Diffusion limits of kinetic models *

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Abstract

This paper reports on recent developments in diffusive limits of kinetic systems. Usually, collision operators in kinetic theory exhibit multiple relaxation scales and before a full relaxation towards a Maxwellian equilibrium has been achieved, the system passes through a series of states that can be described by partial equilibria. The paper describes various models describing the dynamics of these partial equilibria, namely the SHE (Spherical Harmonics Expansion) and the ET (Energy Transport) models. Various examples of applications of these models to plasma problems are presented. models.

Key words: Boltzmann equation, diffusive limits, Spherical Harmonics Expansion model, Energy Transport model, Drift-Diffusion model.

AMS Subject classification:

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1 Introduction

Let us consider a particle system (e.g a gas, a plasma, or a semi-conductor, etc). A fundamental question is how the elementary particles properties translate into the large-scale behaviour of the system. This question is an old problem (which goes back, to Maxwell, Boltzmann, Einstein, ...). However, recent developments have revived this question as it is central to the problem of model reduction in many scientific or industrial areas. Indeed, the use of microscopic models of kinetic type is too expensive. However, resorting to standard macroscopic models such as the drift-diffusion model leads to unacceptable inacurracy, as the account for the microscopic physics in such model is too coarse.

The aim of this paper is to present a strategy in order to derive a hierarchy of intermediate models which provide a better description of the physics at still an affordable numerical cost.

The outline of the paper is as follows. In section 2, we shall review the standard derivation of macroscopic diffusion models from microscopic kinetic models. Then, section 3 will present a first 'improved' model: the Spherical Harmonics Expansion (SHE) model. Section 4 will deal with a second such 'improved' model: the Energy-Transport model (ET) model, which is derived from the SHE model by a suitable averaging. Then, we shall conclude in section 5.

2 From micro to macro: standard theory and Drift-Diffusion limits

The microscopic description of particle systems uses the concept of distribution function (or phase-space density) $f(x, v, t)$, which describes the number of particles at position x, with velocity v, at time t. Each individual particle moves according to a succession of free flights in a given (or self-consistent) force field $F(x, t)$ and of collisions (among themselves and with surrounding obstacles). During their free flight, each individual particle follows Newton's equations

$$\frac{dx}{dt} = v \,, \quad m\frac{dv}{dt} = F(x, t) \,,$$

where m is the particle mass.

The time evolution of the distribution function $f$is such that the total time derivative of f while following free-flight trajectories is given by a collision operator $Q(f)$:

$$\frac{d}{dt} f(x(t), v(t), t) = Q(f)|_{(x(t),v(t),t)} \,.$$

The collision operator $Q(f)$ is local in (t, x) (as collisions are supposed instantaneous and the distance travelled by the particle during a collision is supposed small compared with the distance travelled during free-flights). Therefore, $Q(f)$ operates on the velocity variable v only. Applying the chain rule immediately leads to the Boltzmann equation:

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + \frac{1}{m} F \cdot \nabla_v f = Q(f) \,. \tag{2.1}$$

A prototype model for the collision operator, which is a suitable for plasmas or semiconductors is

$$Q(f)|_v = \int_{v'} \phi(v,v')[M(v)f(v') - M(v')f(v)]dv' \,, \tag{2.2}$$

where $\phi(v,v')$ is related to the collision scattering cross section. ϕ contains most of the information about the nature of the interaction which causes the collision event. The function

$$M(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{mv^2}{2k_B T}} \,,$$

is the Maxwellian. It is the equilibrium distribution function towards which the system is going to relax under the influence of the collisions.

The description of a particle system through (2.1), (2.2) is fairly expensive from a numerical view-point because of the high dimensionality of the model (3 space coordinates, 3 velocity coordinates, plus time). In most industrial applications, it is necessary to reduce the dimensionality of the model and to use fluid equations.

The first idea for deriving fluid equations is to use moments. Example of such moments are the density n, the flux j and the energy W, given by:

$$\begin{pmatrix} n \\ j \\ W \end{pmatrix} = \int f \begin{pmatrix} 1 \\ v \\ m|v|^2/2 \end{pmatrix} dv \,.$$

Evolution equations for (n, j, W) can be obtained by integrating the Boltzmann equation with respect to v. However, in such a procedure, one faces a closure problem as the evolution system for n, j, W involves unknown moments, i.e. quantities obtained by integrating f against other functions of v than just $1, v$ or $|v|^2$. Therefore, since f is a priori not known, an arbitrary prescription is necesary to express these moments in terms of n, j and W. Several approahes have been proposed, e.g. [33], [41] and more recently, [37]. However, the so-obtained moment system may be ill-posed. Also, the presription may sometimes seem arbitrary as it is difficult to relate it to some properties of the system and to the associated physical scales.

