Speculation, trading and bubbles

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Bubbles

- History of financial markets dotted with episodes described as *bubbles* - periods in which asset prices seem to vastly exceed fundamentals.
  
- However not much agreement among economists on which economic mechanisms generate such episodes.

  “I don’t even know what a bubble means. These words have become popular. I don’t think they have any meaning.”

  Eugene Fama, The New Yorker
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Introduction
Stylized Facts
Model

Bubbles

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- However not much agreement among economists on which economic mechanisms generate such episodes.
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Bubbles

• Discussions of bubbles often concentrate solely on the behavior of prices.
  • The most common definition of a bubble is as a period in which prices exceed fundamental valuation.
  • Any valuation however depends on a model of fundamentals
  • Valuations are always ex-post wrong.
• Additional empirical regularities help determine “reasonable” mechanisms that generate bubbles.
Plan

1. Present some empirical evidence concerning bubbles.
2. Present a mathematical model of bubbles and argue that it fits this evidence.
3. Present a version for “finite lived” assets that yields a PDE (free boundary value problem)
Three stylized facts

1. Asset price bubbles coincide with increases in trading volume.
2. Asset price bubble implosions seem to coincide with increases in asset supply.
3. Asset price bubbles often coincide with financial or technological innovations.

- Present a class of models that can explain all three facts.
- Today simplified version that aims at explaining the correlation of bubbles and trading volume only.
Bubbles and trading volume: South Sea Bubble

- Extraordinary rise and fall of price of South Sea Company shares and other similar joint-stock companies in 1720.
- \( \sim 2,000 \) transactions per year in Bank of England stock 1717-1719, 6,846 transactions (100% of stocks outstanding) in 1720.
- East India Company and Royal African Company turned over 150% of stock outstanding in 1720.
- Carlos, Neal and Wandschneider (2006)
Bubbles and trading volume: Roaring Twenties

- Accounts of stock-market boom of late 1920s emphasize overtrading in 28-29.
- Annual turnover at NYSE climbs from 100% per annum in 1925-27 to over 140% in 1928 and 1929. (Davis, Neal and White, 2005)
- All-time daily records of share trading volume were reached 10 times in 1928 and 3 times in 1929. New record not set until April 1, 1968, when LBJ announced he would not seek re-election (Hong and Stein, 2006)
Bubbles and trading volume: Internet...

- During the DotCom bubble internet stocks had 3 times the turnover of other similar stocks.
- Lamont and Thaler’s 6 cases of spinoffs average 38% daily turnover.
  - Typical NYSE stock turnover of 100% per year.
- Cochrane (2002) documents cross sectional correlation between the ratio of market value to book value of a stock and that stock’s turnover.
Principal assumptions

- Costly shorting
- Heterogeneous beliefs from overconfidence, the tendency of people to overestimate the precision of their knowledge.
- Far from being standard in economics
  - Economic models typically assume symmetric costs between going long and going short
  - Results showing that rational investors with common priors cannot agree to disagree.
  - No trade theorems: Unless some traders trade for “irrational” reasons, there is no trade. (K. Arrow, *The New Palgrave*)
  - Trading induced by liquidity shocks (Campbell, Grossman and Wang, 1993).
Evidence for costly short-sale

- Some obvious cases
  - Housing
  - CDO’s before the introduction of ABX and synthetic CDO’s.
- Shorting mechanisms for stocks (D’Avolio, 2002)
- Stocks with higher dispersion of earnings forecasts have lower future returns (Diether, Malloy and Scherbina, 2002)
  - It is easier for optimists to express their beliefs in markets.
Evidence of overconfidence

- Documented among: Engineers (Kidd, 1970), Entrepreneurs (Cooper, Woo, and Dukelberg, 1988)...
- Expert political judgment (Tetlock, 2005).
- Ben David, Graham and Harvey, 2010 on CFO predictions of S&P returns.
  - Realized returns are within executives [10%,90%] intervals 33% of the time.
Outline

- A single risky asset in finite supply
- Two large groups of Agents $A$ and $B$.
- Individuals receive signals on the value of an asset in finite supply.
- Different groups of individuals have common information but excess confidence on distinct signals.
  - One group may be rational
- Filtering.
- Buyer acquires (American) option to sell asset.
- Optimal stopping time.
Outline

- Value that buyer is willing to pay today depends on prices he forecasts for the future.
- Equilibrium.
- Solve for infinite horizon model, but can accommodate finite horizon securities.
Payoffs and Information

• Cumulative dividend process $D_t$:
\[ dD_t = f_t dt + \sigma_D dZ_t^D \]

• $f$ is not observable but known to follow:
\[ df_t = -\lambda (f_t - \bar{f}) dt + \sigma_f dZ_t^f \]

