Minimax Current Density Coil Design

M. S. Poole[1], P. Weiss[2], H. Sanchez Lopez[1], M. Ng[3], and S. Crozier[1]
[1]School of Information Technology and Electrical Engineering, University of Queensland, Brisbane, QLD, Australia, [2]Institut de Mathematiques, Universite Paul Sabatier Toulouse 3, Toulouse, France, [3]Department of Mathematics, Hong Kong Baptist University, Kowloon Tong, Hong Kong

Introduction Gradient and shim coils are required to generate a magnetic field of particular spatial form when energised with electric current. The design of these coils is a well studied inverse problem [1] that is inherently ill-conditioned with an infinite number of solutions. In order to obtain a single solution, the system must be regularised, which is commonly achieved by minimising the stored energy or power dissipation of the coil [1]. In some cases, this results in a solution that possesses regions of high current density. This may cause problems with construction, localised heating and maximum field strength. In this work we design coils with minimum, maximum current density magnitude, or minimax$|j|$.

Methods The design of minimax$|j|$ coils is general and does not restrict the method by which the coils are modelled. We have employed three different boundary element methods (BEMs); one in which the current density is assumed to be a finite sum of truncated sinusoids on a cylindrical surface [2,3], a second designs coils on any surface described by triangular elements with uniform current density in each [4] and a that method assumes only rotational symmetry about one axis [5]. Each of these methods uses a finite number of weights, ψ, to define the coil. Therefore matrices can be generated for each of the coil design parameters and numerical techniques may be applied. The functional that we optimise in the present work is

$$\underbrace{\min}_{\psi\in\Psi}\left\{U(\psi)=f(\psi)+\alpha e(\psi)+\beta W(\psi)+\gamma P(\psi)+\delta\,\|\,j(\psi)\,\|_\infty\right\} \tag{1}$$

f is the primary field term, e is the eddy current field term, W is the stored energy, P is the power dissipation, j is the current density magnitude, Ψ is the set of all solutions with zero net torque and α, β, γ and δ are user-definable weights which ≥ 0. $\|\cdot\|_\infty$ is the ℓ^∞-norm which finds the maximum value of its argument. When $\delta \neq 0$, U is non-differentiable with respect to ψ. We use an accelerated descent algorithm of Nesterov [6] on the smoothed norm dual to solve (1). Many minimax$|j|$ coils were designed with various BEMs and parameters. In the limited space of this abstract we present the wire paths of sum-of-sinusoids min(P) and minimax$|j|$ X-gradient coils with 5% max field error side-by-side for comparison. These X-gradients illustrate the principal difference between the designs. Small scale coils were milled into flexible PCB with 18 micron-thick copper for testing. R and L were scaled up to reflect full-sized coils with 3mm thick copper tracks.

Results Below are one quadrant each of the wire-paths and the important performance data for the two example X-gradient coils. The coil on the left is the min(P) (α=0, β=0, γ=6.9×10^{-11} and δ=0) X-gradient and the minimax$|j|$ coil (α=0, β=0, γ=5×10^{-12} and δ=3.33×10^{-9}) is on the right.

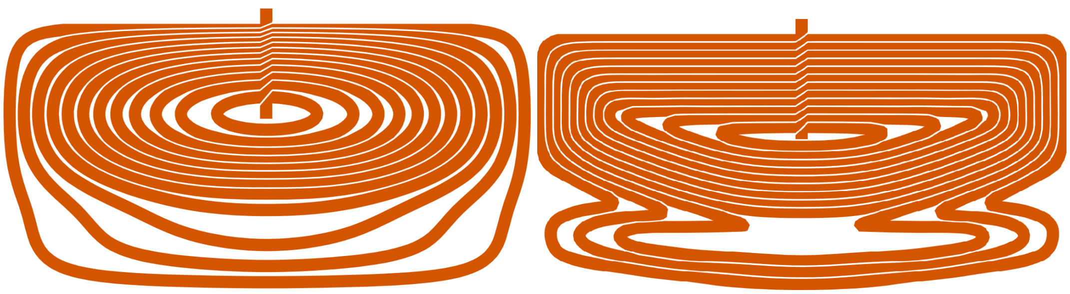

Figure 1. Wire-patterns of one quadrant each for the min(P) (left) and minimax|j| (right) X-gradient coils

	min(P)	minimax$|j|$
Radius (m)	0.38	0.38
Length (m)	1.2	1.2
η (μTm^{-1}A^{-1})	90.3	89.6
L (μH)	260	299
R (mΩ)	445	552
Wire spacing (mm)	9.5	18.0

Table 1. Performance properties of the two coils shown in Fig. 1.

Discussion and Conclusions The results shown above exemplify the marked difference between the wire-patterns of the minimax$|j|$ coil and min(P) coil. The region of high current density at the top of the coil is dispersed and maximally so. The spacing between the closest wires of the design is almost doubled. Consequently, there was some increase in the total resistance and inductance of the coils. The field error is the same for both and the efficiency is slightly reduced in the case of the minimax$|j|$ coil. None of this should be surprising as the objective of the coil design has been shifted away from minimal stored energy or power dissipation to maximally spread the wires. All other coils behaved in a similar manner to a greater or lesser extent. It is important to note also, that the coil designer may choose any solution between minimax$|j|$ and min(P). In some cases this may be desirable since extra loops appear in the coils with extremely spread wires.

Designing coils with more spread wires may help with construction of the coils since the gap that is cut has a minimum size dependant on the method of manufacture. Also, it may help isolate each turn from its neighbouring turn to reduce the risk of sparks when very high voltages are being used. Another motivation for lowering the maximum current density is to reduce the temperature of the hot spots of the coil. Although overall more heat is being generated, the reduced temperature of the hot spots may permit more current or duty cycle without reaching dangerous localised temperatures in the coil that could cause failure of the gradient coil.

Using the Nesterov scheme allows us to efficiently solve (1). A previous approach [7] showed reduced maximum current density, but was not optimal. Minimax$|j|$ coils can be considered as using all the available surface. This means that they have the highest possible efficiency for a given wire spacing limit and coil surface since more turns may be added to the coil. This can improve the strength of coils in which neither stored energy nor power dissipation is an issue.

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