To overcome the closure problem, a second idea is to consider the physical scales. Indeed, the Boltzmann equation is suitable for the microscopic scale, where the time scale is fixed by the mean time between collisions while the space scale is connected with the distance travelled by particles between two collisions (the so-called mean free path). The macroscopic description however deals with larger scales.

In our context, it is appropriate to introduce a diffusion rescaling. Let us introduce the small parameter η being the ratio of the mean free path to the typical macroscopic distance. The assumption that we are interested in the large scale behaviour of the system translates into $\eta \ll 1$. We now introduce the following change of variables:

$$x' = \eta x \,, \quad t' = \eta^2 t \,, \quad F = \eta F' \,. \tag{2.3}$$

The η^2 in the time rescaling is characteristic of diffusion rescaling, where diffusion occurs on time scales much larger than transport (or hydrodynamic) phenomena.

With this change of variables, the Boltzmann equation is now written:

$$\eta^2 \frac{\partial f^\eta}{\partial t} + \eta \left(v \cdot \nabla_x f^\eta + \frac{1}{m} F \cdot \nabla_v f^\eta \right) = Q(f^\eta) \tag{2.4}$$

The limit $\eta \to 0$ in (2.4) describes the large scale behaviour of the system. The following presentation will stay at a formal level. We refer the interested reader to [42], [32] for rigorous proofs.

The determination of the $\eta \to 0$ limit depends crucially on the Properties of Q, as it appears in the leading order term in (2.4). We list below the most important of these properties:

(i) Conservation of particle number: $\int Q(f) dv = 0$

(ii) Null set $N(Q)$ (equilibria) :

$$Q(f) = 0 \Longleftrightarrow \exists n \in \mathbb{R} \text{ such that } f = nM(v)$$

(iii) $-Q$ is a self-adjoint non-negative operator:

(iv) $-Q$ is Positive-definite and invertible on $N(Q)^\perp$

Property (iii) is related to entropy decay and states that the manifold of equilibria (i.e. the Maxwellians) is stable

The formal derivation of the limit $\eta \to 0$ follows easily from these properties and leads to the so-called Drift-Diffusion (DD) model. More precisely, as $\eta \to 0$, $f^\eta \longrightarrow n(x,t)M(v)$ where $n(x,t)$ satisfies the system:

$$\frac{\partial n}{\partial t} + \nabla_x \cdot j = 0 \, , \tag{2.5}$$

$$j = -A(\nabla_x n - nF) \, . \tag{2.6}$$

The first eq. is called the Continuity equation while the second one is the Current equation.

The Diffusion tensor A is related to Q and ϕ according to

$$A = - \int v Q^{-1}(vM) \, dv \, ,$$

where Q^{-1} denotes the pseudo-inverse (acting from $N(Q)^\perp$ into itself), $Q^{-1}(vM)$ denotes the vector of components $Q^{-1}(v_i M)$ and $vQ^{-1}(vM)$ is a tensor product. As a consequence of (iv), A is a symmetric, positive-definite tensor.

The proof of this result follows the following steps: First, we note that if $f^\eta \to f$ smoothly, then , by (2.4), $Q(f) = 0$, i.e. by property (ii), $f = n(x,t)M(v)$ where the function $n(x,t)$ is still to be determined. Then, the continuity eq. (2.5) follows from integrating eq. (2.4), with respect to v, using the Conservation of particle number (property (i)) and letting $\eta \to 0$. Finally, to derive the current eq (2.6), it is necessary to use a perturbation approach and to write

$$f^\eta = nM + \eta f^1 + o(\eta) \, , \tag{2.7}$$

from which, $j = \int f^1 v\, dv$ is obtained. f^1 may be viewed as the microscopic response of the system to gradients of macroscopic variables. To obtain f^1, one inserts the Hilbert expansion (2.7) into (2.4) and restricts to the leading order term. This leads to an equation for $Q(f^1)$ which is solved, thanks to property (iv). Details can be found in the above-cited references.

The Drift-Diffusion model is widely used by the engineers and has motivated considerable studies in both the semiconductor, plasma or gas discharge physics contexts (see e.g. [38], [44], [27]. However, it has proved unsuitable to describe strongly non equilibrium phenomena. One major reason for this is that it supposes instantenous relaxation of the particle temperature to the background temperature T. This assumption fails for fast phenomena in which energy relaxation may be slower than the time evolution of the system.

The question which we are going to investigate in the next sections is the derivation through the same kind of methodology, of other more elaborate macroscopic models with a broader range of physical applicability.

