Introduction to mathematical finance

Sheet 9 HS 2014

Hand-in your solutions until 26.11.2014, in Martina Dal Borgo's mailbox on K floor.

Exercise 1

In an arbitrage free two period ($T = 2$ years) binomial model, consider an American option with payoff

$$X_t = \min\left\{\max\{S_t, K_1\}, K_2\right\}.$$

The parameters of the model are

$$u = 2, \quad d = \frac{1}{2}, \quad , S_0 = 1, \quad K_1 = 1, \quad K_2 = 2, \quad r = \frac{1}{2}.$$

i) Determine the process of the American option price.

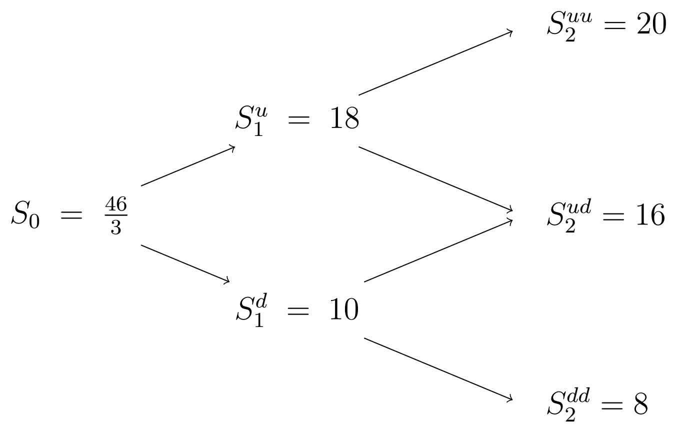
ii) When is exercising optimal from the buyer's point of view?

iii) Show that, if the customer does not exercise the option at time $t = 1$ (for instance consider the case where $S_1 = S_1^u$), a risk profit for the seller results, equal to $\frac{8}{9}$.

In the following exercises, we will consider non-homogenous models described by trees. Formally speaking, the state space Ω of all possible scenarios is identified with the *paths* in the tree. A probability $\mathbb{Q}$ on $(\Omega, \mathcal{P}(\Omega))$ is completely specified by the probability of each edge on the tree. This way, one realises that the model is simply obtained by gluing many binomial 1-period models, with different parameters.

Exercise 2 (*non homogeneous model*)

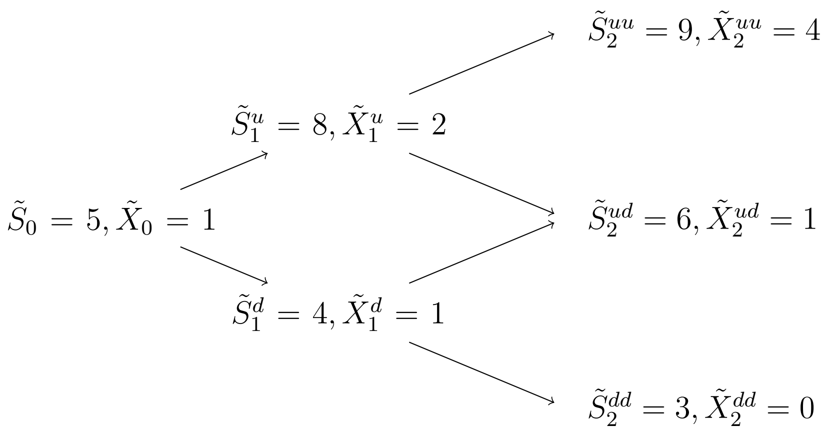
We consider a two period (2 years) arbitrage free financial discrete market model consisting of a bond and a stock with price process $S = (S_0, S_1, S_2)$. We introduce a European call option on S with maturity $T = 2$ years and strike $K = 15$. Assume $r = 0$ and that the process S evolves according to the following tree, where each scenario has a positive probability



Compute the initial price of the European Call. You are advised to compute risk-neutral transition probabilities and present them on the edges of the tree.

Exercise 3 (*non homogeneous model*)

We consider a two period arbitrage free financial market model consisting of a bond and a stock with *discounted* price process $\tilde{S} = (\tilde{S}_0, \tilde{S}_1, \tilde{S}_2)$. We introduce an American option with *discounted* payoff process $\tilde{X} = (\tilde{X}_0, \tilde{X}_1, \tilde{X}_2)$. Assume that the processes evolve according to the following tree, where each scenario has a positive probability



i) compute the minimal capital requirement that is necessary to perfectly super-replicate the American option.

ii) When is exercising optimal?

iii) Determine the knots in which the seller of the option gets a riskless profit if the buyer exercises the option.