Computing In-Service Aircraft Reliability

December 1st, 2006

L. Saintis, E. Hugues, C. Bès, M. Mongeau

***Key Words*—Aircraft reliability, fault trees, reliability modeling, repairable system.**

***Summary & Conclusions*—This paper deals with the modeling and computation of in-service aircraft reliability at the preliminary design stage. This problem is crucial for designers because it enables them to evaluate in-service interruption rates, in view of system under design and optimizing aircraft support. In the context of a sequence of flight cycles, standard reliability methods are not computationally conceivable in terms of industrial timing constraints. Based on analytic developments, we introduce a methodology that provides an efficient algorithm for computing good reliability bounds. Finally, we show the relevance of our approach on real-world cases provided by Airbus.**

ACRONYMS

ADM	Degraded Mode Accepted
BA	Bound Algorithm
DM	Degraded Mode
OR	Operational Reliability
RDM	Refused Degraded Mode
TL	Total Loss of the system

L. Saintis is based at Airbus France, 31060 Toulouse cedex 3, France (+33-5-61-18-99-84; laurent.s.saintis@airbus.com).
E. Hugues is based at Airbus France, 31060 Toulouse cedex 3, France (+33-5-61-18-88-06; emmanuel.hugues@airbus.com).
C. Bès, is based at Université Paul Sabatier, LGMT, 31062 Toulouse cedex 9, France (+ 33-5-61-55-81-77; cbes@cict.fr).

M. Mongeau is based at Université de Toulouse, LAAS-CNRS, 31077 Toulouse cedex 4, France (+ 33-5-61-33-63-13; mongeau@cict.fr).

NOTATION

T Index of cycle

S The set of all components within the system

k Number of minimal cuts

MC^i i^{th} Minimal Cut, $1 \le i \le k$

MC_T^i The event that all components of the Minimal Cut MC^i, are failed at the end of cycle T

p Number of minimal paths

MP^j j^{th} Minimal Path, $1 \le j \le p$

MP_T^j The event that all components of the Minimal Path MP^j, work at the end of cycle T

TL_T The event that Total Loss occurs at cycle T

xD_T The event that component x works at the Departure (beginning) of cycle T

xF_T The event that component x Fails during cycle T

ADM_T The event that the airline Accepts the Degraded Mode for take-off at cycle $T+1$

RDM_T The event that the airline Refuses the Degraded Mode for take-off at cycle $T+1$

$UB(E)$ Upper Bound for the probability of event E

$LB(E)$ Lower Bound for the probability of event E

I. INTRODUCTION

In-service aircraft reliability relates to aircraft availability and punctuality. It measures the frequency of unscheduled service interruptions caused by technical failures and associated required maintenance. The different interruption types are:

- delays at take-off (the aircraft departs later than the scheduled departure time),

- flight cancellations (the aircraft does not depart at all),
- air diversions (the aircraft has to land at an airport different from its destination),
- in-flight turn-backs (the aircraft has to return to its departure airport).

For the airlines, these unscheduled service interruptions induce high direct costs related to the aircraft (fuel consumption, airport taxes, flight crew accommodation / duty time, passenger accommodation, financial compensation…). They also induce high indirect costs: loss of image, impact on customer loyalty, etc… So, in-service aircraft reliability is closely monitored by airlines and therefore also by aircraft manufacturers. As a consequence, in-service aircraft reliability has become a major target for aircraft designers.

We shall examine how this issue is addressed at the preliminary design stage. Predicting accurate levels of an aircraft future in-service reliability at the preliminary design stage is a key issue, because this ability allows an improved system design for targeted support performances. This prediction involves computing system failure probabilities, which requires the modeling and analyzing of dynamic process using a fault-tree analysis at each flight cycle [1]. Previous methods for computing these failure probabilities include the following: Markov processes [2], Monte-Carlo simulation, dynamic fault trees and multi-state systems. Because of the explosion of the number of possible states of the system, Markov processes (see [3] for an aircraft application) cannot be considered here. On the other hand, Monte-Carlo simulation [4, 5] requires too many simulations to obtain sufficient accuracy. Indeed, in our context the interruption rate probability is between 10^{-7} and 10^{-4} per take-off. Despite recent progress on dynamic fault trees [6] and multi-state systems [7], these two approaches cannot be applied because the CPU time required to extract all the minimal sequences is unmanageable (there can be up to 1,600 flight cycles during one year of aircraft use). The lack of general tractable methods for large-scale dynamic models has yielded analytical developments for specific problems; see for instance [8,9]. However, these results do not apply to our aeronautical reliability problem, which involves a long *sequence* of flight cycles with *dependencies* between component states.

This paper focuses on three aspects of in-service aircraft reliability: modeling, analytical resolution and validation. It is organized as follows. In Section II, we describe both the different failure modes of an aircraft system during its successive flight and ground phases, and the way airlines manage these failures. From this operational context we introduce the associated mathematical framework. In Section III, we derive analytical probability equations for the basic failure mode events. Then, we propose an efficient algorithm that provides relevant bounds for system reliability. Section IV reports very encouraging computational experiments on Airbus real-world cases that show the efficiency and accuracy of our approach. Finally, we provide a conclusion in Section V.

II. Model Formulation

In service, an aircraft is subject to a sequence of cycles with each cycle consisting of a flight phase, followed by a ground phase (which then precedes the next flight). Here we consider an aircraft system made up of a number of various components. During any phase, a component failure may occur. Due to the low probabilities involved for such a failure in the aeronautics context, we can assume that at most one failure will occur during each cycle (see assumption C2 below). Figure 1 illustrates the main events that may happen in a sequence of cycles.

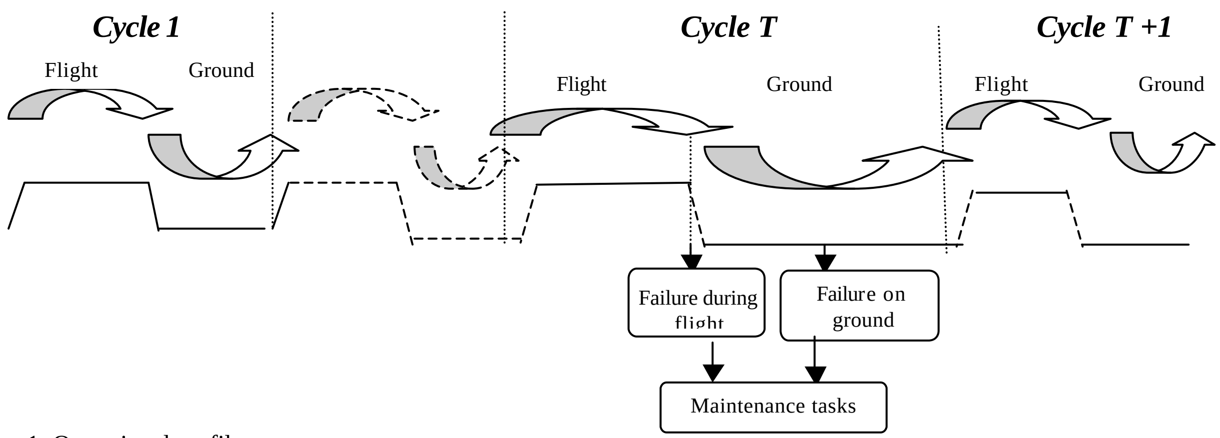

Figure 1: Operational profile

When a component *x* fails during cycle T, it may cause the system to fail, so-called *total loss* (TL). The occurrence of the TL event is represented by a fault tree, which is based solely on the component states (working or failed). In the case of TL during cycle T, the airline must repair *all* the components in a state of failure. For the degraded mode, one of two possible decision is made by the airline. Either the airline decides to take off in a so-called *Accepted Degraded Mode* (ADM), or it refuses the degraded mode (RDM) and then repairs the component that has just failed at cycle T, and does not repair any previously failed component. It is important to note that, if a degraded mode is accepted, some minor maintenance tasks configure the component that has just failed for the Degraded Mode (DM) state. Figure 2 illustrates all of these different scenarios in detail.

