On plasma expansion in vacuum

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Abstract. In this paper, we propose a model describing the expansion of a plasma in vacuum. Our starting point consists in a two-fluids Euler system coupled with the Poisson equation. Since numerical simulations of this model are very expensive, we investigate a quasi-neutral limit of it. We show that electron emission happens at the plasma-vacuum interface. This emission is well modeled by a Child-Langmuir law. The difficulty consists in accounting for the motion of the plasma-vacuum interface. In this paper, we formally and numerically justify why electron emission produces a reaction pressure which slows down the plasma expansion.

1. Introduction

This paper is devoted to the study of a quasi-neutral plasma expansion in vacuum, and more particularly to the description of the motion of the plasma-vacuum interface. This configuration occurs in some diodes like those mentioned in [10]. So, let us consider an external electric field applied between two electrodes and assume that a quasi-neutral plasma constituted of ions and electrons is emitted from the cathode. This plasma undergoes a thermal expansion towards the anode while the electrons are emitted from the plasma-vacuum interface according to a Child-Langmuir law [3, 7]. The gap between the interface and the anode is decreasing with time and the extracted electron current is simultaneously increasing in accordance with the Child-Langmuir law.

Our starting point is a one-dimensional Euler model for each species (ions and electrons) coupled with the Poisson equation. However, due to the large plasma densities, numerical simulations of this model are very expensive in practice. So, we rather investigate a one-fluid Euler model obtained as a formal quasi-neutral limit of the initial two-fluids model. Moreover, during the thermal expansion of the plasma, electrons are emitted from the plasma-vacuum interface to the anode. The quasi-neutral model is unable to describe their motion between the plasma-vacuum interface and the anode but by means of a rescaling of the system, we show that this emission is well modeled by a Child-Langmuir law. Then, the boundary conditions at the interface $X(t)$, which connect the two models, are determined through two additional assumptions validated by numerical simulations.

i) The electron emission is governed by the Child-Langmuir law with a maximal extracted electron current.

ii) Electron acceleration in vacuum produces a reaction pressure which acts on the plasma-vacuum interface and slows down its motion.

A similar asymptotic analysis is performed in [4] for the isentropic Euler system. Here we extend it to the full Euler system. We detail assumptions i) and ii) and numerically justify them comparing simulations obtained on the one hand with the two-fluids Euler system and on the other hand with the quasi-neutral model.

2. Two-fluids Euler-Poisson model

Let us consider a pair of electrodes: the cathode is located at $\bar{x} = 0$ and the anode at $\bar{x} = L$. A plasma constituted of ions and electrons is emitted from the cathode. Let i and e be the index for ions and electrons. Then, $m_{i,e}$ are the mass of the particles, $N_{i,e}$ their density, $U_{i,e}$ their velocity and $T_{i,e}$ their temperature. The total energy $W_{i,e}$ is defined by

$$W_e = \frac{1}{\gamma - 1} P_e + \frac{m_e N_e U_e^2}{2}, \qquad W_i = \frac{1}{\gamma - 1} P_i + \frac{m_i N_i U_i^2}{2}$$

where $\gamma = 5/3$ and the pressure laws are given by $P_{i,e} = N_{i,e}\, k_B\, T_{i,e}$ where k_B is the Boltzmann constant. We start with the Euler system for each species, coupled with the Poisson equation. Thus, the system for ions is given by:

$$\begin{cases} N_{i\,\bar{t}} + (N_i\, U_i)_{\bar{x}} = 0, \\ m_i\, \left((N_i\, U_i)_{\bar{t}} + (N_i\, U_i^2)_{\bar{x}}\right) + P_{i\,\bar{x}} = -q\, N_i\, \Phi_{\bar{x}}, \\ W_{i\,\bar{t}} + (W_i\, U_i)_{\bar{x}} + (P_i\, U_i)_{\bar{x}} = -q\, N_i\, U_i\, \Phi_{\bar{x}}, \end{cases}$$

while electrons are described by:

$$\begin{cases} N_{e\,\bar{t}} + (N_e\, U_e)_{\bar{x}} = 0, \\ m_e\, \left((N_e\, U_e)_{\bar{t}} + (N_e\, U_e^2)_{\bar{x}}\right) + P_{e\,\bar{x}} = q\, N_e\, \Phi_{\bar{x}}, \\ W_{e\,\bar{t}} + (W_e\, U_e)_{\bar{x}} + (P_e\, U_e)_{\bar{x}} = q\, N_e\, U_e\, \Phi_{\bar{x}}, \end{cases}$$

where $q > 0$ is the elementary charge.