3 The Spherical Harmonics Expansion (SHE) model

Our approach relies on the observation that most collision operators exhibit a wide range of Relaxation scales. Indeed, collisions may relax some moments faster than other ones: typically, momentum relaxation is usually faster than energy relaxation: This is particularly the case in plasmas, becaue of the very small electron mass compared with the ions mass, or in semiconductors, wheree the phonon energy is much smaller than the electron energy.

Therefore, our aim is to derive a model for the intermediate scale, i.e. after momentum relaxation has occurred but Before energy relaxation has taken place. In such a case, we can expect that the distribution function f is isotropic with respect to v as a consequence of momentum relaxation, but is not yet Maxwellian.

Our method is to split the collision operator Q into an elastic operator $\mathcal{E}$ (the one which is going to produce momentum relaxation) and an inelastic operator $\mathcal{I}$ (which is responsible for energy relaxation), and then, to consider situations such that $\mathcal{E} \gg \mathcal{I}$. Then we shall proceed like in the first section, with the elastic operator $\mathcal{E}$ replacing Q.

The splitting is performed through a passage to spherical coordinates in velocity space $v = (\varepsilon, \omega)$, where $\varepsilon = m|v|^2/2$ is the energy and $\omega = v/|v|$ is the velocity angle). We then decompose the 'real' collision $v' = (\varepsilon', \omega') \to v = (\varepsilon, \omega)$ into 2 fictitious processes $(\varepsilon', \omega') \to (\varepsilon, \omega')$ and $(\varepsilon, \omega') \to (\varepsilon, \omega)$, introducing a fictitious state (ε, ω'). The first process changes the energy and gives rise to the inelastic operator while the second one only changes the velocity angle and gives rise to the elastic operator. We shall not detail the expressions of these operators and refer to [11] for details. We note $Q(f) = \mathcal{E}(f) + \mathcal{I}(f)$ the resulting decomposition of the collision operator into elastic and inelastic operators.

We now suppose that elastic scattering dominates over inelastic scattering: $\mathcal{E} \gg \mathcal{I}$. We are therefore entitled to rescale the operators according to

$$\mathcal{E} = O(1)\,, \quad \mathcal{I} = O(\eta^2)\,,$$

and we use again a diffusive scaling (2.3) for the variables x, t and F. Linking the scaling of the inelastic operator to the parameter η can be viewed as somewhat arbitrary. However, other scaling hypotheses lead to similar models [21].

The resulting rescaled Boltzmann equation is now written:

$$\eta^2 \frac{\partial f^\eta}{\partial t} + \eta \left(v \cdot \nabla_x f^\eta + \frac{1}{m} F \cdot \nabla_v f^\eta \right) = \mathcal{E}(f^\eta) + \eta^2 \mathcal{I}(f^\eta) \tag{3.1}$$

Again, the investigation of the limit $\eta \to 0$ requires the determination of the properties of the leading term, namely the collision operator $\mathcal{E}$. First, we note that $\mathcal{E}$ is given by

$$\mathcal{E}(f)|_{(\varepsilon,\omega)} = \int_{\omega'} \psi(\varepsilon;\omega,\omega')[f(\varepsilon,\omega') - f(\varepsilon,\omega)] d\omega'$$

where $\psi(\varepsilon;\omega,\omega')$ is related to ϕ by integration with respect to ε'. In particular, it is seen that $\mathcal{E}$ operates on the ω variable only . Now we have

(i) Conservation of particle number on each energy surface:

$$\int \mathcal{E}(f)|_{(\varepsilon,\omega)} d\omega = 0\,, \quad {}_m box \forall \varepsilon.$$

(ii) The null set $N(\mathcal{E})$ consists of constant functions with respect to ω (i.e. all functions of the energy only: $f(v) = G(\varepsilon)$)

(iii) $-\mathcal{E}$ is a self-adjoint non-negative operator and is positive-definite and invertible on $N(\mathcal{E})^\perp$

