Sheet 2

Warm up

Exercise 1. [(2.5) in [1]]
Let G be a finite group acting on a set X and consider V the associated permutation representation.
Prove that for $g \in G$ the character $\chi_V(g)$ is given by the number of fixed points of g acting on X.
Application: Give the character of the (left-)regular representation of G.

On the symmetric group S_3

Recall that S_3, the symmetric group acting on 3 elements, has three irreducible representations known as:

   a) $V(triv)$: The trivial representation.

   b) $V(alt)$: The alternate representation.

   c) $V(st)$: The standard representation.

Exercise 2. [(2.7) in [1]]
Decompose the representation $V(st)^{\otimes n}$ into irreducibles.

Exercise 3.
Decompose the following representations into irreducibles:

   - $Sym^2(V(st))$

   - $\Lambda^2(V(st))$

   - $\left(Sym^2(V(st)) \oplus V(triv)\right) \otimes V(alt)$

   - $\left(Sym^2(V(st)) \oplus V(triv)\right) \otimes V(st)$

References

[1] [FH] Fulton, Harris. Representation theory: A first course. Springer 1991.