The Poisson equation is used to take into account the evolution of the electric field, so that:

$$-\Phi_{\bar{x}\bar{x}} = \frac{q}{\varepsilon_0}\, (N_i - N_e),$$

where Φ is the electric potential and ε_0 is the vacuum permittivity.

We assume that a quasi-neutral plasma is emitted at $\bar{x} = 0$, with the same velocities and temperatures for the two species. So, the boundary conditions for the plasma are given by: $N_i(\bar{x} = 0) = N_e(0) = N_0$, $U_i(0) = U_e(0) = U_0$, $T_i(0) = T_e(0) = T_0$, and those for the electric potential are: $\Phi(0) = 0$, $\Phi(L) = \Phi_L$.

3. Quasi-neutral limit in the plasma region

Now, in order to get a quasi-neutral model, we scale these equations with the following characteristic sizes: the gap between the two electrodes L, the thermal ionic velocity $U_{th} = \sqrt{T_0/m_i}$, the time $\tau = L/U_{th}$, the density of the emitted plasma N_0, the temperature at the cathode T_0, the anode potential Φ_L and the internal energy at the cathode $W_0 = N_0 T_0$. This leads us to introduce some parameters:

- the mass ratio: $\varepsilon = \dfrac{m_e}{m_i}$,
- the ratio of the internal energy at the cathode to the applied potential energy: $\eta = \dfrac{T_0}{q\Phi_L} << 1$,
- the ratio of the applied potential energy to the coulombian interaction energy: $\lambda = \dfrac{\varepsilon_0 \Phi_L}{q N_0 L^2}$,
- the ratio of the emission velocity at the cathode to the thermal ionic velocity: $\alpha = \dfrac{U_0^2}{U_{th}^2}$.

Next, writing the Euler-Poisson system with the scaled variables: x, t, ϕ, $n_{i,e}$, $u_{i,e}$, $w_{i,e}$ and $p_{i,e}$ where $\bar{x} = Lx$, $\bar{t} = \tau\, t$, $\Phi = \phi_L \phi$, $N_{i,e} = N_0\, n_{i,e}$, $U_{i,e} = U_0\, u_{i,e}$, $W_{i,e} = W_0\, w_{i,e}$ and $P_{i,e} = N_0\, T_0\, p_{i,e}$, we get:

$$\begin{cases} {n_i}_t + (n_i\, u_i)_x = 0, \\ (n_i\, u_i)_t + (n_i\, u_i^2)_x + {p_i}_x = -\dfrac{n_i\, \phi_x}{\eta}, \\ {w_i}_t + (w_i\, u_i + p_i\, u_i)_x = -\dfrac{n_i\, u_i\, \phi_x}{\eta}, \end{cases} \tag{1}$$

$$\begin{cases} {n_e}_t + (n_e\, u_e)_x = 0, \\ \varepsilon\left((n_e\, u_e)_t + (n_e\, u_e^2)_x\right) + {p_e}_x = \dfrac{n_e\, \phi_x}{\eta}, \\ {w_e}_t + (w_e\, u_e + p_e\, u_e)_x = \dfrac{n_e\, u_e\, \phi_x}{\eta}, \end{cases} \tag{2}$$

$$-\lambda\, \phi_{xx} = n_i - n_e, \tag{3}$$

with

$$w_i = \frac{1}{\gamma - 1}\, p_i + \frac{1}{2}\, n_i\, u_i^2, \quad w_e = \frac{1}{\gamma - 1}\, p_e + \frac{\varepsilon}{2}\, n_e\, u_e^2, \tag{4}$$

and the boundary conditions given by:

$$\begin{cases} n_i(x=0) = n_e(0) = 1, & u_i(0) = u_e(0) = \sqrt{\alpha}, \\ w_i(0) = \left(\dfrac{1}{\gamma - 1} + \dfrac{\alpha}{2}\right), & w_e(0) = \left(\dfrac{1}{\gamma - 1} + \dfrac{\varepsilon\, \alpha}{2}\right), \\ p_i(0) = p_e(0) = 1, & \phi(0) = 0, \quad \phi(1) = 1. \end{cases} \tag{5}$$

We are now interested in the limit $\eta \to 0$ with $\lambda = O(1)$ and $\alpha = O(1)$. These values agree with those computed in some high current diodes [10]. Because the forthcoming analysis is independent of whether ε is assumed to be small or not, we assume $\varepsilon = O(1)$.