Then, the limit $\eta \to 0$ follows in the same way as in the previous section. We have: $f^\eta \longrightarrow G(x,\varepsilon,t)$ solving the SHE model:

$$N(\varepsilon) \frac{\partial G}{\partial t} + \tilde{\nabla} J = \tilde{\mathcal{I}}(G)\,, \tag{3.2}$$

$$J = -D(\varepsilon) \tilde{\nabla} G\,, \tag{3.3}$$

where $\tilde{\nabla}$ denotes the operator $\tilde{\nabla} = \nabla_x + F \frac{\partial}{\partial \varepsilon}$. Eq. (3.2) is a continuity equation in the 'extended space' (x,ε) while eq. (3.3) is a current equation. The diffusion tensor $D(\varepsilon)$ is related with $\mathcal{E}$ through

$$D(\varepsilon) = -\sqrt{2\varepsilon} \int v \mathcal{E}^{-1}(v)\, d\omega\,,$$

while $N(\varepsilon) = \sqrt{2\varepsilon}$, is related with the geometry of the energy surfaces. Finally, $\tilde{\mathcal{I}}$ is the contribution of the inelastic collision operator $\mathcal{I}$ and is given by

$$\tilde{\mathcal{I}} = \sqrt{2\varepsilon} \int \mathcal{I}(G)\, d\omega\,.$$

The SHE model has received considerable attention lately. On can find for instance in [7], [8] a derivation of the model via stochastic processes. It is currently used by semiconductor engineers on a routine basis [30], [31]. The present derivation which does not make use of any expansion of the Boltzmann eq. in spherical harmonics can be extended to non spherically symmetric situations, which are common in semiconductor physics. It has first been derived in this way by [29] for simplified relaxation time collision operators. This derivation has later been extended to arbitrary collision operators in [2] (see also [10] for a review in the semiconductor physics context) and by [43] (where additionally, an approximation of the inelastic collision operator in terms of a Fokker-Planck operator is proposed). The proof of convergence of solutions from the Boltzmann to the SHE model is done in [11] in a slightly more general setting.

The SHE model is particularly attractive where relaxation towards an isotropic state results from collisions with the boundary. Indeed, the process in which particles are reflected by the boundary with conservation of energy and randomization of the velocity angle bears similarities with elastic collisions as described above. If particles undergo some confinement along the boundary which makes encounters with the boundary extremally frequent, then, the lateral dynamics (in the directions parallel to the boundary) can be described by a SHE type model. In this case, the passage from Boltzmann to SHE not only saves 2 velocity dimensions (the coordinates of the velocity angle), but also one space dimension (the coordinate in the direction normal to the boundary) and is particularly efficient.

Two situations have been investigated in which such an occurrence happens: first the motion between two parallel plates seperated by a very small gap. Second, the case of a confinement force which reflects the particles back to the boundary. Both cases correspond to situations of practical interest. The first case is a model for electron transport in Stationary Plasma Thrusters, a device used in satellite propulsion. The precise physical model is described in [18] and [36] as well as in [35]. As a matter of example of the efficiency of the SHE model, we show in figure 1 a comparison between the SHE model and the solution of the original Boltzmann equation by means of a standard Monte-Carlo simulation. The SHE model is about a hundred times faster.

The case where the confinement is due to a confining force has been investigated in [12]. The resulting SHE model has been used to derive an Energy-Transport model (see next section) for certain kinds of surface discharges that occur for instance on satellite solar pannels [25]. Indeed, in physically complex situations, the SHE model, with its additional energy variable, is still too costly compared with a standard fluid model. This is why it is attractive to investigate if the SHE model can be reduced to such a standard model. This is the aim of the next section.

4 The Energy-Transport (ET) model

We are going to consider a relaxation limit of the SHE model in which the inelastic operator $\tilde{\mathcal{I}}$ tends to relax $G(x, \varepsilon, t)$ to a thermodynamical equilibrium. Such a relaxation mechanism may be due to e.g. binary collision, turbulence, etc ... For that purpose, we assume that $\tilde{\mathcal{I}}$ satisfies the following hypotheses:

(i) The Null Set $N(\tilde{\mathcal{I}})$ consists of Maxwellian equilibria, with arbitrary temperature T:

$$G_{\mu,T}(\varepsilon) = e^{-((\varepsilon-\mu)/k_B T)}\,, \tag{4.1}$$

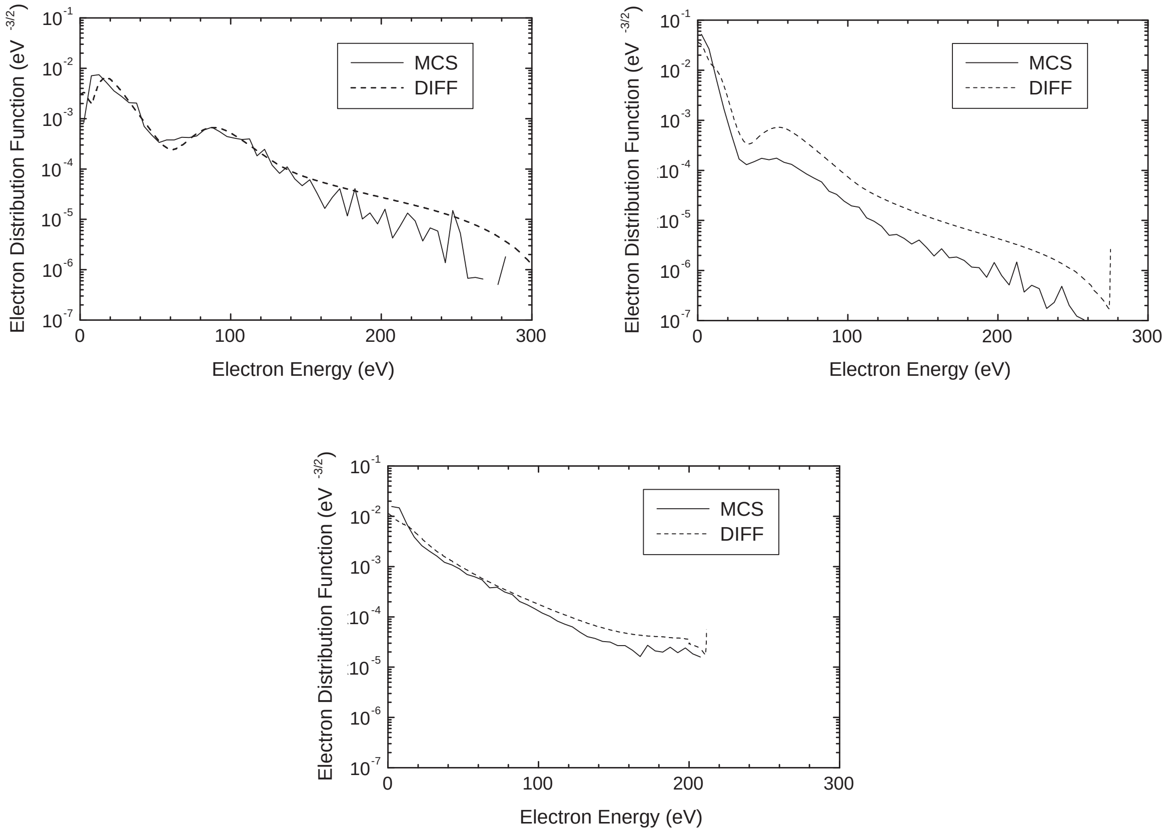

Figure 1: Electron distribution function as a function of the energy at various locations of a plasma thruster. Solid line: Monte-Carlo simulation; dashed line: SHE model. From left to right and top to bottom: respectively at 0.1 cm, 2 cm and 3.4 cm from the anode. The total length of the device is 4 cm (see [18] and [36])

where μ is the chemical potential (related with the density).

(ii) $\tilde{\mathcal{I}}$ decreases the entropy (i.e. it relaxes G to the equilibrium $G_{\mu,T}$)

(iii) $\tilde{\mathcal{I}}$ preserves the local density and energy, i.e.

$$\int \tilde{\mathcal{I}}(G) d\varepsilon = 0\,, \quad \int \tilde{\mathcal{I}}(G) \varepsilon d\varepsilon = 0$$

Now, we rescale the SHE model as follows: let α denote the ration of the inelastic mean free path to the macroscopic scale ($\alpha \ll 1$). We change to the macroscopic variables:

$$x' = \alpha x\,, \quad t' = \alpha^2 t\,, \quad F = \alpha F'\,, \quad J = \alpha J'\,,$$

and obtain the rescaled SHE model, which reads (after dropping the primes):

$$\alpha^2 \left(N(\varepsilon) \frac{\partial G^\alpha}{\partial t} + \tilde{\nabla} J^\alpha \right) = \tilde{\mathcal{I}}(G^\alpha)\,, \tag{4.2}$$

$$J^\alpha = -D(\varepsilon) \tilde{\nabla} G^\alpha\,, \quad \tilde{\nabla} = \nabla_x + F \frac{\partial}{\partial \varepsilon}\,. \tag{4.3}$$

Now, the formal limit $\alpha \to 0$ of (4.2), (4.3) lead to the Energy-Transport model:

$$G^\alpha \to \text{ equilibrium Maxwellian} G_{\mu,T}\,,$$

where the Chemical potential μ and the temperature T satisfy the Energy-Transport system:

$$\frac{\partial}{\partial t} \begin{pmatrix} n(\mu,T) \\ W(\mu,T) \end{pmatrix} + \nabla \cdot \begin{pmatrix} j_n \\ j_W \end{pmatrix} + \begin{pmatrix} 0 \\ -F \cdot j_n \end{pmatrix} = 0\,, \tag{4.4}$$

$$\begin{pmatrix} j_n \\ j_W \end{pmatrix} = -\mathbb{D} \begin{pmatrix} \nabla(\mu/T) - F/T \\ \nabla(-1/T) \end{pmatrix}\,. \tag{4.5}$$

$$\tag{4.6}$$

In (4.4) The density n and the Energy W are local functions of μ and T, given by the integration of the Maxwellian (4.1) with respect to the energy ε, premultiplied by $N(\varepsilon)$ and $\varepsilon N(\varepsilon)$ respectively, while j_n and j_W are the particle and energy fluxes respectively.

