Mini-courses

P. Cardaliaguet
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Title. *Introduction to the homogenization of Hamilton-Jacobi equations.*
Abstract. Hamilton-Jacobi equations with periodic or random coefficients are used, for instance, in the formalization of fronts propagating in inhomogeneous media. After introducing the notion of viscosity solutions of Hamilton-Jacobi equations, we will discuss homogenization in periodic and random media for these equations. Then we will explicit estimates for the error between the oscillating solution and its homogenized limit. It turns out that, in the random setting, the techniques used for this analysis have very much to do with methods developed for “first passage percolation models”. The results discussed so far rely in a crucial way on a “coercivity condition” on the Hamiltonian; for instance, in the front propagation problems, this condition means that the region enclosed by the front is increasing. We will complete the presentation by the analysis of the G-equation, which is a simple model for flame propagation where the coercivity condition does not hold.

S. Méléard
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Title. *Modeling in Adaptive Biology*
Abstract. We present different large population scalings of an individual-based model with mutation and selection. Depending on the scaling, we will obtain either non linear integro-differential equations or generalized Fisher-KPP (fractional) equations or measure-valued stochastic processes. We will emphasize the modeling of adaptive dynamics where biological assumptions yield a separation of time scales and the appearance of evolutionary phenomena.

L. Ryzhik
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Title. *Fisher-KPP: steady states, invasions and Bramson delay in periodic media.*
Abstract. The Fisher-KPP equation is probably the simplest model of biological invasions that proved capable over nearly 80 years of both leading to interesting mathematics and producing interesting “ballpark” predictions in applications. In these introductory
lectures we will describe the qualitative behavior of its solutions in spatially periodic media. The main focus will be on the existence of periodic steady states, the Freidlin-Gartner formula for the invasion speed and the Bramson long time asymptotics for the Fisher-KPP front position.

D. Slepcev
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Title: Nonlocal-interaction equations and models of collective behaviour.
Abstract. The aim of these lectures is to present a brief introduction to nonlocal-interaction equations and their application to modeling collective behavior of interacting agents.

Introduction. Agents in nature sense each other at a distance via sight, sound and adjust their movement accordingly. We will start with the simplest models of collective behavior where the agents interact at a distance via pairwise potentials. The interaction is typically attractive at large distances, and repulsive at short distances. Nonlocal-interaction equations arise as a continuum description of these systems and serve as one of the basic models of biological aggregation.

Basic features of nonlocal-interaction equations. We will discuss properties of nonlocal-interaction equations, their gradient flow structure with respect to the Wasserstein metric and weak measure solutions which allow simultaneous treatment of discrete and continuum models.

After introducing the basic theory the lectures will focus on a variety of phenomena observed and the mathematical structures that arise in understanding them.

Existence and stability of steady states. The systems with long-range attractive and short-range repulsive interactions exhibit steady states of a variety of shapes and patterns. We will introduce sharp conditions on the existence of steady states and discuss their connections to statistical mechanics (Ruelle’s H-stability). We will discuss nonlinear stability of steady states and how it depends on the properties of the interaction potential.

Nonlocal equations with nonlinear diffusion. In some cases the anti-crowding mechanism is modeled by a nonlinear diffusion, instead of the short-range repulsion. We show that this can lead to phase separation: the density profile develops interfaces between a near-constant-density aggregate state and the empty space. We will explain why the interfaces evolve under surface-tension-like “forces”.

Evolution in heterogeneous environments and environments with boundaries. We will consider systems in which the mobility of individuals depends on their location. The space dependent mobility gives a Riemannian structure to the underlying space which leads us to study the equations on manifolds. This raises a number of challenges but also gives rise to new and interesting phenomena. For example if there is a steady state suddenly affected by a unidirectional potential (say the individuals all learn where the food is) in the homogeneous environment this results in an orderly translation, while in some natural heterogeneous environments this leads to rolling swarms, not unlike those seen in swarming locust.

Refined models for interaction. We will discuss a new model for biological aggregation in which the response of individuals to stimuli depends on the whole configuration. It takes
into account that the influence of individual an B on an individual A depends on whether there are other individuals near A. In particular its influence will be diminished if there are other individuals near A. What is surprising about this model is that it still has a gradient flow structure as the original model. We will discuss properties of solutions and consequence of the geometric nature of the equation.

O. Zeitouni  
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Title. Classical and modified models of branching brownian motions.  
Abstract. I will develop the second moment method in the context of branching random walks and branching brownian motions, and show how it yields classical results concerning fluctuation of the front, as well as modifications needed for handling different models of changing environments.