• Two extra signals:
\[ ds_t^A = f_t dt + \sigma_s dZ_t^A \]
\[ ds_t^B = f_t dt + \sigma_s dZ_t^B \]

• $(Z^D, Z^f, Z^A, Z^B)$ a 4-d Brownian motion
• Group A agents believe:

\[ ds_t^A = f_t dt + \sigma_s \phi dZ_t^f + \sigma_s \sqrt{1 - \phi^2} dZ_t^A \]

Group B agents believe:

\[ ds_t^B = f_t dt + \sigma_s \phi dZ_t^f + \sigma_s \sqrt{1 - \phi^2} dZ_t^B \]

• \( 0 \leq \phi \leq 1 \), agents views on correlations of innovations is public information.

• Agents use signals and \( D \) to forecast \( f \).

• Stationary variance,

\[
\gamma \equiv \left[ \sqrt{\lambda + \phi \sigma_f / \sigma_s}^2 + (1 - \phi^2) \left( \frac{2 \sigma_f^2}{\sigma_s^2} + \frac{\sigma_f^2}{\sigma_D^2} \right) - (\lambda + \phi \sigma_f / \sigma_s) \right] / \left( \frac{1}{\sigma_D^2} + \frac{2}{\sigma_s^2} \right)
\]

decreases with \( \phi \).
\begin{align*}
\text{Introduction} \\
\text{Stylized Facts} \\
\text{Model} \\
\text{“Beliefs”}
\end{align*}

\begin{align*}
\hat{f}^A &= -\lambda (\hat{f}^A - \bar{f}) dt + \frac{\phi \sigma_s \sigma_f + \gamma}{\sigma_s^2} (ds^A - \hat{f}^A dt) \\
&\quad + \frac{\gamma}{\sigma_s^2} (ds^B - \hat{f}^A dt) + \frac{\gamma}{\sigma_D^2} (dD - \hat{f}^A dt) \\
\hat{f}^B &= -\lambda (\hat{f}^B - \bar{f}) dt + \frac{\gamma}{\sigma_s^2} (ds^A - \hat{f}^B dt) \\
&\quad + \frac{\phi \sigma_s \sigma_f + \gamma}{\sigma_s^2} (ds^B - \hat{f}^B dt) + \frac{\gamma}{\sigma_D^2} (dD - \hat{f}^B dt)
\end{align*}
Difference in beliefs

- $g^C = \hat{f}^C - \hat{f}^C$
- To $A$ investors: $g^A = \hat{f}^B - \hat{f}^A$

$$dg^A = -\rho g^A dt + \sigma_g dW^A_g$$

- $W^A$ a BM for $A$ investors
- For $B$ investors...
- Difference in beliefs is a state variable.
- $\sigma_g = \sqrt{2\phi}\sigma_f$

- Larger $\phi$ increases volatility and decreases mean-reversion.
Trading

- Cost $c$ per trade.
- All agents are risk neutral, fixed rate of interest $r$.
- Finite supply and large number of agents of each type guarantee that buyers pay reservation price.
- Price that $C \in \{A, B\}$ is willing to pay is

$$p_t^C = \max_{\tau \geq 0} E_t^C \left\{ \int_t^{t+\tau} e^{-r(s-t)} \left[ \bar{f} + e^{-\lambda(s-t)} (\hat{f}_t^C - \bar{f}) \right] ds + e^{-r\tau} (p_{t+\tau} - c) \right\}.$$
Guess: Demand price of current owner is:

\[ p_t^o = p^o(\hat{f}_t^o, g_t^o) = \frac{\bar{f}}{r} + \frac{\hat{f}_t^o - \bar{f}}{r + \lambda} + q(g_t^o). \]  

with \( q > 0 \) and \( q' > 0 \).

\[ p_t^o = p^o(\hat{f}_t^o, g_t^o) = \frac{\bar{f}}{r} + \frac{\hat{f}_t^o - \bar{f}}{r + \lambda} + \sup_{\tau \geq 0} \mathbb{E}_t^o \left[ \left( \frac{g_{t+\tau}}{r + \lambda} + q(g_{t+\tau}) - c \right) e^{-r\tau} \right]. \]

\[ q(g_t^o) = \sup_{\tau \geq 0} \mathbb{E}_t^o \left[ \left( \frac{g_{t+\tau}}{r + \lambda} + q(g_{t+\tau}) - c \right) e^{-r\tau} \right]. \]
The option problem

\[ q(x) \geq \frac{x}{r + \lambda} + q(-x) - c \]  \hspace{1cm} (2)

\[ \frac{1}{2} \sigma_g^2 q'' - \rho x q' - rq \leq 0, \]  \hspace{1cm} (3)

with equality in (3) if (2) strict,

• Guess: Stop if \( \{x : x \geq k^*\} \), \( k^* \geq 0 \).