So, let $X(t)$ be the plasma-vacuum interface position at time t. Then, when η goes to 0, we formally obtain that $n_e \to n$, $n_i \to n$, $u_e \to u$, $u_i \to u$ and $(w_i + w_e) \to w$. Furthermore, n, u and w, the density, the velocity and the total energy of the quasi-neutral fluid, satisfy:

$$\begin{cases} n_t + (n\,u)_x = 0, \\ (1+\varepsilon)\,\big((n\,u)_t + (n\,u^2)_x\big) + p_x = 0, \\ w_t + (w\,u + p\,u)_x = 0, \\ \phi(x) = 0, \end{cases} \tag{6}$$

for all $x \in [0, X(t)]$, where

$$w = \frac{1}{\gamma - 1}\,p + (1+\varepsilon)\,\frac{n\,u^2}{2} \quad \text{and} \quad p = p_i + p_e,$$

with the following boundary conditions:

$$\begin{cases} n(x=0) = 1, \quad u(0) = \sqrt{\alpha}, \\ w(0) = \dfrac{2}{\gamma - 1} + \dfrac{\alpha\,(1+\varepsilon)}{2}. \end{cases}$$

Let us notice that only the total pressure $p = p_i + p_e$ is given. To compute each pressure term, p_e for example, we need to solve the additional non-conservative equation (see [1] for more details):

$$p_{e\,t} + u_e\,p_{e\,x} + \gamma\,p_e\,u_{e\,x} = 0.$$

Moreover, the quasi-neutral model (6) deduced from (1, 5) in the limit $\eta \to 0$, is only valid when the density n is strictly positive. A rigorous proof of this limit was given in [2] assuming the density strictly greater than a positive constant. The same condition appears for the quasi-neutral limit of the drift-diffusion model in [8]. So, the quasi-neutral model (6) is no more valid close to the interface $X(t)$ where ions and electrons are moving apart. Therefore, a different asymptotic regime must be investigated in order to describe the behavior of the electrons in the region $[X(t), 1]$ (the “beam”).

4. Child-Langmuir limit in the beam region

Under the action of the anode potential, the electrons are accelerated while the ions are slowed down. Beyond the interface $X(t)$, there are no ions. The electrons are described by equations (2, 3) with $n_i = 0$. Due to the external electric field, the electron velocity is increasing to reach a value of the order of $(\varepsilon\,\eta)^{-1/2}$. In order to investigate the behavior of electrons in the beam, we must rescale the electron velocity. So for all $x \in [X(t), 1]$, we set $u_e = \bar{u}_e/\sqrt{\varepsilon\,\eta}$. This scaling transforms the system (2, 3) into

$$\begin{cases} \bar{n}_{e\,t} + \dfrac{1}{\sqrt{\varepsilon\,\eta}}\,(\bar{n}_e\,\bar{u}_e)_x = 0, \\ \varepsilon\left(\dfrac{1}{\sqrt{\varepsilon\,\eta}}\,(\bar{n}_e\,\bar{u}_e)_t + \dfrac{1}{\varepsilon\,\eta}\,(\bar{n}_e\,\bar{u}_e^2)_x\right) + p_{e\,x} = \dfrac{\bar{n}_e\,\bar{\phi}_x}{\eta}, \\ \dfrac{\eta\sqrt{\varepsilon\,\eta}}{\gamma-1}\,p_{e\,t} + \dfrac{\sqrt{\varepsilon\,\eta}}{2}\,\left(\bar{n}_e\,\bar{u}_e^2\right)_t + \dfrac{\gamma\,\eta}{\gamma-1}\,(p_e\,\bar{u}_e)_x + \dfrac{1}{2}\,(\bar{n}_e\,\bar{u}_e^3)_x = \bar{n}_e\,\bar{u}_e\,\bar{\phi}_x, \\ -\lambda\,\bar{\phi}_{xx} = -\bar{n}_e, \end{cases}$$

with the boundary conditions $\phi(X(t)) = 0$, $\phi(1) = 1$ and $u_e(X(t)) = 0$.