The Energy Transport (ET) model is a nonlinear diffusion system. The diffusion matrix $\mathbb{D}$ is symmetric, positive-definite (and tranlates a well-known thermodynamic property, known as Onsager's reciprocity relation). It is derived from the SHE-diffusivity D by integration w.r.t. ε [3], [2].

The ET model expresses the time evolution of conservative variables (the density n and the energy W) in terms of flux j_n and j_W which in turn are expressed in terms of gradients of the state (or entropic) variables μ and T. There exists a one-to-one correspondance between (n, W) and (μ, T) through the entropy (more precisely, between (n, W) and $(\mu/T, -1/T)$) (see e.g. [10] and [14]). This relation can be exploited to obtain existence results (see e.g. [15], [16]). The ET system can also be seen as a relaxation limit of the Navier-Stokes system [3].

The ET model is increasingly used for semiconductor or plasma discharge modeling. The present derivation stems from [3], [2]. A rigorous convergence result of the Boltzmann solution to the ET model has been proved in [5]. A formal derivation in the plasma physics context has been carried through in [22] and leads to the coupling of Euler equations for the ions with the ET model for electrons.

As an example of the use of the ET model, we present an application to certain kinds of surface discharges which occur on satellite solar panels. This model is derived from the SHE model obtained through collisions with the boundary and confinement along the boundary by a confining force, as presented in [12]. In this case, the occurence of the discharge and the confinement along the boundary originates from the charging of the satellite by the spatial environement like e.g. the solar wind, the impact by UV photons, etc. The model is adapted from a scenario originally proposed by M. Cho in [9]. The impact of the electrons on the boundary also generate secondary emission, thus increasing the number of elecrons involved in the process and eventually leading to an avalanche phenomenon. Additionally, secondary emission leaves positive charges along the boundary which increase the confinement force, the collision frequency with the boundary, leading to a positive feedback which enhances the electron avalanche. Let us stress that, through the derivation of the diffusion constants, the ET model retains information about the hopping dynamics of the electrons along the boundary. The model and the results are described in more detail in [25]. Here, we only show in figure 2 the electron density as a function of the distance along the boundary for various times. The picture clearly the avalanche breakdown phenomenon.

Another example of use of the ET model in plasma modeling is for 2-fluid plasmas. According to the model derived in [22] (see also [28]), a convenient model for plasmas is obtained by coupling the ET model for electrons with the Euler equations for the ions. Compared with a more standard 2-fluid Euler model for both electrons and ions, the present model is obtained by letting the ratio of the electron to ion mass tend to zero. This amounts to assuming that the electron collisions against the ions are almost elastic, which they are up to terms of the order of this mass ratio. The advantage in doing such an approximation is that no time-scale limitation in the computation is linked with the plasma frequency, which is fortunate since the plasma frequency is a very short scale phenonmenon. We refer to [28] for details about this point.

Such a model is very useful for the modeling of devices such as the Plasma Opening Switch. In these devices, the plasma exhibits a strong departure from quasi-neutrality which makes the use of a 2-fluid model necessary. The particle dynamics is coupled with Maxwell equations in order to describe the interaction of the plasma with an electromagnetic wave. Simulations of such devices can be found in [39], [34] as well as references therein. However, in many instances, the ion to electron mass ration has been artificially modified, in order to cope with the very different time scales associated with the two species of particles. By using an ET model for the electrons, we directly place ourselves in the time scale of the ion dynamics and avoid unphysical results that could result from a artificial change of the mass ratio.