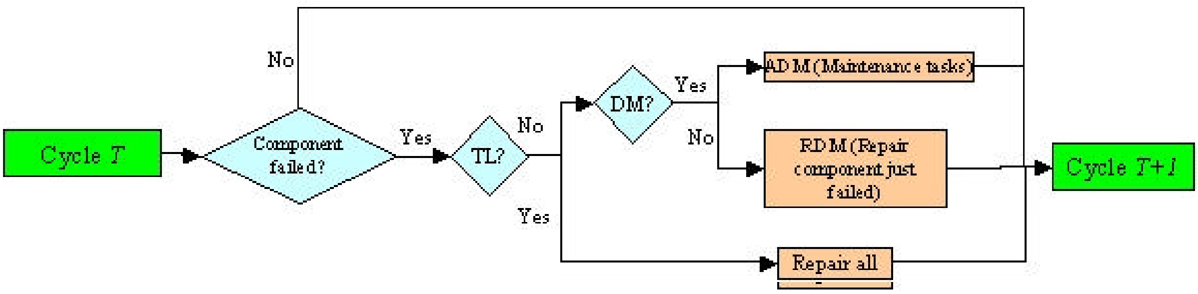

Figure 2: The different possible scenarios during a cycle

Let us now enumerate precisely the constraints and assumptions based on airline maintenance strategy and aircraft systems. We will consider the following points:

C1. Component failure probabilities are independent.

C2. There is at most one component that can fail during each single cycle (due to high component reliability).

C3. A component is repaired only in the following cases:

  a. Total Loss

  b. Failure during the current cycle and refusal of the degraded mode.

C4. In case of TL, all failed components are repaired.

C5. Acceptance of a degraded mode (ADM) at cycle *T* only depends upon a component failure during cycle T (i.e. it does not depend upon failures during previous cycles).

C6. When a degraded mode is refused (RDM) at cycle T, the only component repaired is the one that has just failed at cycle *T*.

Remark: Validity of C2 assumption has been studied in [10].

From the previous constraints and assumptions we can now present a tree of all events that may occur within cycle *T*, for a given component *x* (Figure 3)..

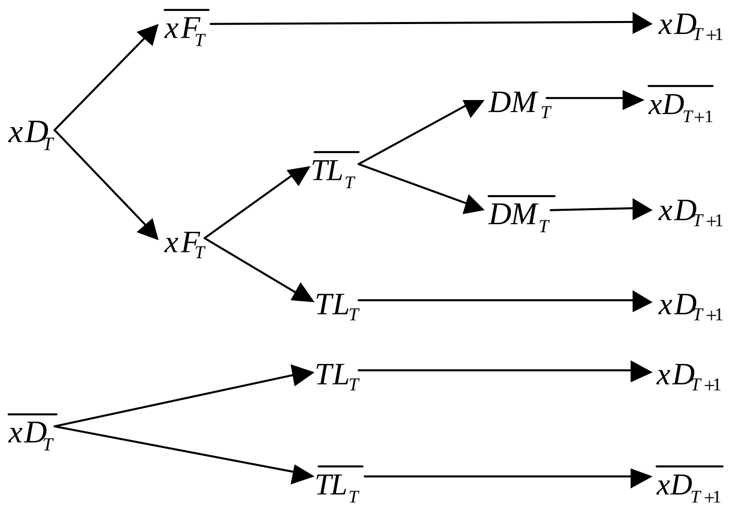

Figure 3: The event tree for component *x* at cycle *T*.

The fact that a given component *y* is in a failed state at the beginning of cycle *T* does not lead to an increased tendency for another component *x* to fail at the beginning of cycle *T*. Therefore we can formalize this using the following property:

Property P (positive dependency):

Let $x, y \in S, x \neq y$ be two components. Then $\Pr\{\overline{xD_T} \cap \overline{yD_T}\} \leq \Pr\{\overline{xD_T}\} \times \Pr\{\overline{yD_T}\}$.

More generally, let $x \in S$, $A \subset S$ with $x \notin A$. Then $\Pr\left\{\overline{xD_T} \cap \left(\bigcap_{y\in A}\left(\overline{yD_T}\right)\right)\right\} \leq \Pr\left\{\overline{xD_T}\right\} \times \Pr\left\{\bigcap_{y\in A}\left(\overline{yD_T}\right)\right\}$.

To conclude this section, here is the input data (known quantities) of our problem:

- A coherent fault tree of the system, issued by designers from the system architecture.
- $\Pr\left\{xF_T \middle| xD_T\right\}$, the probability that component *x* fails during cycle *T*, given that *x* works at the beginning of cycle *T*. This quantity is a function of the component failure rate, which is provided by the component manufacturer.
- $\Pr\left\{ADM_T \middle| \overline{TL_T} \cap xF_T \cap xD_T\right\}$, the probability of accepting the degraded mode, given that *x* fails during cycle *T* and no TL occurs. This quantity is a function of the pilot's behavior and the maintenance airline strategy.
- The initial condition (the state or the probability $\Pr\left\{xD_1\right\}$ of each component *x* at the beginning of cycle 1).

III. Analytic developments

The objective of this section is to compute at each cycle *T* the probabilities of the main events TL, ADM, and RDM (see Figure 2), which are our in-service aircraft reliability indicators at the preliminary design stage. The proposed methodology for computing these probabilities will be presented in four steps. In Subsection A, we develop recursive analytical formulae (from cycle *T* to cycle *T+1*) for the three main event probabilities $\Pr\left\{TL_T\right\}$, $\Pr\left\{ADM_T\right\}$, and $\Pr\left\{RDM_T\right\}$. However, these formulae rely on two probabilities that are not computationally tractable for real-world aeronautical systems. Thus, in Subsection B we develop astute bounds on probabilities related to minimal sets of the TL fault tree. These bounds, in turn, allow us in Subsection C to derive a bounding methodology for the above main event probabilities. Finally, we put these results together in Subsection D to derive an overall iterative scheme based on the initial condition (at cycle 1) and input data.

A. Probabilities of main events:

In this subsection, assuming that probability $\Pr\left\{xD_T\right\}$ is given for all components *x* in *S*, we show how to calculate the main event probabilities: $\Pr\left\{TL_T\right\}$, $\Pr\left\{ADM_T\right\}$, $\Pr\left\{RDM_T\right\}$, and consequently $\Pr\left\{xD_{T+1}\right\}$ for all *x* in *S*. The latter probability will enable us to restart the iterative process.

When a TL occurs at cycle *T*, a component must have failed during this cycle. Therefore, we have:

$$TL_T = \bigcup_{x \in S} \left(TL_T \cap xF_T \cap xD_T \right).$$

Because the events $\{xF_T\}_{x \in S}$ are disjointed (see C2), we obtain:

$$\text{(1)} \Pr\{TL_T\} = \sum_{x \in S} \Pr\{TL_T | xF_T \cap xD_T\} \times \Pr\{xF_T | xD_T\} \times \Pr\{xD_T\}.$$

Similarly, for the second main event ADM_T, we have: $ADM_T = \bigcup_{x \in S} \left(ADM_T \cap \overline{TL_T} \cap xF_T \cap xD_T \right)$ (see Figure 3).