When η goes to 0, we formally get a stationary model (which only depends on time through the interface position: $X(t)$) such that for all $x \in [X(t), 1]$:

$$\begin{cases} (\bar{n}_e\,\bar{u}_e)_x = 0, \\ (\bar{n}_e\,\bar{u}_e^2)_x = \bar{n}_e\,\bar{\phi}_x, \\ \dfrac{1}{2}\,(\bar{n}_e\,\bar{u}_e^3)_x = \bar{n}_e\,\bar{u}_e\,\bar{\phi}_x, \\ -\lambda\,\bar{\phi}_{xx} = -\bar{n}_e. \end{cases} \tag{7}$$

Note that the third equation can be easily deduced from the other ones. This system is known as the Child-Langmuir model (see [3, 7]). Its solution depends on a free parameter, the current $\bar{j} = \bar{n}_e\,\bar{u}_e$, where $\bar{j}$ ranges in the interval $[0, \bar{j}_{CL}]$. The maximal value of $\bar{j}$, called the Child-Langmuir current, is given by the relation

$$\bar{j}_{CL} = \frac{4\sqrt{2}\,\lambda}{9\,(1-X)^2},$$

and is associated with the additional boundary condition $\bar{\phi}_x(X(t)) = 0$. We shall assume that the emitted current coincides with the Child-Langmuir current, and shall verify numerically that this hypothesis seems fulfilled.

5. A model for the plasma-beam interface

Now, we have to connect the quasi-neutral model (6) and the Child-Langmuir model (7). The boundary conditions at $X(t)$ result from the dynamics of the particles confined about the interface. A formal asymptotic analysis will be developed in future work (see [5]). In the present work we shall rely on some hypotheses. We set that the total momentum is preserved while electrons are crossing the interface. The acceleration of the electrons towards the anode generates a reaction pressure which slows down the motion of the plasma-vacuum interface.

So, let us determine this pressure term. Let $\sigma = dX/dt$ be the interface velocity. We denote by $[f]$ the jump of the function f across the interface, by f_- the limit of f on the left of $X(t)$ and by f_+ the one on the right. The Rankine-Hugoniot relations applied to the conservation of the ion density across the interface leads to:

$$[n_i\,(u_i-\sigma)]=0. \tag{8}$$

Similarly, using the total momentum equation, we have:

$$\left[\left(n_i\,u_i^2+\varepsilon\,n_e\,u_e^2+p_e+p_i-\frac{\lambda}{2\,\eta}\,|\phi_x|^2\right)-\sigma\,\left(n_i\,u_i+\varepsilon\,n_e\,u_e\right)\right]=0. \tag{9}$$

Now, from equations (1, 3), the conservation of total energy can be written as:

$$(w_i+w_e)_t+(w_i\,u_i+p_i\,u_i+w_e\,u_e+p_e\,u_e)_x=-\frac{\lambda}{\eta}\,(\phi_x)_t\,\phi_x=-\frac{\lambda}{2\eta}(\phi_x^2)_t,$$

and applying the Rankine-Hugoniot conditions, we get:

$$\left[(w_i\,u_i+p_i\,u_i+w_e\,u_e+p_e\,u_e)-\sigma\,\left(w_i+w_e+\frac{\lambda}{2\eta}|\phi_x|^2\right)\right]=0. \tag{10}$$

Moreover, we suppose that the quasi-neutral limit is still valid up to the interface $X(t)$, such that:

$$n_{e-}=n_-,\quad n_{i-}=n_-,\quad u_{i-}=u_-,\quad u_{e-}=u_-, \tag{11}$$

where n_- and u_- denote the limit of the quasi-neutral quantities on the left of the interface.

There are no ions beyond the interface, hence

$$n_{i+}=0,\quad \text{and}\quad w_{i+}=0. \tag{12}$$

With the condition of maximal current given by the Child-Langmuir law and the quasi-neutrality of the plasma, we get: $\phi_{x-}=\phi_{x+}=0$. Next, assuming that the electron density has no jump across the interface, we obtain: $n_{e+}=n_-$. We have seen that the electron flow in the gap between $X(t)$ and the anode is given by the Child-Langmuir current such that: $j_{CL}=\bar{j}_{CL}/\sqrt{\varepsilon\,\eta}=n_{e+}\,(u_{e+}-\sigma)$. From (8) we deduce the interface velocity:

$$\frac{dX}{dt}=\sigma=u_-. \tag{13}$$

and (9) reduces to:

$$\varepsilon\,[n_e\,u_e\,(u_e-\sigma)]+[p_e+p_i]=0.$$

Therefore, since $u_{e-}=u_-=\sigma$, $n_{e+}\,(u_{e+}-\sigma)=j_{CL}$, $p_{i+}=0$ and $n_{e+}=n_-$, we have:

$$p_-=p_{e-}+p_{i-}=\varepsilon j_{CL}u_{e+}+p_{e+}=\varepsilon\,j_{CL}\left(\frac{j_{CL}}{n_-}+\sigma\right)+p_{e+}. \tag{14}$$