Results of simulations of the POS by means of Particle-in-Cell type for the coupled electron-ET and ions-Euler system and with difference discretization of the Maxwell equations are presented in figures 3 and 4. They clearly show the appearence of a non quasineutral region

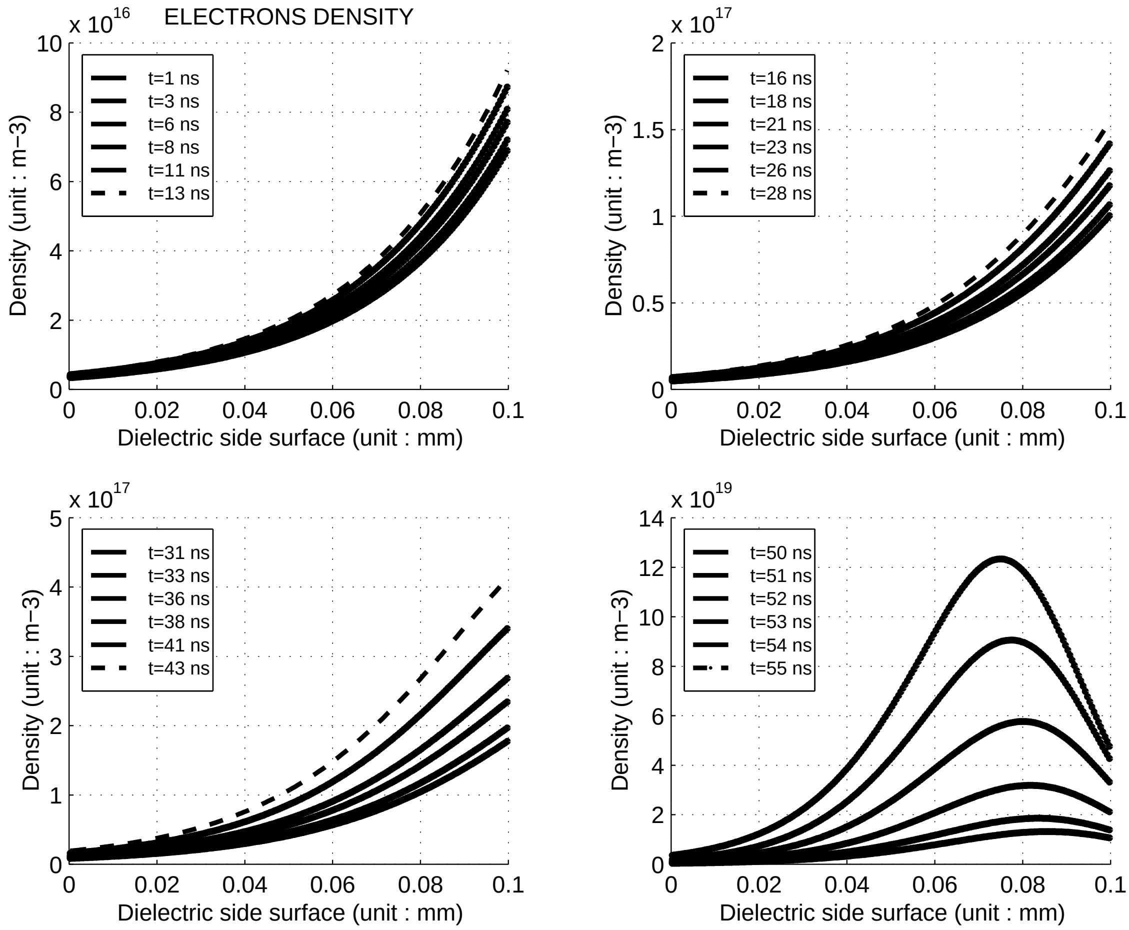

Figure 2: Electron density along the dielectric surface of a solar cell during a discharge event. Snapshots at times ranging from 1 ns up to 55 ns are shown and displayed in 4 seperate pictures for clarity. Note that the density scale changes by 3 orders of magnitude between the begining and the end of the simulation, clearly showing that an avanlanche is occuring. Details are found in [25].

close to the cathode (sheath region). The test problem and the numerical results are presented in detail in [28].

