Propagation enhancement by a line of fast diffusion

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Goal: Give a mathematical content to the following statement: transportation networks enhance biological invasions...

Example 1: Displacement directed by the habitat

- *Seismic lines:* straight lines (width $\sim 5m$) used by oil companies to detect oil.

Figure: 1. Seismic lines in Alberta

Motivation

- GPS observations (McKenzie et al., 2012): wolves concentrate and move preferentially along seismic lines.
- Derivation by Hillen-Painter (2011) of a diffusive model and simulations.

Example 2: Spreading of invasive species

Figure: 2. Tiger mosquito

Figure: 3. Processionary pine moth

Some invasive species use road networks to spread faster than theoretical speed.

Example 3: Propagation of black plague in Europe (1347-1352)

Historic data: the pandemics spreads quickly along the main roads and spreads inland from them.

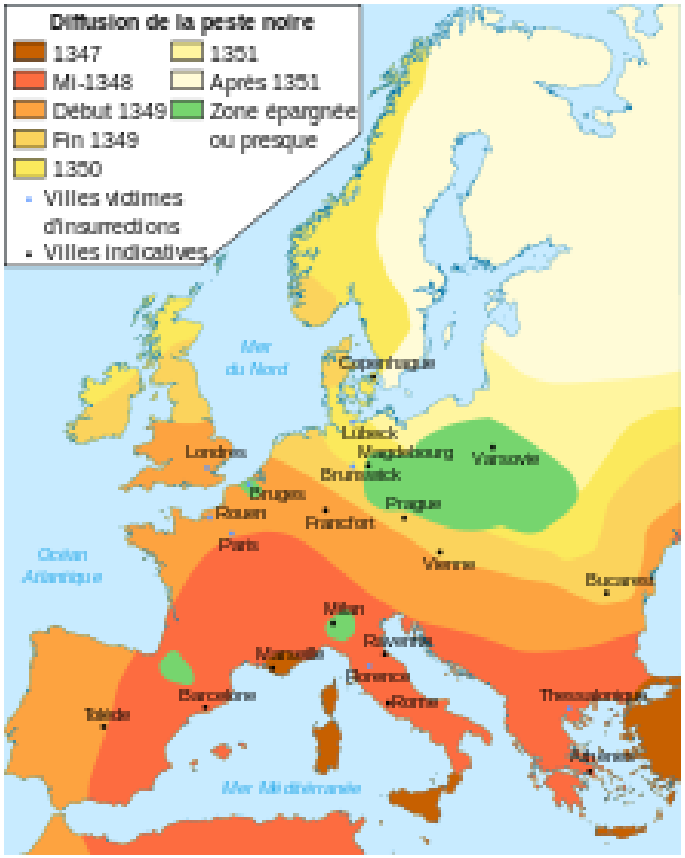
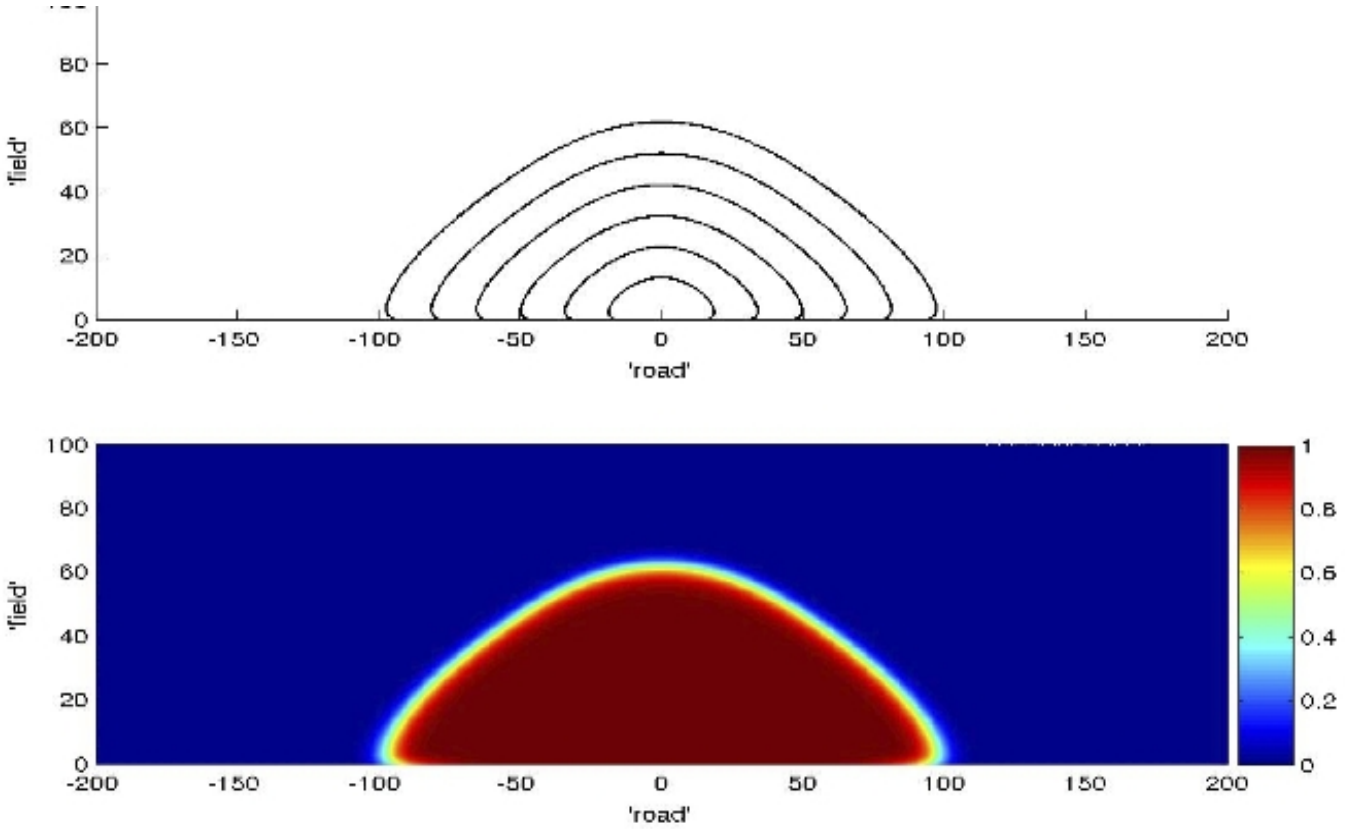