Using $w_{i+}=0$, $u_{i-}=\sigma$ and $\phi_{x-}=\phi_{x+}=0$, (10) becomes

$$[p_i\,u_i+w_e\,u_e+p_e\,u_e-\sigma\,w_e]=0.$$

Now from (11) and (12), we deduce:

$$p_-u_-=p_{i-}u_{i-}+p_{e-}u_{e-}=w_{e+}\,(u_{e+}-\sigma)+p_{e+}u_{e+}\,,$$

then thanks to the definition of w_{e+} and to the relation $n_{e+}(u_{e+}-\sigma) = j_{CL}$ we get

$$p_- u_- = p_{e+}\left(\frac{\gamma}{\gamma-1}\frac{j_{CL}}{n_-}+\sigma\right)+\frac{\varepsilon}{2}j_{CL}\left(\frac{j_{CL}}{n_-}+\sigma\right)^2 . \qquad (15)$$

Solving the system (14) and (15) we determine p_{e+} and p_-. The pressure term p_{e+} is given by:

$$p_{e+} = \frac{\varepsilon}{2}\left(\frac{\gamma-1}{\gamma}\right) n_-\left(u_-^2-\left(\frac{j_{CL}}{n_-}\right)^2\right).$$

Therefore, substituting this expression in (14) , we can deduce the momentum flux p_- in the plasma resulting from the electronic emission at the interface and finally the energy flux $p_- u_-$.

6. Numerical validation of the asymptotic model

This section is devoted to the numerical study of the quasi-neutral model for the plasma (6) coupled with the Child-Langmuir model (7) for the "beam" by the momentum and energy fluxes p_- and $p_- u_-$. So, let us consider the quasi-neutral system (6) written in the conservative form (see [9],[11]):

$$\frac{\partial V}{\partial t}+\frac{\partial F(V)}{\partial x} = 0, \quad V(0,x)=0, \quad V(t,0)=V_0 \qquad (16)$$

where $V = (n,\ (1+\varepsilon)nu,\ w)^T$ and $F(V) = \big(nu,\ (1+\varepsilon)nu^2+p,\ wu+pu\big)^T$. Let us consider a uniform grid constituted by N cells $(m_k)_{k=1,\cdots,N}$ of size Δx. We denote by Δt the time step and we set $t^n = n\Delta t$ for all $n \geq 0$. Then for $n \geq 0$ and $k \in \{1,\cdots,N\}$, V_k^n is an approximation of V at time t^n on the cell m_k and X^n is that of the plasma-vacuum interface position X at time t^n. Let us denote by m_{k_0} ($k_0 \in \{1,\cdots,N\}$) the cell such that $X^n \in m_{k_0}$. Using the evolution equation of the interface velocity (13), we get: $X^{n+1} = X^n + \Delta t u_{k_0}^n$. The system (16) is discretized with a classical finite volume method (see [11]). So, we describe the scheme only in the last cell occupied by the plasma: m_{k_0}.

Using the Green formula and taking into account the interface motion, we have

$$\left(X^{n+1}-X_{k_0-1/2}\right)\tilde{V}-\left(X^n-X_{k_0-1/2}\right)V_{k_0}^n = \Delta t\left(g_{k_0-1/2}^n-g_+^n\right)$$

where $g_{k_0-1/2}^n$ is the numerical flux through the interface between the cells m_{k_0-1} and m_{k_0} at time t^n. It is computed using classical schemes like the HLLE scheme or the "polynomial" upwind scheme (see [6],[11]). Moreover, g_+^n is the numerical flux through the interface, we set $g_+^n = \left(0,(p_-)^n,(p_-u_-)^n\right)^T$. Finally if $X^{n+1} \in m_{k_0}$, that is if the plasma-vacuum interface did not leave the cell m_{k_0} during the time step, we set $V_{k_0}^{n+1} = \tilde{V}$, else if $X^{n+1} \in m_{k_0+1}$ we set $V_{k_0}^{n+1} = V_{k_0+1}^{n+1} = \tilde{V}$.