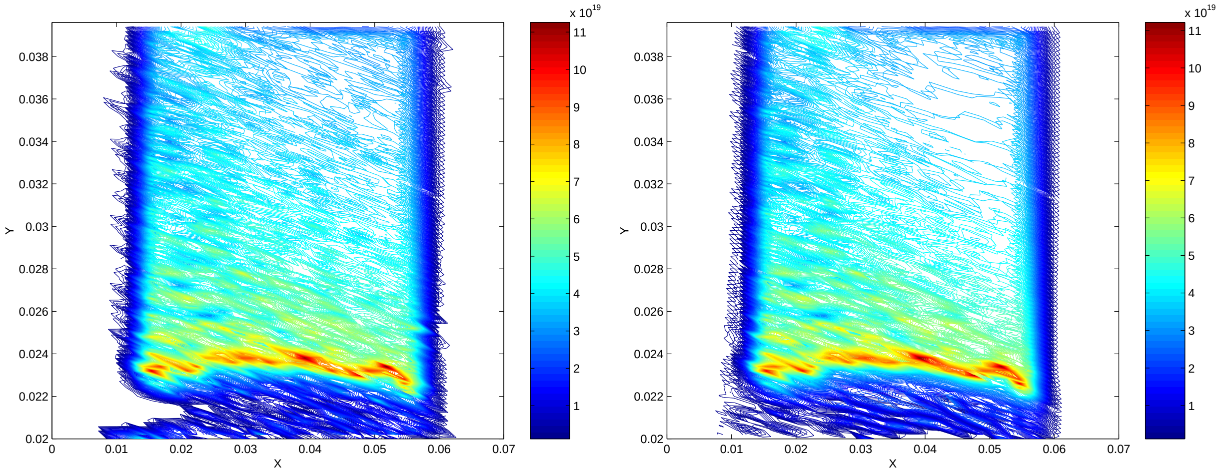

Figure 3: Electron (left) and Ion (right) densities in the post during the erosion of the plasma by the electromagnetic wave. Differences in density clearly show that quasineutrality breaks down close to the cathode (lower boundary). See also [28].

5 Conclusion

In this paper, we have summarized the derivation of intermediate models between kinetic and fluid models from diffusion limits of kinetic models. The key ide is to use 'incomplete' relaxation by the physical dissipation mechanisms (such as e.g. collisions). More precisely, a first intermediate model, the SHE model, has been derived under the assumption of dominant elastic scaling. Then, the SHE model can be futher reduced to a second intermediate model, the Energy-Transport model, which appears as a diffusion system for the fluid density and internal energy. These model have been practically implemented in physically realistic situation and they have proven to be good compromises between physical accuracy and solution efficiency

Further developments have been or are currently being undertaken concerning boundary conditions and kinetic boundary layers [24], coupling with quantum models [13], scattering by interfaces [23], homogenization [4], [26], [1] (see also [40] and references therein).

Future research directions concern finding 'new' applications of these models, their careful numerical validation [6], the design of new 'artificial' collision operators in order to enrich the set of available macroscopic models.

Note that these systems have their counterpart in the context of Hydrodynamic limits [19] and can be applied to the kinetic modeling of turbulence [20]. Also, these model can

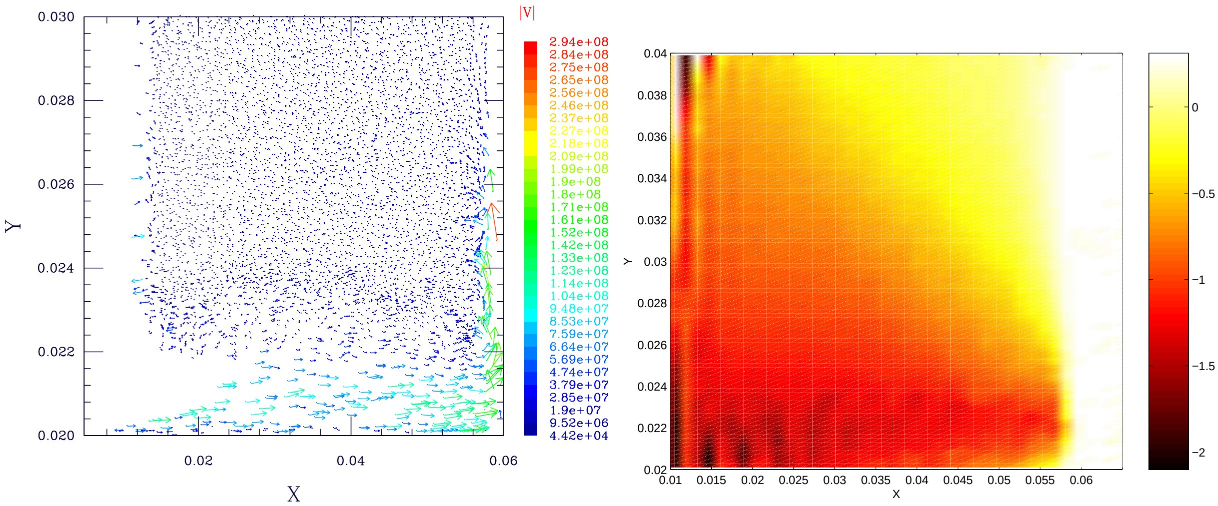

Figure 4: Electron velocity (left) and magnetic field intensity (right) during the erosion phase of the POS.

themselves be 'Improved' e.g. by considering SHE models with energy levels coupling in the diffusion operator [11] or by considering collisions with no principle of detailed balance [17] or [40].

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