This implies

$$\text{(2)} \Pr\{ADM_T\} = \sum_{x \in S} \Pr\{ADM_T | \overline{TL_T} \cap xF_T \cap xD_T\} \times \left(1 - \Pr\{TL_T | xF_T \cap xD_T\}\right) \times \Pr\{xF_T | xD_T\} \times \Pr\{xD_T\}.$$

Again, from Figure 3, $RDM_T = \bigcup_{x \in S} \left(RDM_T \cap \overline{TL_T} \cap xF_T \cap xD_T \right)$, and thus:

$$\text{(3)} \Pr\{RDM_T\} = \sum_{x \in S} \Pr\{\overline{ADM_T} | \overline{TL_T} \cap xF_T \cap xD_T\} \times \left(1 - \Pr\{TL_T | xF_T \cap xD_T\}\right) \times \Pr\{xF_T | xD_T\} \times \Pr\{xD_T\}.$$

Finally, in accordance with Figure 3, we have: $\overline{xD_{T+1}} = \left(\overline{xD_T} \cap \overline{TL_T} \right) \cup \left(ADM_T \cap \overline{TL_T} \cap xF_T \cap xD_T \right)$.

Hence, the probability of component x not working at the next cycle, $T+1$, is:

$$\text{(4)} \begin{aligned} \Pr\{\overline{xD_{T+1}}\} &= \Pr\{\overline{xD_T}\} - \Pr\{TL_T \cap (\overline{xD_T})\} \\ &+ \Pr\{ADM_T | \overline{TL_T} \cap xF_T \cap xD_T\} \times \left(1 - \Pr\{TL_T | xF_T \cap xD_T\}\right) \times \Pr\{xF_T | xD_T\} \times \Pr\{xD_T\}. \end{aligned}$$

Except for $\Pr\{TL_T \cap \overline{xD_T}\}$ and $\Pr\{TL_T | xF_T \cap xD_T\}$, all values involved in the above formulae are known from inputs (see Section III) and from probabilities $\Pr\{xD_T\}_{x \in S}$. The next two subsections will address the issue of bounding / approximating these two unknown probabilities.

B. Bounds related to minimal sets

In this subsection, we derive two types of recursive bounds. The first type is related to minimal cuts. More precisely, in Theorem 1 we provide bounds on $\Pr\{MC_T^i | xF_T \cap xD_T\}$ and $\Pr\{MC_T^i \cap yF_T \cap yD_T \cap \overline{xD_T}\}$ for all i, $1 \le i \le k$ and for all $x, y \in S$, $y \ne x$. The second type given by Theorem 2 relates to minimal paths and provides bounds on $\Pr\{MP_T^j | xF_T \cap xD_T\}$ and $\Pr\{MP_T^j \cap yF_T \cap yD_T \cap \overline{xD_T}\}$ for all j, $1 \le j \le p$ and for all $x, y \in S$, $y \ne x$. It should be noted that all bounds given by Theorems 1 and 2 can be computed from input data and probabilities $\Pr\{xD_T\},\ x\in S$. We shall use these bounds in subsection C.

Theorem 1:

Consider the i[th] minimal cut, $1\le i\le k$, and two components $x, y\in S$ with $x\neq y$. We then obtain:

i) $$\Pr\left\{MC_T^i \middle| xF_T\cap xD_T\right\}\le\begin{cases}\dfrac{\prod\limits_{\substack{y\in MC^i\\ y\neq x}}\Pr\left\{\overline{yD_T}\right\}}{\Pr\{xD_T\}}, & if\ \ x\in MC^i\\ 0, & otherwise;\end{cases}$$

ii) $$\Pr\left\{MC_T^i\cap yF_T\cap yD_T\cap\overline{xD_T}\right\}\le\begin{cases}\Pr\left\{yF_T\middle|yD_T\right\}\times\left(\prod\limits_{\substack{z\in MC^i\\ z\neq x,\ z\neq y}}\Pr\left\{\overline{zD_T}\right\}\right)\times\Pr\left\{\overline{xD_T}\right\}, & if\ \ y\in MC^i\\ 0, & otherwise.\end{cases}$$

Proof:

i) If we consider the **minimal cuts** for a cut MC^i to occur at *T*, a component $x\in MC^i$ must fail during the current cycle (only one according to our hypothesis C2), and the other components have to be lost at the beginning of the cycle *T*. Thus, this event can be rewritten as

$$MC_T^i\cap xF_T\cap xD_T=\begin{cases}\left(\bigcap\limits_{\substack{y\in MC^i\\ y\neq x}}\overline{yD_T}\right)\cap xF_T\cap xD_T, & if\ \ x\in MC^i\\ \varnothing, & otherwise.\end{cases}$$

Let $x\in MC^i$,

$$\Pr\left\{MC_T^i\cap xF_T\cap xD_T\right\}=\Pr\left\{\left(\bigcap\limits_{\substack{y\in MC_i\\ y\neq x}}\overline{yD_T}\right)\cap xF_T\cap xD_T\right\}$$

$$=\Pr\left\{xF_T\middle|\left(\bigcap\limits_{\substack{y\in MC_i\\ y\neq x}}\overline{yD_T}\right)\cap xD_T\right\}\times\Pr\left\{\left(\bigcap\limits_{\substack{y\in MC_i\\ y\neq x}}\overline{yD_T}\right)\cap xD_T\right\}$$

The fact that $\Pr\left\{xF_T\middle|xD_T\cap\left(\bigcap\limits_{\substack{y\in MC_i\\ y\neq x}}\overline{yD_T}\right)\right\}=\Pr\left\{xF_T\middle|xD_T\right\}$ is due to the probability independence of failure (C1).

We use the inclusion of events $\left(\bigcap_{\substack{y\in MC^i\\ y\neq x}}\overline{yD_T}\right)\cap xD_T \subseteq \left(\bigcap_{\substack{y\in MC^i\\ y\neq x}}\overline{yD_T}\right)$ and the overvaluation of their probabilities that can be deduced. Hence, with Property P we obtain:

$$\Pr\left\{MC_T^i \cap xF_T \cap xD_T\right\} \leq \Pr\left\{xF_T \mid xD_T\right\}\times \Pr\left\{\left(\bigcap_{\substack{y\in MC_i\\ y\neq x}}\overline{yD_T}\right)\right\}$$
$$\leq \Pr\left\{xF_T \mid xD_T\right\}\times \prod_{\substack{y\in MC_i\\ y\neq x}}\Pr\left\{\overline{yD_T}\right\}$$

Consequently, with the equality $\Pr\left\{MC_T^i \cap xF_T \cap xD_T\right\} = \Pr\left\{MC_T^i \middle| xF_T \cap xD_T\right\}\times \Pr\left\{xF_T \cap xD_T\right\}$, we conclude.

ii) An analogous development of the previous proof is used to demonstrate the second overvaluation.

Let $y \notin MC^i$, $x\in S$. The cut cannot occur and we have $MC_T^i \cap yF_T \cap yD_T \cap \overline{xD_T} = \varnothing$.

