Sheet 3

Standard facts

Exercise 1. [The unitary trick - (1.14) in [1]] Let V be an irreducible representation of the finite group G. Show that, up to scalars, there is a *unique* Hermitian inner product $\langle , \rangle$ on V invariant under G:

$$\forall g \in G, \forall (x, y) \in V^2, \langle g \cdot x, g \cdot y \rangle = \langle x, y \rangle$$

Exercise 2. [(2.34) in [1]] Let V and W be irreducible representations of G and $L_0 : V \to W$ be any linear mapping. Define $L : V \to W$ by:

$$L(v) = \frac{1}{|G|} \sum_{g \in G} g^{-1} \cdot L_0(g \cdot v)$$

Show that $L = 0$ if V and W are not isomorphic, and that L is a multiplication by $\frac{trace(L_0)}{\dim V}$ if $W = V$.

On the dihedral group

Exercise 3. [(3.7) and (3.8) in [1]] The dihedral group D_{2n} is the group of isometries of a regular n-gon in the plane. It is made of n rotations and n reflections. The purpose of this exercise is to understand the representation theory of this group.

- Method 1: Consider an arbitrary representation V. Notice that Γ, the subgroup of rotations is abelian. The space V breaks up into eigenspaces for the action of Γ. Analyze the action of reflections on these eigenspaces, and explain how to decompose the representation V.
- Method 2: Using character theory, give a character table.

References

[1] Fulton, Harris. Representation theory: A first course. Springer 1991.