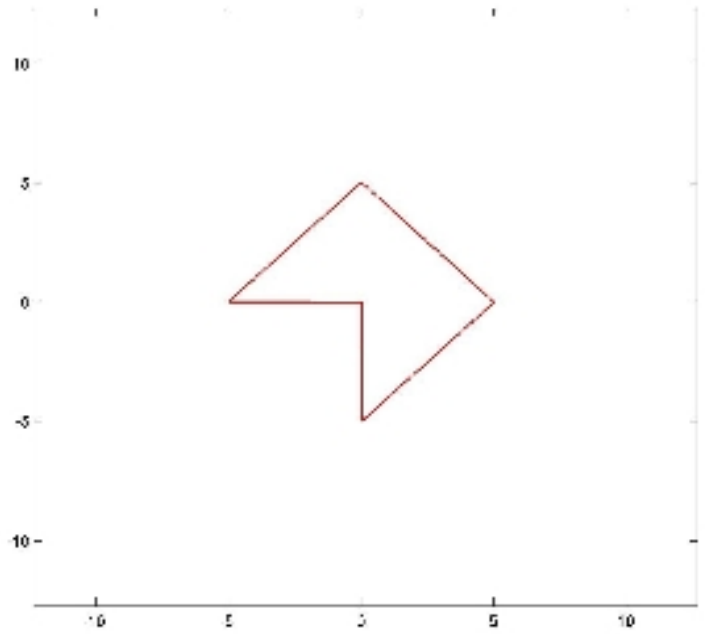
Figure: 4. Black plague in Europe

The case $L = -D\partial_{xx}$: level sets of v



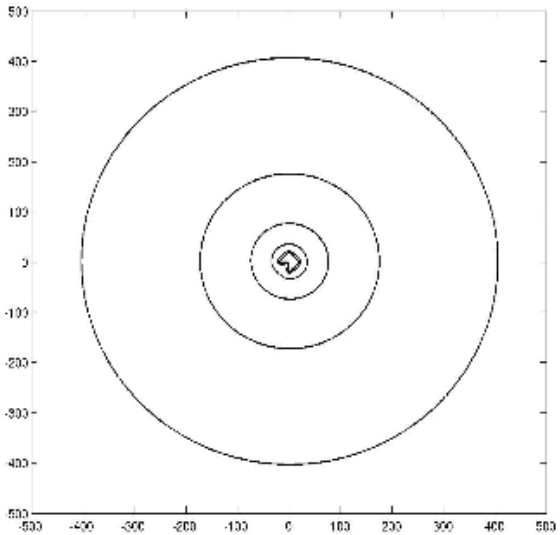
Numerics (A.-C. Coulon)

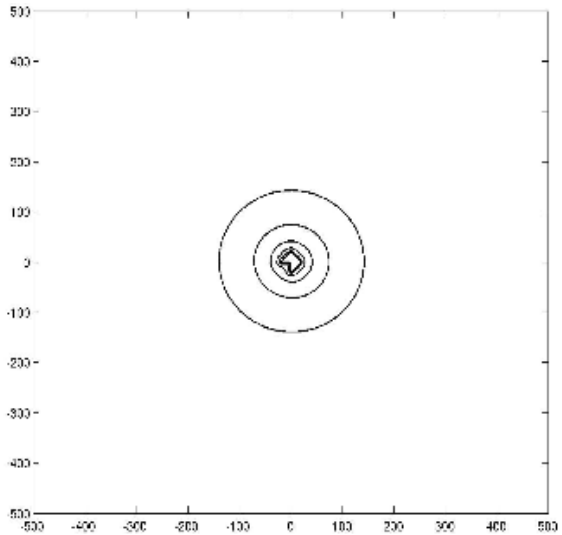
$u_t + (-\Delta)^\alpha u = u - u^2$: **level sets of** u_0



Numerics (A.-C. Coulon)

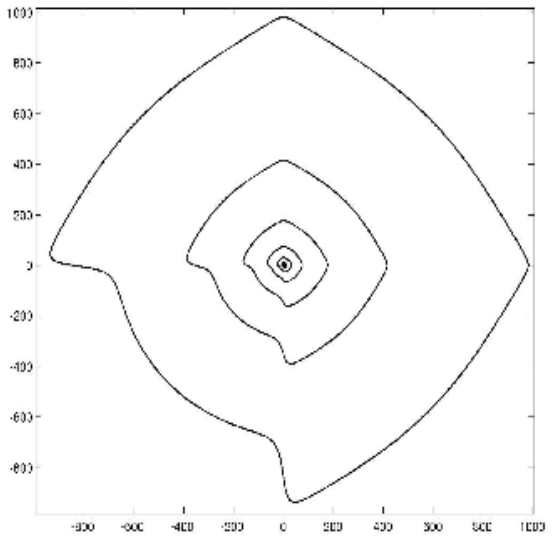
$u_t + (-\Delta)^\alpha u = u - u^2$: **level sets of** u **when** $u_0(x) = O(e^{-|x|})$.