Let $y \in MC^i$, $x\in S$. The cut will occur if all the other components are in a failure state at the beginning of the cycle. Thus, we have:
$$MC_T^i \cap yF_T \cap yD_T \cap \overline{xD_T} = \left(\bigcap_{\substack{z\in MC^i\\ z\neq x, z\neq y}}\overline{zD_T}\right)\cap yF_T \cap yD_T \cap \overline{xD_T}$$

Hence, if $y \in MC^i$, $x\in S$,

$$\Pr\left\{MC_T^i \cap yF_T \cap yD_T \cap \overline{xD_T}\right\} = \Pr\left\{yF_T \middle| \bigcap_{\substack{z\in MC^i\\ z\neq x, z\neq y}}\overline{zD_T} \cap yD_T \cap \overline{xD_T}\right\}\times \Pr\left\{\bigcap_{\substack{z\in MC^i\\ z\neq x, z\neq y}}\overline{zD_T} \cap yD_T \cap \overline{xD_T}\right\}$$
$$\leq \Pr\left\{yF_T \middle| yD_T\right\}\times \Pr\left\{\bigcap_{\substack{z\in MC^i\\ z\neq x, z\neq y}}\overline{zD_T} \cap \overline{xD_T}\right\}$$

The previous development relies on the fact that $\Pr\left\{yF_T \middle| \bigcap_{\substack{z\in MC^i\\ z\neq x, z\neq y}}\overline{zD_T} \cap yD_T \cap \overline{xD_T}\right\} = \Pr\left\{yF_T \middle| yD_T\right\}$ (from the probability independence of failures), and on the inclusion of the following events $yD_T \cap \left(\bigcap_{\substack{z\in MC^i\\ z\neq y}}\overline{zD_T}\right)\cap \overline{xD_T} \subset \left(\bigcap_{\substack{z\in MC^i\\ z\neq x, z\neq y}}\overline{zD_T}\right)\cap \overline{xD_T}$.

Hence, with Property P we conclude:

$$\Pr\left\{MC_T^i \cap yF_T \cap yD_T \cap \overline{xD_T}\right\} \le \Pr\left\{yF_T \middle| yD_T\right\} \times \left(\prod_{\substack{z \in MC^i \\ z \ne x, z \ne y}} \Pr\left\{\overline{zD_T}\right\} \right) \times \Pr\left\{\overline{xD_T}\right\}.$$ **Q.E.D**

Theorem 2:

Consider the j[th] minimal path, $1 \le j \le p$, and two components $x, y \in S$ with $x \ne y$. Then, we have:

i) $$\Pr\left\{MP_T^j \middle| xF_T \cap xD_T\right\} \le \begin{cases} 0, & if \;\; x \in MP^j \\ \prod_{y \in MP^j} \Pr\left\{yD_T\right\}, & otherwise. \end{cases}$$

$$\Pr\left\{MP_T^j \cap yF_T \cap yD_T \cap \overline{xD_T}\right\}$$
$$\le \begin{cases} 0, & if \;\; x \in MP^j \;\; or \;\; y \in MP^j \\ \min\left(\Pr\left\{yF_T \middle| yD_T\right\} \times \left(\prod_{z \in MP^j} \Pr\left\{zD_T\right\} \right) \times \Pr\left\{yD_T\right\}, \Pr\left\{yF_T \middle| yD_T\right\} \times \Pr\left\{\overline{xD_T}\right\} \right), & otherwise \end{cases}$$

ii)

Proof:

According to minimal cuts, in order to have overvaluations related to minimal paths, the following lemma will be used in the proof of the next theorem.

Lemma 1:

For all pairs of components $x, y \in S$ ($x \ne y$), Property P implies the following overvaluation:

$$\Pr\left\{xD_T \cap yD_T\right\} \le \Pr\left\{xD_T\right\} \times \Pr\left\{yD_T\right\}.$$

Proof of Lemma 1:

Let $x, y \in S$ with $x \ne y$, we have

$$\begin{aligned} \Pr\left\{xD_T\right\} * \Pr\left\{yD_T\right\} - \Pr\left\{xD_T \cap yD_T\right\} &= \Pr\left\{xD_T\right\} \times \left(1 - \Pr\left\{\overline{yD_T}\right\}\right) - \left(\Pr\left\{xD_T\right\} - \Pr\left\{xD_T \cap \overline{yD_T}\right\}\right) \\ &= \Pr\left\{xD_T \cap \overline{yD_T}\right\} - \Pr\left\{xD_T\right\} \times \Pr\left\{\overline{yD_T}\right\} \\ &= \left(\Pr\left\{\overline{yD_T}\right\} - \Pr\left\{\overline{xD_T} \cap \overline{yD_T}\right\}\right) - \left(1 - \Pr\left\{\overline{xD_T}\right\}\right) \times \Pr\left\{\overline{yD_T}\right\} \\ &= \Pr\left\{\overline{xD_T}\right\} \times \Pr\left\{\overline{yD_T}\right\} - \Pr\left\{\overline{xD_T} \cap \overline{yD_T}\right\} \end{aligned}$$

and we use Property P to conclude.

i) If we consider the **minimal paths**, we exploit the observation that one component must fail during the current cycle in order to prevent all minimal paths occurring. Again, using the assumption (C2), the other components necessarily work at the beginning of the cycle. Thus, for any component $x \in S$, we have:

$$MP_T^j \cap xF_T \cap xD_T = \begin{cases} \varnothing, & \text{if } x \in MP^j \\ \left(\bigcap_{y \in MP^j} yD_T \right) \cap xF_T \cap xD_T, & \text{otherwise.} \end{cases}$$

The next development relies on Property P (through Lemma 1) and the fact that $\Pr\left\{ xF_T \middle| xD_T \cap \left(\bigcap_{y \in MP^j} yD_T \right) \right\} = \Pr\{xF_T | xD_T\}$ (from the probability independence of failures C1).

$$\Pr\{MP_T^j \cap xF_T \cap xD_T\} \le \begin{cases} 0, & \text{if } x \in MP^j, \\ \Pr\{xF_T | xD_T\} \times \prod_{y \in MP^j} \Pr\{yD_T\} \times \Pr\{xD_T\}, & \text{otherwise.} \end{cases}$$

ii) An analogous development of the previous proof is used to demonstrate the second overvaluation.

The event is divided as follows

$$MP_T^j \cap yF_T \cap yD_T \cap \overline{xD_T} = \begin{cases} \varnothing, & \text{if } x \in MP^j \text{ or } y \in MP^j \\ \left(\bigcap_{z \in MP^j} zD_T \right) \cap yF_T \cap yD_T \cap \overline{xD_T} & \text{otherwise,} \end{cases}$$

but we cannot apply Property P to conclude directly.

On the other hand, with $x, y \notin MP^j$, we have the following two inclusions:

$$MP_T^j \cap yF_T \cap yD_T \cap \overline{xD_T} \subset \left(\bigcap_{z \in MP^j} zD_T \right) \cap yF_T \cap yD_T,$$

$$MP_T^j \cap yF_T \cap yD_T \cap \overline{xD_T} \subset yF_T \cap yD_T \cap \overline{xD_T}.$$

From the first inclusion, with Property P, we obtain the following overvaluation:

$$\Pr\{MP_T^j \cap yF_T \cap yD_T \cap \overline{xD_T}\} \le \Pr\{yF_T | yD_T\} \times \left(\prod_{z \in MP^j} \Pr\{zD_T\} \right) \times \Pr\{yD_T\}.$$

From the second one, we use the inclusion $yD_T \cap \overline{xD_T} \subset \overline{xD_T}$ to obtain the following overvaluation:

$$\Pr\{MP_T^j \cap yF_T \cap yD_T \cap \overline{xD_T}\} \le \Pr\{yF_T | yD_T \cap \overline{xD_T}\} \times \Pr\{yD_T \cap \overline{xD_T}\} \le \Pr\{yF_T | yD_T\} \times \Pr\{\overline{xD_T}\}.$$

We take the minimum of the two overvaluations to conclude. **Q.E.D**

Remark: For highly reliable systems, the second overvaluation is more efficient than the first one. It corresponds to the following inequality:

$$\Pr\{yD_T\}\times\prod_{z\in MP^j}\Pr\{zD_T\}\geq \Pr\{\overline{xD_T}\}.$$

C. Bounds related to Total Loss

Here, we show how to bound the unknown probabilities $\Pr\{TL_T\cap\overline{xD_T}\}$ and $\Pr\{TL_T\,|\,xF_T\cap xD_T\}$ of Subsection A. More precisely, upper bounds will be derived in Theorem 3, using a minimal cut-set decomposition and Theorem 1. Then, lower bounds will be given by Theorem 4, using a minimal path-set decomposition and using Theorem 2. It should be noted that all bounds given by Theorems 3 and 4 can be computed from input data and probabilities $\Pr\{xD_T\},\ x\in S$.

