* Δ is $w(K,\Delta) = \sup_{\sigma \in K} \mathrm{Tr}(\Delta(\sigma - Id/n))$.

• The *mean-width of K* is the average of $w(K,\cdot)$ over the Hilbert-Schmidt unit sphere of $n \times n$ Hermitians, equipped with the uniform probability measure.

It is equivalently defined as $w(K) = \mathbf{E}\, w(K,G)/\gamma_n$, where G is a $n \times n$ GUE matrix and $\gamma_n = \mathbf{E}\,\|G\|_{HS} \sim_{n \to +\infty} n$.

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Theorem (Wigner's semicircle law)

On $\mathbf{C}^n$, the mean-width of the set of all states is asymptotically $2/\sqrt{n}$.

Mean-width of the set of separable states

Theorem (Aubrun/Szarek)

Denote by $\mathcal{S}$ the set of separable states on $\mathbf{C}^d \otimes \mathbf{C}^d$.
There exist universal constants c, C such that $c/d^{3/2} \leqslant w(\mathcal{S}) \leqslant C/d^{3/2}$.

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Proof idea :

• Upper-bound : Approximate $\mathcal{S}$ by a polytope with "few" vertices, and use that $\mathbf{E} \sup_{i \in I} Z_i \leqslant C\sqrt{\log |I|}$ for $(Z_i)_{i \in I}$ a finite bounded Gaussian process (Pisier).
• Lower-bound : Estimate the *volume-radius* of $\mathcal{S}$ by "geometric" considerations, and use that $vrad \leqslant w$ (Urysohn).

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Fix $k \geqslant 2$ and denote by $\mathcal{E}_k$ the set of k-extendible states on $\mathbf{C}^d \otimes \mathbf{C}^d$.
Asymptotically, $w(\mathcal{E}_k) = 2/\sqrt{k}d$.

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Proof strategy : $\sup_{\sigma\ k-\mathrm{ext}} \mathrm{Tr}(G(\sigma - Id/d^2))$ may be expressed as $\|\widetilde{G}\|_\infty$ for some suitable $\widetilde{G}$. So one has to estimate $\mathbf{E}\,\|\widetilde{G}\|_\infty$ for the "modified" GUE matrix $\widetilde{G}$. This is done by computing the p-order moments $\mathbf{E}\,\mathrm{Tr}\,\widetilde{G}^p$, and identifying the limiting spectral distribution (after rescaling by d/k) : a centered semicircular distribution $\mu_{SC(k)}$. The latter has $2\sqrt{k}$ as upper-edge.

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Random induced states

System space $\mathrm{H} \equiv \mathbf{C}^n$. Ancilla space $\mathrm{H}' \equiv \mathbf{C}^s$.
Random mixed state model on H : $\rho = \mathrm{Tr}_{\mathrm{H}'} |\psi\rangle\langle\psi|$ with $|\psi\rangle$ a uniformly distributed pure state on $\mathrm{H} \otimes \mathrm{H}'$ (*quantum marginal*).

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Question : Fix $d \in \mathbf{N}$ and consider ρ a random state on $\mathbf{C}^d \otimes \mathbf{C}^d$ induced by some environment $\mathbf{C}^s$.
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"typically" = "with overwhelming probability as d grows". Hence 2 steps :
(i) Identify the range of s where ρ is, on average, separable/k-extendible.
(ii) Show that the average behaviour is generic in high dimension (concentration of measure).

Separability of random induced states

Theorem (Aubrun/Szarek/Ye)

Let ρ be a random state on $\mathbf{C}^d \otimes \mathbf{C}^d$ induced by $\mathbf{C}^s$. There exists a threshold s_0 satisfying $cd^3 \leqslant s_0 \leqslant Cd^3 \log^2 d$ for some constants c, C such that, if $s < s_0$ then ρ is typically entangled, and if $s > s_0$ then ρ is typically separable.

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Intuition : If $s \leqslant d^2$ then ρ is uniformly distributed on the set of states of rank at most s, therefore generically entangled. If $s \gg d^2$ then ρ is expected to be close to Id/d^2, therefore separable.
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Proof idea : Convex geometry + Comparison of random matrix ensembles.

k-extendibility of random induced states

Theorem

Let ρ be a random state on $\mathbf{C}^d \otimes \mathbf{C}^d$ induced by $\mathbf{C}^s$. If $s < \frac{(k-1)^2}{4k} d^2$ then ρ is typically not k-extendible.

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Proof strategy : If $\sup_{\sigma\ k-\text{ext}} \operatorname{Tr}(\rho\sigma) < \operatorname{Tr}(\rho^2)$, then ρ is not k-extendible. To identify when such is the case, one should characterize when $\mathbf{E} \sup_{\sigma\ k-\text{ext}} \operatorname{Tr}(W\sigma) < \mathbf{E} \operatorname{Tr}(W^2) / \mathbf{E} \operatorname{Tr} W$ for W a (d^2, s)-Wishart matrix.

- RHS : In the limit $d, s \to +\infty$, $\mathbf{E} \operatorname{Tr}(W^2) = d^4 s + d^2 s^2$ and $\mathbf{E} \operatorname{Tr} W = d^2 s$.
- LHS : Write $\sup_{\sigma\ k-\text{ext}} \operatorname{Tr}(W\sigma) = \|\widetilde{W}\|_\infty$, and estimate $\mathbf{E} \|\widetilde{W}\|_\infty$ for the "modified" Wishart matrix $\widetilde{W}$. This may be done by computing the p-order moments $\mathbf{E} \operatorname{Tr} \widetilde{W}^p$, and identifying the limiting spectral distribution (after rescaling by s/k) : a Marčenko-Pastur distribution $\mu_{MP(ks/d^2)}$. The latter's support has $(\sqrt{ks/d^2} + 1)^2$ as upper-edge.
- For $s < (k-1)^2 d^2 / 4k$, $(\sqrt{ks/d^2} + 1)^2 < (d^2 + s) k / s$.

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- What happens when k is not fixed, but instead grows with d ?

A few references

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