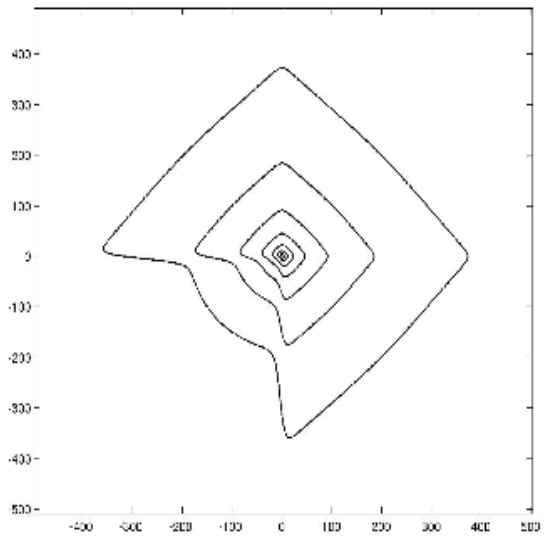




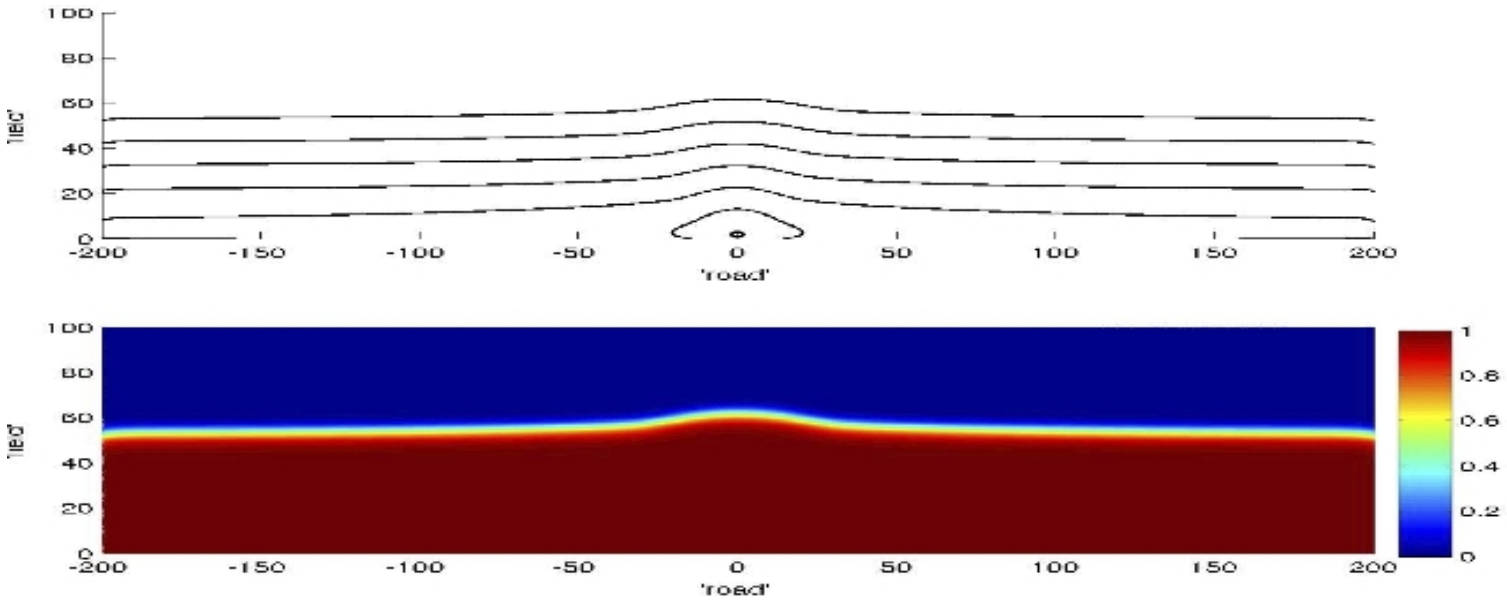
Numerics (A.-C. Coulon)

The case $L = (-\partial_{xx})^{\alpha}$: $u_0(x) \sim |x|^{-(N+2\alpha)}$ in upper right quadrant.





The case $L = (-\partial_{xx})^{\alpha}$: level sets of $v(t, x, y)$.



The case $L = (-\partial_{xx})^{\alpha}$: level sets of $\tilde{u}(t,x) = u(t, e^{-t/1+2\alpha} t^{3/2} x)$.