Theorem 3:

Consider a component $x\in S$. We then obtain:

i) $$\Pr\{TL_T\,|\,xF_T\cap xD_T\}\leq 1-\prod_{\substack{i=1\\ x\in MC^i}}^{k}\left(1-\frac{\prod_{\substack{y\in MC^i\\ y\neq x}}\Pr\{\overline{yD_T}\}}{\Pr\{xD_T\}}\right)\qquad(5)$$

ii) $$\Pr\{TL_T\cap\overline{xD_T}\}\leq\sum_{\substack{y\in S\\ y\neq x}}\Pr\{yF_T\,|\,yD_T\}\times\left(1-\prod_{\substack{i=1\\ y\in MC^i}}^{k}\left(1-\frac{\prod_{\substack{z\in MC^i\\ z\neq x\\ z\neq y}}\Pr\{\overline{zD_T}\}}{\Pr\{yD_T\}}\right)\right)\times\Pr\{\overline{xD_T}\}.\ (6)$$

Proof:

For this purpose, we derive extensions of the Esary-Proschan (EP) bounding method [11] for conditional probabilities.

The EP bounding method is based on the following overvaluation:

For any $1 \le i \le k$, we have $\Pr\left\{\overline{MC_T^i} \middle| \bigcap_{l=1}^{i-1} \overline{MC_T^l}\right\} \ge \Pr\left\{\overline{MC_T^i}\right\}$.

The inequality holds with equality if the minimal cuts have no component in common (which is only the case for series systems). Generally, the inequality is strict because the component that renders MC^i active may simultaneously render MC^l active if it is a component of both MC^i and MC^l for some $l < i$.

Let us now add the failure of a specific component x at the current cycle to the intersection of the minimal cut-sets. Let $1 \le i \le k$ and $x, y \in S$ with $x \ne y$. We have

$$\Pr\left\{\overline{MC_T^i} \middle| \bigcap_{l=1}^{i-1} \overline{MC_T^l} \cap xF_T \cap xD_T\right\} \ge \Pr\left\{\overline{MC_T^i} \middle| xF_T \cap xD_T\right\}$$

$$\Pr\left\{\overline{MC_T^i} \middle| \bigcap_{l=1}^{i-1} \overline{MC_T^l} \cap yF_T \cap yD_T \cap \overline{xD_T}\right\} \ge \Pr\left\{\overline{MC_T^i} \middle| yF_T \cap yD_T \cap \overline{xD_T}\right\}$$

i) Let us consider the failure of a component x. In order to have a TL, at least one minimal cut which contains x must occur. Thus, for any component $x \in S$, we have:

$$\begin{aligned}\Pr\{TL_T \cap xF_T \cap xD_T\} &= \Pr\left\{xF_T \cap xD_T \cap \left(\bigcup_{\substack{i=1 \\ x \in MC^i}}^{k} MC_T^i\right)\right\} \\ &= \Pr\{xF_T \cap xD_T\} - \Pr\left\{\bigcap_{\substack{i=1 \\ x \in MC^i}}^{k} \left(\overline{MC_T^i} \cap xF_T \cap xD_T\right)\right\}.\end{aligned}$$

And, with the extensions of EP method, we obtain:

$$\begin{aligned}\Pr\left\{\bigcap_{\substack{i=1 \\ x \in MC^i}}^{k} \overline{MC_T^i} \middle| xF_T \cap xD_T\right\} &\ge \prod_{\substack{i=1 \\ x \in MC^i}}^{k} \Pr\left\{\overline{MC_T^i} \middle| xF_T \cap xD_T\right\} \\ &\ge \prod_{\substack{i=1 \\ x \in MC^i}}^{k} \left(1 - \Pr\left\{MC_T^i \middle| xF_T \cap xD_T\right\}\right) \\ &\ge \prod_{\substack{i=1 \\ x \in MC^i}}^{k} \left(1 - \frac{\prod_{\substack{y \in MC^i \\ y \ne x}} \Pr\left\{\overline{yD_T}\right\}}{\Pr\{xD_T\}}\right)\end{aligned}$$

from Theorem 1.

Hence, we obtain an upper bound for the TL probability given failure of a component $x \in S$ at cycle *T*:

$$\Pr\{TL_T | xF_T \cap xD_T\} \leq 1 - \prod_{\substack{i=1 \\ x \in MC^i}}^{k} \left(1 - \frac{\prod_{\substack{y \in MC^i \\ y \neq x}} \Pr\{\overline{yD_T}\}}{\Pr\{xD_T\}} \right).$$

Of course, we can also obtain an upper bound for the probability of TL using the previous overvaluation and the equation (1).

ii) If the failure of component *x* at cycle T causes a TL, then $TL_T \cap \overline{xD_T} = \emptyset$ and therefore $\Pr\{TL_T \cap \overline{xD_T}\} = 0$. Based on the formulation (1) of TL_T event, an analogous development is applied to the event $TL_T \cap \overline{xD_T}$, and we obtain the following overvaluation:

$$\Pr\{TL_T \cap \overline{xD_T}\} = \sum_{\substack{y \in S \\ y \neq x}} \Pr\{yF_T \cap yD_T \cap TL_T \cap \overline{xD_T}\}$$

$$\leq \sum_{\substack{y \in S \\ y \neq x}} \Pr\{yF_T | yD_T\} \times \left(1 - \prod_{\substack{i=1 \\ y \in MC^i}}^{k} \left(1 - \frac{\prod_{\substack{z \in MC^i \\ z \neq x,y}} \Pr\{\overline{zD_T}\}}{\Pr\{yD_T\}} \right) \right) \times \Pr\{\overline{xD_T}\},$$

with the remarks:

- $\Pr\{yF_T | yD_T \cap TL_T \cap \overline{xD_T}\} = \Pr\{yF_T | yD_T\}$
- $yD_T \cap \overline{xD_T} \subset \overline{xD_T}$, which implies the overvaluation. **Q.E.D**

Theorem 4:

Consider a component $x \in S$. We then obtain:

i) $\Pr\{TL_T | xF_T \cap xD_T\} \geq \prod_{\substack{j=1 \\ x \notin MP^j}}^{p} \left(1 - \prod_{y \in MP^j} \Pr\{yD_T\} \right)$ (7)

$$\text{ii)}\Pr\{TL_T \cap \overline{xD_T}\} \geq \sum_{\substack{y\in S\\ y\neq x}} \prod_{\substack{j=1\\ x,y\notin MP^i}}^{p} \left(\begin{array}{l} \Pr\{yF_T | yD_T\} \times \Pr\{yD_T\} \times \Pr\{\overline{xD_T}\} \\ -\min\left(\Pr\{yF_T | yD_T\} \times \left(\prod_{z\in MP^j} \Pr\{zD_T\} \right) \times \Pr\{yD_T\}, \Pr\{yF_T | yD_T\} \times \Pr\{\overline{xD_T}\} \right) \end{array} \right) \quad (8)$$

Proof:

To apply the method to the **minimal paths**, we use the following overvaluations instead, in an analogous way, for $1 \leq j \leq p$ and $x, y \in S$, with $x \neq y$:

$$\Pr\left\{ \overline{MP_T^j} \middle| \left(\bigcap_{l=1}^{i-1} \overline{MP_T^l} \right) \cap xF_T \cap xD_T \right\} \geq \Pr\{\overline{MP_T^j} | xF_T \cap xD_T\},$$

$$\Pr\left\{ \overline{MP_T^j} \middle| \left(\bigcap_{l=1}^{i-1} \overline{MP_T^l} \right) \cap yF_T \cap yD_T \cap \overline{xD_T} \right\} \geq \Pr\{\overline{MP_T^j} | yF_T \cap yD_T \cap \overline{xD_T}\}$$

i) If we consider the failure of a component x, to have a TL, all the minimal paths not containing x, do not occur at beginning of the cycle. So we have:

$$xF_T \cap xD_T \cap TL_T = xF_T \cap xD_T \cap \bigcap_{\substack{j=1\\ x\notin MP^j}}^{p} \overline{MP_T^j}.$$