•

\[ \frac{1}{2} \sigma_g^2 H'' - \rho x H' - rH = 0. \]  \hspace{1cm} (4)
• Solutions $H$ to (4) with $H > 0$ and $H' > 0$ in $(-\infty, 0)$ are of the form $\beta_1 h$, $h$ convex, 
  \[ \lim_{x \to -\infty} h(x) = 0, \quad \lim_{x \to -\infty} h'(x) = 0. \]
  • An “explicit” formula for $h$.
  • Kummer functions.
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\[ q(x) = \begin{cases} 
\beta_1 h(x), & \text{for } x < k^* \\
\frac{x}{r+\lambda} + \beta_1 h(-x) - c, & \text{for } x \geq k^*. 
\end{cases} \] \tag{5}

“Smooth pasting” \(\Rightarrow\)

\[ \beta_1 = \frac{1}{[h'(k^*) + h'(-k^*)](r + \lambda)}, \] \tag{6}

\[ [k^* - c(r + \lambda)][h'(k^*) + h'(-k^*)] + h(-k^*) - h(k^*) = 0 \] \tag{7}
Proposition

There exists unique $k^* = k^*(c)$ that solves (7). $k^*(0) = 0$ and $k^*(c) > c(r + \lambda)$ if $c > 0$. $q$ defined by (5) is an equilibrium option value function. The optimal policy consists of exercising immediately if $g^o \geq k^*$, otherwise wait until first time in which $g^o \geq k^*$.

- Bubble: If $x \in (-\infty, k^*)$, $q(x)$ the difference between owner's demand price and his fundamental valuation.

\[ b(c) = \frac{h(-k^*(c))}{[h'(k^*(c)) + h'(-k^*(c))](r + \lambda)} \]

- Mean duration between trades: $E[\tau(-k^*, k^*)]$
Properties of equilibria

- **Bubble**
  - $b(0) = \frac{h(0)}{2(r+\lambda)h'(0)} > 0$.
  - Bubble increases with overconfidence $\phi$, volatility of fundamentals $\sigma_f$, and decreases with interest rate $r$.

- **Trading volume**
  - If $c = 0$, $k^* = 0$ and $E[\tau(-k^*, k^*)] = 0$.
  - $E[\tau(-k^*, k^*)]$ varies continuously with $c$.
    - Large trading volume for small $c$.
  - $\frac{dk^*}{dc}(0) = \infty$ but $b'(0)$ is finite.
  - Trading increases with overconfidence $\phi$, volatility of fundamentals $\sigma_f$, and decreases with interest rate $r$. 
Price and turnover

- Risk-neutrality allows a separate model for each asset.
- Turnover and size of bubble are equilibrium values.
- No causality.
- Cochrane: Cross-sectional regression of market value / book value on share turnover for stocks in NASDAQ 96-00.
- China’s A and B shares. (MSX)
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Bubble with finite horizon asset

- China’s put warrants (Xiong and Yu, 2010).
  - Panel of prices, trading volume etc... of 18 put warrants trading in 2005-2007.
  - Price much higher than value justified by fundamentals.
    - Black-Scholes price
    - Looser upper bounds
  - Bubble declines as expiration approaches.
  - Bubble positively related to trading volume in panel.
Finite horizon version

- $T$ terminal time, $\lambda = 0$.
- Guess: Demand price of current owner is:

$$p_t^o = p^o(\hat{f}_t^o, g_t^o, t) = \frac{\hat{f}_t^o(1 - e^{-r(T-t)})}{r} + q(g_t^o, t).$$  \hfill (8)

with $q(x, t) > 0$ and $q_x > 0$, $q_t < 0$.

$$q(g_t^o, t) = \sup_{\tau \geq 0} \left[ E_t^o \left( \frac{g_{t+\tau}^o(1 - e^{-r(T-(t+\tau))})}{r} + q(g_{t+\tau}^o, t+\tau) - c \right) e^{-r\tau} \right].$$
The option problem: Finite horizon

- \( q(x, t) \geq \frac{x(1 - e^{-r(T-t)})}{r} + q(-x, t) - c \) \hspace{1cm} (9)

- \( q_t + \frac{1}{2} \sigma^2 q_{xx} - \rho x q_x - rq \leq 0, \) \hspace{1cm} (10)
  with equality in (10) if (9) strict

- \( q(x, T) = 0. \)

- Exercise boundary

\[ \mathcal{E} = \left\{ (x, t) : q(x, t) = \frac{x(1 - e^{-r(T-t)})}{r} + q(-x, t) - c \right\} \]