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An introduction to stochastic filtering [1]

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Abstract: Stochastic filtering deals with the problem of estimating evolving dynamical system, the signal, customarily modelled by a stochastic process. The signal process can not be measured directly. However, a partial measurement of the signal can be obtained, modelled by another continuous time process called the observation process. The aim of stochastic filtering is to compute the conditional distribution of the current state of the signal given the information supplied by the observation process.

Course synopsis:

Part I. We review the framework for the filtering problem and deduce some of the main theoretic results of the subject: the Kallianpur-Striebel formula, the Zakai equation and the Kushner-Stratonovitch equation. Finally we deduce the Kalman filter (the linear filter) equations from the Kushner-Stratonovitch equation. Sections:

1. The Framework
2. The Stochastic Process π
3. The *Innovation* Process I
4. The Unnormalised Conditional Distribution
5. The Zakai Equation
6. The Kushner-Stratonovitch Equation
7. The Kalman-Bucy Filter

Part II. We start with a brief overview of the main computational methods currently available for solving the filtering problem. As expected of a topic with such a diversity of applications, numerous algorithms for solving the filtering problem have been developed. We continue with a detailed study of continuous time particle filters. Particle filters (also known as sequential Monte Carlo methods) are some of the most successful methods for the numerical approximations of the solution of the filtering problem. Recent particle filters based on cubature methods are also discussed. Sections:

1. Numerical Methods for Solving the Filtering Problem (The Extended Kalman Filter, Finite Dimensional Nonlinear Filters, The Projection Filter and Moments Methods, Partial Differential Equations Methods, The Spectral Approach, Particle Methods)
2. The Kusuoka-Lyons-Victoir cubature particle filter.
3. The Implementation of the KLV particle filter
4. Convergence Results

Reference: A. Bain, D. Crisan, *Fundamentals of Stochastic Filtering*, Series: Stochastic Modelling and Applied Probability , Springer Verlag, Vol. 60, 2009.

[1] Euro-Mediterranean Research Center for Mathematics and its Applications (EMRCMA): Second school, 2nd to 4th of April, 2012.