With the extensions of the EP bounding method and the use of Theorem 2, the probability of the event is developed as follows:

$$\Pr\{TL_T \cap xF_T \cap xD_T\} = \Pr\left\{ \bigcap_{\substack{j=1\\ x\notin MP^j}}^{p} \overline{MP_T^j} \middle| xF_T \cap xD_T \right\} \times \Pr\{xF_T \cap xD_T\}$$

$$\geq \left(\prod_{\substack{j=1\\ x\notin MP^j}}^{p} \Pr\{\overline{MP_T^j} | xF_T \cap xD_T\} \right) \times \Pr\{xF_T \cap xD_T\}$$

$$\geq \left(\prod_{\substack{j=1\\ x\notin MP^j}}^{p} \left(1 - \Pr\{MP_T^j | xF_T \cap xD_T\}\right) \right) \times \Pr\{xF_T | xD_T\} \times \Pr\{xD_T\}$$

$$\geq \left(\prod_{\substack{j=1\\ x\notin MP^j}}^{p} \left(1 - \prod_{y\in MP^j} \Pr\{yD_T | xF_T \cap xD_T\} \right) \right) \times \Pr\{xF_T | xD_T\} \times \Pr\{xD_T\}$$

$$\geq \left(\prod_{\substack{j=1\\ x\notin MP^j}}^{p} \left(1 - \prod_{y\in MP^j} \Pr\{yD_T\} \right) \right) \times \Pr\{xF_T | xD_T\} \times \Pr\{xD_T\}.$$

Hence, we obtain a lower bound for the TL probability given the failure of a component $x \in S$ at cycle T:

$$\Pr\left\{TL_T \left| xF_T \cap xD_T \right.\right\} \geq \prod_{\substack{j=1 \\ x \notin MP^j}}^{p} \left(1 - \prod_{y \in MP^j} \Pr\left\{yD_T\right\}\right).$$

ii) Based on the formulation (1) of the TL event, an analogous argument is applied to the event $TL_T \cap \overline{xD_T}$, and we obtain the following development:

$$TL_T \cap yF_T \cap yD_T \cap \overline{xD_T} = \left(\bigcap_{\substack{j=1 \\ x,y \notin MP_T^j}}^{p} \overline{MP_T^j}\right) \cap yF_T \cap yD_T \cap \overline{xD_T} \ .$$

The fact that $\Pr\left\{yF_T \left| yD_T \cap \overline{xD_T} \right.\right\} = \Pr\left\{yF_T \left| yD_T \right.\right\}$ (from the probability of independent failures), and the application of Theorem 2 yield the following development:

$$\begin{aligned}
&\Pr\left\{TL_T \cap yF_T \cap yD_T \cap \overline{xD_T}\right\} \\
&\qquad = \Pr\left\{\bigcap_{\substack{j=1 \\ x,y \notin MP^j}}^{p} \overline{MP_T^j} \cap yF_T \cap yD_T \cap \overline{xD_T}\right\} \\
&\qquad = \Pr\left\{\bigcap_{\substack{j=1 \\ x,y \notin MP^j}}^{p} \overline{MP_T^j} \left| yF_T \cap yD_T \cap \overline{xD_T} \right.\right\} \times \Pr\left\{yF_T \cap yD_T \cap \overline{xD_T}\right\} \\
&\qquad = \Pr\left\{\bigcap_{\substack{j=1 \\ x,y \notin P^j}}^{p} \overline{MP_T^j} \left| yF_T \cap yD_T \cap \overline{xD_T} \right.\right\} \times \Pr\left\{yF_T \cap yD_T \cap \overline{xD_T}\right\} \\
&\qquad \geq \left(\prod_{\substack{j=1 \\ x,y \notin MP^j}}^{p} \Pr\left\{\overline{MP_T^j} \left| yF_T \cap yD_T \cap \overline{xD_T} \right.\right\}\right) \times \Pr\left\{yF_T \cap yD_T \cap \overline{xD_T}\right\} \\
&\qquad \geq \left(\prod_{\substack{j=1 \\ x,y \notin MP^j}}^{p} \left(1 - \min\left(\Pr\left\{yF_T \left| yD_T \right.\right\} \times \left(\prod_{z \in MP^j} \Pr\left\{zD_T\right\}\right) \times \Pr\left\{yD_T\right\}, \Pr\left\{yF_T \left| yD_T \right.\right\} \times \Pr\left\{\overline{xD_T}\right\}\right)\right)\right) \\
&\qquad\qquad \times \Pr\left\{yF_T \left| yD_T \right.\right\} \times \Pr\left\{yD_T\right\} \times \Pr\left\{\overline{xD_T}\right\}
\end{aligned}$$

This, together with (2), implies the lower bound for the probability $\Pr\left\{TL_T \cap \overline{xD_T}\right\}$. **Q.E.D**

Note: Our methodology for obtaining bounds is based on an extension of the Esary-Proschan (EP) method [11]. As aeronautical systems are highly reliable, it is well known that only the upper bound is likely to be relevant [12]. In our in-service aircraft reliability context, only upper bounds are crucial in guaranteeing system performance.

D. *Summary of the bounds of probabilities by iteration*

Based on the inequalities of the previous sections, we give the iterative methodology to compute bounds of the three main event probabilities $\Pr\{TL_T\}, \Pr\{ADM_T\}$, and $\Pr\{RDM_T\}$, for each cycle *T*. These bounds are given by the following formulae:

$$UB(TL_T) = \sum_{x\in S} UB(TL_T | xF_T \cap xD_T) \times \Pr\{xF_T | xD_T\} \times UB(xD_T)$$

$$LB(TL_T) = \sum_{x\in S} LB(TL_T | xF_T \cap xD_T) \times \Pr\{xF_T | xD_T\} \times LB(xD_T)$$

$$UB(ADM_T) = \sum_{x\in S} \Pr\{ADM_T | \overline{TL_T} \cap xF_T \cap xD_T\} \times (1 - LB(TL_T | xF_T \cap xD_T)) \times \Pr\{xF_T | xD_T\} \times UB(xD_T)$$

$$LB(ADM_T) = \sum_{x\in S} \Pr\{ADM_T | \overline{TL_T} \cap xF_T \cap xD_T\} \times (1 - UB(TL_T | xF_T \cap xD_T)) \times \Pr\{xF_T | xD_T\} \times LB(xD_T)$$

$$UB(RDM_T) = \sum_{x\in S} (1 - \Pr\{ADM_T | \overline{TL_T} \cap xF_T \cap xD_T\}) \times (1 - LB(TL_T | xF_T \cap xD_T)) \times \Pr\{xF_T | xD_T\} \times UB(xD_T)$$

$$LB(RDM_T) = \sum_{x\in S} (1 - \Pr\{ADM_T | \overline{TL_T} \cap xF_T \cap xD_T\}) \times (1 - UB(TL_T | xF_T \cap xD_T)) \times \Pr\{xF_T | xD_T\} \times LB(xD_T),$$

where:

$$UB(TL_T | xF_T \cap xD_T) = 1 - \prod_{\substack{i=1 \\ x\in MC^i}}^{k} \left(1 - \frac{\prod_{\substack{y\in MC^i \\ y\neq x}} UB(\overline{yD_T})}{LB(xD_T)} \right)$$ is obtained from inequality (5) and

$$LB(TL_T | xF_T \cap xD_T) = \prod_{\substack{j=1 \\ x\notin MP^j}}^{p} \left(1 - \prod_{y\in MP^j} UB(yD_T) \right)$$ is obtained from inequality (7).