Research talks

G. Barles  
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Title. Reaction-Diffusion Equations and Front Propagations Problems.  
Abstract. The large time behavior of reaction - diffusion equations and particle systems is generally mainly described in terms of the front propagations they generate. The aim of this talk is to present several types of arguments which were used to rigourously establish the connections between these equations/systems and the ?geometrical? law of propagation of the associated moving fronts (fronts with constant normal velocities, motion by mean curvature...). This will be done through three basic examples : KPP with Large Deviations arguments, ZFK (a not so well understood case) and Allen-Cahn with the “geometrical approach” based on the famous “level sets approach”.

J. Berestycki  
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Title. BBM with absorption and critical drift.  
Abstract. From joint works with N. Berestycki and J. Schweinsberg. We consider branching Brownian motion on the real line with absorption at zero, in which particles move according to independent Brownian motions with the critical drift of $2^{1/2}$. Kesten (1978) showed that almost surely this process eventually dies out. I will present results concerning the behavior of the process before the extinction time, as the position x of the initial particle tends to infinity. We first obtain bounds on the probability of survival until time $t$ large which improve upon results of Kesten (1978). We then study how the system behaves during its life-time (number of particles, position of the right-most particle, typical configuration of particles).
The probability of survival of this system satisfies a one-sided KPP equation with boundary condition in which the unstable phase invades the stable one. The existence of a limiting front (whose position at time \( t \) would be at \( ct^{1/3} \)) is open.

**N. Berestycki**
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**Title.** Shape of multidimensional Brunet–Derrida particle systems.

**Abstract.** In this talk I will consider particle systems in one or more dimensions in which particles perform branching Brownian motion and the population size is kept constant equal to \( N \), through the following selection mechanism: at all times only the \( N \) fittest particles survive, while all the other particles are removed. Fitness is measured with respect to some given score function. I will present results, obtained jointly with Lee Zhuo Zhao, on the asymptotic motion and shape of the resulting cloud of particles. I will also discuss a number of open problems and the relationship to some questions in biology concerning the role of recombination.

**J. Carrillo**
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**Title.** Stability and Pattern formation in Nonlocal Interaction Models

**Abstract.** I will review some recent results for first and second order models of swarming in terms of patterns, stationary states, and qualitative properties. I will discuss the stability of these patterns for the continuum and discrete particle cases. These non-local models appear in collective behavior for animals, control engineering, and molecular structures among others. We first concentrate in the spatial shape of these patterns and the dynamics when inertia terms are neglected. The mathematical question behind consists in finding properties about local minimizers of the total interaction energy. Concerning 2nd order models, we will discuss particular properties of two patterns: flocks and mills. We will discuss the stability of these patterns in the discrete case. In both cases, we will describe the properties obtained for the continuum limits.

**F. Hamel**
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**Title.** Bistable and monostable transition fronts.

**Abstract.** The standard notions of reaction-diffusion waves and fronts can be viewed as examples of generalized transition waves. These notions involve uniform limits, with respect to the geodesic distance, to a family of hypersurfaces which are parametrized by time. The existence of transition waves has been proved in various contexts where the standard notions of waves make no longer sense. Even for homogeneous equations, fronts with various non-planar shapes or with varying speeds are known to exist. In this talk, I will report on some recent existence results and qualitative properties of transition fronts for monostable and bistable homogeneous and heterogeneous equations. I will also discuss their mean speed of propagation.
A. Kiselev  
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**Title.** Chemotaxis and Reactions in Biology  
**Abstract.** Chemical attraction plays an important role in enhancing a wide range of biological reactions. One example is reproduction processes; another is attraction of monocytes (blood killer cells) to the source of infection. We consider a system modeling this process and provide estimates for the enhancement effect. One of the elements involved in the proof are sharp estimates for convergence to ground state of Fokker-Planck operators with logarithmic potential. These estimates are based on a new weighted Poincare-type inequality.

A. Stevens  
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**Title.** Cell motion and self-organization: from discrete/stochastic to continuum  
**Abstract.** In this talk a partial differential equation of chemotaxis-type, coupled with an ODE is considered w.r.t. its qualitative behavior. Connections to self-attracting reinforced random walks are discussed as well as limits of interacting stochastic many particle systems towards these kinds of equations. These systems arise in the context of cell motion along self-reinforced attractive cues.