MONTE-CARLO METHODS IN FINANCE

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B. BOUCHARD

This lecture aims at studying Monte-Carlo methods in finance. In financial models without frictions or constraints, the price of a European option, written on some underlying asset $X=(X_t)_{t\geq 0}$, and with discounted payoff g(X), can be written as the expectation under some risk-neutral probability measure of its liquidation value : $u(0,x) =E[g(X) \mid X_0=x]$. Monte-Carlo methods consist in estimating this expecation by sampling a large number of independent copies $\{g(X^{(n)})\}_{1\leq n\leq N}$ of g(X) and then taking the mean $\hat{u}(0,x)=\sum g(X^{(n)})/N$, The convergence of $\hat{u}(0,x)$ to u(0,x) follows from the law of large numbers.

This lecture is divided in four parts :

I- Generalities on Monte-Carlo methods and main sampling technics
1 Moment estimators and main convergence results
2 Uniform law samplers
3 Sampling of other laws
4 Law discrepency sequences

II- Simulation of processes and payoffs
1 Black-Scholes model
2 Discrete time approximation of SDEs
3 Brownian bridges and application to barrier and lookback options

III- Variance reduction technics
1 Anthitetic control
2 Payoff regularization
3 Control variate
4 Importance sampling

IV- Sensitivity computation (Greeks)
1 Finite differences
2 Intergration by parts in the Black-Scholes
3 Tangent process and Greeks