The above bounds on the three main event probabilities can be computed from input data, because $UB(\overline{xD_T}), UB(xD_T)$, $LB(\overline{xD_T})$ and $LB(xD_T)$ for all x in S are easily obtained by the following formulae:

$$UB(\overline{xD_{T+1}}) \equiv UB(\overline{xD_T}) - LB(TL_T \cap \overline{xD_T}) + \Pr\{DM_T | \overline{TL_T} \cap xF_T \cap xD_T\} \times (1 - LB(TL_T | xF_T \cap xD_T)) \times \Pr\{xF_T | xD_T\} \times UB(xD_T)$$

$$LB\left(\overline{xD_{T+1}}\right) \equiv LB\left(\overline{xD_T}\right) - UB\left(TL_T \cap \overline{xD_T}\right) + \Pr\left\{DM_T \middle| \overline{TL_T} \cap xF_T \cap xD_T\right\} \times \left(1 - UB\left(TL_T \middle| xF_T \cap xD_T\right)\right) \times \Pr\left\{xF_T \middle| xD_T\right\} \times LB\left(xD_T\right),$$

where:

$$UB\left(TL_T \cap \overline{xD_T}\right) \equiv \sum_{\substack{y \in S \\ y \neq x}} \left(1 - \prod_{\substack{i=1 \\ y \in MC^i}}^{k} \left(LB\left(yD_T\right) - \prod_{\substack{z \in MC^i \\ z \neq x \\ z \neq y}} UB\left(\overline{zD_T}\right) \times UB\left(\overline{xD_T}\right) \right) \right) \times \Pr\left\{yF_T \middle| yD_T\right\},$$

$$LB\left(TL_T \cap \overline{xD_T}\right) \equiv \sum_{\substack{y \in S \\ y \neq x}} \left(\prod_{\substack{j=1 \\ x, y \notin MP^i}}^{p} \left(1 - UB\left(MP_T^j\right)\right) \times \Pr\left\{yF_T \middle| yD_T\right\} \right) \times LB\left(yD_T\right) \times LB\left(\overline{xD_T}\right),$$

from (6) and (8), and bearing in mind that, with the complement of xD_T and the relation $LB\left(\overline{xD_T}\right) \leq \Pr\left\{\overline{xD_T}\right\} \leq UB\left(\overline{xD_T}\right)$, we have: $UB\left(xD_T\right) = 1 - LB\left(\overline{xD_T}\right)$ and $LB\left(xD_T\right) = 1 - UB\left(\overline{xD_T}\right)$.

The algorithm that enables us to compute upper and lower bounds for the main events TL, ADM, and RDM will be referred to in the sequel as the Bound Algorithm (BA). The complexity of BA for a horizon of T cycles is $O\left(T.n^2\left(mL_c + pL_p\right)\right)$, where n is the number of components in S, L_c is the maximum length of a minimal cut, and L_p is the maximum length of a minimal path.

IV. Computational Experiments

In view of its industrial implementation, we have to evaluate the practical relevance of our Bound Algorithm (BA). We will compare BA with the Markov approach. The Markov approach has the advantage of providing the exact solution, allowing us to evaluate the accuracy of BA and its relative efficiency. Implementing the Markov approach, for our model, involves three states per component: “working”, “failed in current cycle”, and “failed from previous cycles”. Due to the excessive CPU time required by the Markov approach, we restrict this comparison to a horizon of 100 cycles. This horizon is sufficient for industrial validation purposes because, in practice, aircraft reliability models generally do not consider periods greater than 100 cycles.

We choose $\Pr\left\{xF_T \middle| xD_T\right\} = 10^{-4}$ for any component x in S, a typical component failure rate in aircraft reliability. Moreover, we start the process with all components working, i.e. $\Pr\left\{xD_1\right\} = 1$ for any x in S. The BA and Markov approaches are programmed in MATLAB. We perform all computational experiments on a 256 Mb PC Pentium III running under Windows 2000 (except for some CPU-intensive runs of the Markov approach that are performed on a Sun SF6800 with 900 MHz CPU under Unix Solaris 8).

Let us now report numerical results on two aircraft applications: standard k-out-of-n:F examples and a typical Air Data Inertial Reference System (with fictitious data).

A. *k-out-of-n:F systems*

In this application, the components of each k-out-of-n:F system are assumed to be i.i.d. The worst-case complexity for k-out-of-n:F systems is: $O(n^3.2^n)$ for B.A, against $O(n.4^n)$ for the Markov approach. We shall study all pairs (k,n) with $2 \leq k \leq n \leq 12$. These values cover a wide range of k-out-of-n aircraft system instances.

In the computational experiments that follow, averages are taken over 100 cycles (i.e. $T = 1,2...100$). Table 1 displays the average BA absolute error of the upper bound for $\Pr\{TL_T\}$. The maximal absolute error value of Table 1 is 5.3E-4. This error is not significant because it lies completely within the reliability bounds of data uncertainties. The analogous results on ADM and RDM are not presented because they derive directly from those on TL, and are even much better in terms of performance (accuracy). In terms of CPU time, BA requires at most 258 seconds, whereas the exact Markov approach needs up to 3,000 seconds. It is of interest to compare the unreliability values of TL estimated by BA with the exact result provided by the Markov model. Indeed, state dependency has a strong impact on the probabilities of TL, while the first contributor to the indicators ADM_T and RDM_T is component unreliability.

N =	2	3	4	5	6	7	8	9	10	11	12
k = 2	1.64E-08	3.08E-06	1.21E-05	2.94E-05	5.72E-05	9.74E-05	1.51E-04	2.21E-04	3.07E-04	4.11E-04	5.33E-04
k = 3		6.26E-08	3.88E-07	1.30E-06	3.23E-06	6.72E-06	1.24E-05	2.10E-05	3.32E-05	5.00E-05	7.22E-05
k = 4			1.64E-09	1.10E-08	4.11E-08	1.15E-07	2.67E-07	5.47E-07	1.02E-06	1.77E-06	2.91E-06
k = 5				3.82E-11	2.79E-10	1.15E-09	3.53E-09	8.95E-09	1.99E-08	4.01E-08	7.49E-08
k = 6					8.24E-13	6.63E-12	3.00E-11	1.00E-10	2.75E-10	6.59E-10	1.43E-09
k = 7						1.68E-14	1.48E-13	7.33E-13	2.65E-12	7.87E-12	2.03E-11
k = 8							3.25E-16	3.16E-15	1.70E-14	6.65E-14	2.12E-13
k = 9								5.75E-18	6.13E-17	3.57E-16	1.51E-15
k = 10									4.17E-19	4.53E-18	2.68E-17
k = 11										5.87E-21	6.95E-20
k = 12											8.21E-23

Table 1: Absolute error of the upper bound for the k-out-of-n:F systems

B. *Real aircraft system*

To illustrate the efficiency of the algorithm, we consider a typical aircraft avionic system. The fault tree in Figure 4 models this. It is a large system with 7 different types of components that correspond to 21 components, with 99 minimal cuts, and 3 minimal paths. The running time for the BA is 26.84 seconds in CPU time. The number of states for the Markov model is around 500,000 and its running time about 51 hours in CPU time.

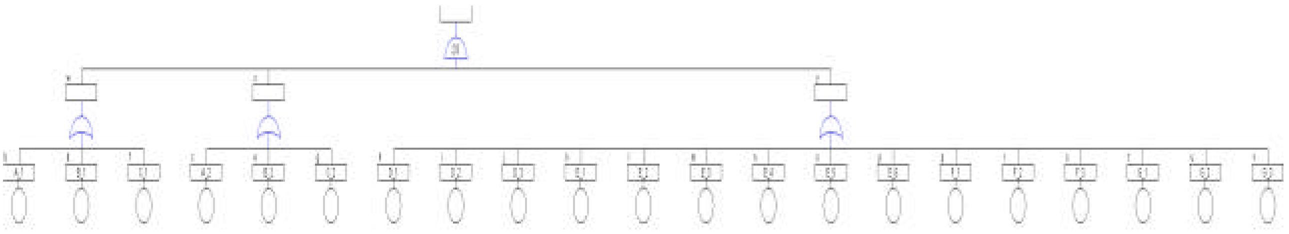

Figure 4: Fault tree modelling the total loss of functions

Table 2 displays the difference between both the BA upper bound and lower bound with the exact Markov results as a function of the number of cycle. Evidently, the maximal error is obtained for 100 cycles. This Upper Bound maximal error is 2.43E-6, and corresponds to a 1.45 % relative error. Due to real-world data uncertainties, these approximations are completely satisfactory for predicting in-service aircraft reliability. In particular, this Air Data Inertial Reference System case shows that BA is a promising method in terms of accuracy and efficiency when applied to aircraft systems.

Cycle *T*	1	2	3	10	30	50	70	100
$UB(TL_T) - \Pr\{TL_T\}$	0	1.14E-09	3.08E-09	3.79E-08	3.14E-07	7.90E-07	1.39E-06	2.43E-06
$LB(TL_T) - \Pr\{TL_T\}$	0	-1.34E-09	-3.87E-09	-5,37E-08	-4,74E-07	-1.23E-06	-2.25E-06	-4.12E-06

Table 2: Absolute error of the upper bound for the k-out-of-n:F systems

V. CONCLUSION

In this paper, we introduced a new methodology for modeling and computing in-service aircraft reliability at the preliminary design stage. We demonstrated both efficiency and accuracy of this method on real aircraft systems. For the sake of simplicity, we restricted our presentation to an elementary set of maintenance tasks: "removal" and "damage tolerance". Current industrial implementation at Airbus does include other types of maintenance tasks (planned maintenance, preventive maintenance etc…).

Future related work will attempt to extend our approach (model and algorithm) to the broader problem of aircraft punctuality by computing the probability of operational interruption.

ACKNOWLEDGMENT

This research was prompted by Airbus, which provided the data. It was financially supported by Airbus and Agence Nationale pour la Recherche et la Technologie (ANRT).

REFERENCES

[1] E. Hugues, L. Saintis, A. Cabarbaye, "ORA, model & tool for Operational Reliability Prediction within Airbus", in Proc. *14th Lambda-Mu Fiabilité Maintenabilité Conference*, 2004, pp 448-451.

[2] E. Hugues, E. Charpentier, A. Cabarbaye, "Application of Markov processes to predict aircraft operational reliability", in Proc. *3rd European Systems Engineering Conference (EUSEC)*, 2002, pp. 231-235.

[3] L. Warrington, J.A Jones, "Modeling of Maintenance within Discrete Event Simulation", in Proc. *48th RAMS*, 2002, pp. 260-265.

[4] J. Loman, W. Wang, "On Reliability Modeling and Analysis of Highly-Reliable Large Systems", in Proc. *48th RAMS*, 2002, pp. 456-459.

[5] L. Warrington, J.A Jones, "Representing Complex Systems within Discrete Event Simulation for Reliability Assessment", in Proc. *49th RAMS*, 2003, pp. 487-492.

[6] Z. Tang, JB. Dugan, "Minimal Cut Set/Sequence Generation for Dynamic Fault Trees", in Proc. *50th RAMS*, 2004, pp. 207-213.

[7] Y. Ma, KS. Trivedi, "An Algorithm for Reliability Analysis of Phased-Mission Systems", *Reliability Engineering and System Safety* 66, 1999 page 157-170.

[8] S. Amari, G. Dill, E. Howald, "A New Approach to Solve Dynamic Fault Trees", in Proc. *49th RAMS*, 2003, pp. 374-379.

[9] T. Khoda, K. Inoue, "Probability Evaluation of System-Failure Occurrence Based on Minimal Cut-sets", in Proc. *48th RAMS*, 2002, pp. 456-459.

[10] L. Saintis, E. Hugues, C. Bès, M. Mongeau, "Methods to asses the operational reliability of an aircraft system : Problem of dependency between component states", in Proc. *15th Lambda-Mu Fiabilité Maintenabilité Conference*, 2006.

[11] J.D. Esary, F. Proschan, "Coherent Structures of non-identical components", *Technometrics* 5, 1963, pp. 191-209.

[12] W. Kuo, M. J. Zuo, "Optimal Reliability Modeling: Principles and Applications", Wiley, 2002.

Laurent Saintis taught mathematics at high-school level from 2000 to 2003 in Toulouse and Stockholm. He then received a Masters in Applied Mathematics from Université Paul Sabatier (Toulouse, France) in 2004. He is currently a PhD student in Applied Mathematics and Mechanical Engineering at Université Paul Sabatier in an industrial program in collaboration with Airbus.

Emmanuel Hugues received an Engineering Degree in Aeronautics and a Masters in Applied Mathematics at the École Nationale Supérieure en Aéronautique et Espace (Toulouse, France), then obtained a PhD in Applied Mathematics, from Université Paul Sabatier (Toulouse, France) for his work on the implementation of a Neural Network Workbench for Airbus Maximal Take-Off Weight calculation. Since 1999, he has worked with Airbus in Toulouse and has been involved in several R&D and operational projects dealing with Supportability and Operational Reliability. Since 2002, he manages the research development and deployment of Operational Reliability Processes and Tool within the Airbus Engineering Division.

Christian Bès received a doctorate in Automatics from Université Paul Sabatier (Toulouse, France) in 1984. From 1984 to 1986, he worked in Operations Research as a post-doctoral researcher at the University of Toronto, and at the Institut National de la Recherche en Informatique et en Automatique (Rocquencourt, France). He received a *Habilitation à Diriger des Recherches* in Applied Mathematics from Université Paul Sabatier in 1995. He worked with Airbus in Toulouse from 1986 to 1999. He is currently a Professor in Mechanical Engineering at Université Paul Sabatier. His current research interests include Mechanical Design and Multidisciplinary Optimization.

Marcel Mongeau received his BSc (1985) and MSc (1987) degrees in Mathematics from Université de Montréal, and his PhD (1991) in Combinatorics & Optimization from the University of Waterloo (Canada). He was then a post-doctoral researcher at the Centre de Recherches Mathématiques (Université de Montréal), and at the Institut National de la Recherche en Informatique et en Automatique (Rocquencourt, France). In 1993-94, he was a Research Fellow at the University of Edinburgh. Since 1994, he has been a Maître de Conférences in Mathematics at Université Paul Sabatier (Toulouse, France), where he received a *Habilitation à Diriger des Recherches* in 2003. Dr Mongeau is currently on leave at LAAS-CNRS in Toulouse since October 2004. His research interests include Global Optimization, Numerical Optimization, and Operations Research.

Laurent Saintis
B.P. M0101/5
Airbus France
316 route de Bayonne
31060 Toulouse cedex 3
France

Emmanuel Hugues
B.P. M0101/5
Airbus France
316 route de Bayonne
31060 Toulouse cedex 3
France

Christian Bès
Département de Génie Mécanique
UFR PCA
31062 Toulouse cedex 9
France

Marcel Mongeau
LAAS - CNRS
7 av du Colonel Roche
31077 Toulouse cedex 4
France