We compare simulations given by the two-fluids and the asymptotic models. We use $\Delta x = 2.10^{-4}$, $\gamma = 5/3$, $\varepsilon = 5.10^{-1}$, $\eta = 10^{-4}$, $\lambda = 10^{-3}$ and $\alpha = 1$.

Figures 1 and 2 are devoted to the study of the plasma between the cathode and the interface $X(t)$. Contrary to a classical fluid expansion in vacuum, in our case the ionic fluid is slowing down. The interface velocity given by the asymptotic model seems to be correct in a qualitative way. The values numerically used for the momentum and energy fluxes p_- and p_-u_- are here: βp_- and $\beta p_- u_-$. The parameter β has to be adjusted according to the two-fluids simulations. Here, $\beta = 0.5$, note that without this correction the interface velocity given by the quasi-neutral model is a little lower than that of the two-fluids model but is correct in order of magnitude.

Figure 3 shows the electron dynamics in the beam. Plotting the electron velocity, we can see that the acceleration of electrons from the interface to the anode is well approximated by the Child-Langmuir law.

Figure 4 gives the electric potential computed from the Poisson equation on the one hand and by the Child-Langmuir model on the other hand. We can observe that the potential is equal to zero between the cathode and the interface, which justifies the quasi-neutral approach. We also show (right picture) that considering a constant Child-Langmuir current in the gap between $X(t)$ and the anode is a good approximation.

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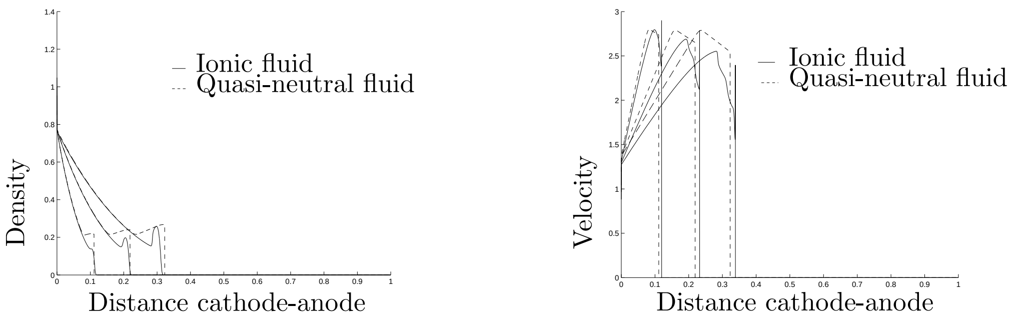

FIGURE 1. Densities and speeds of the ionic fluid given by the 2-fluids model compared to those given by the quasi-neutral model: values observed between the cathode $x = 0$ and the interface at times $t = 0.04\tau$, $t = 0.08\tau$, $t = 0.12\tau$.

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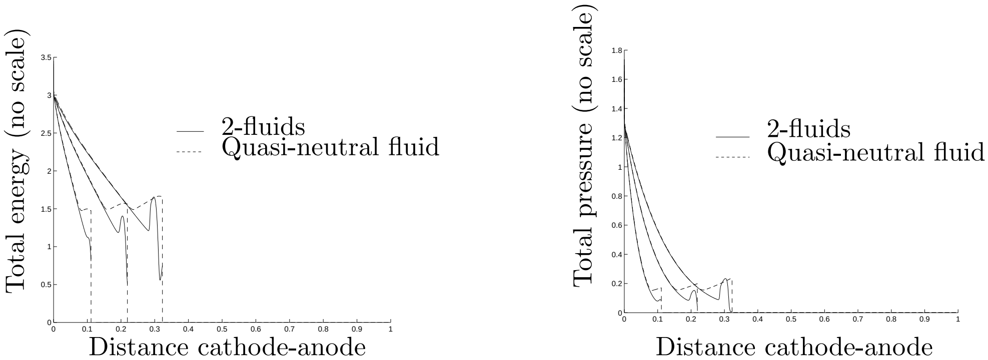

FIGURE 2. Total energy $(w_i + w_e)$ and pressure $(p_i + p_e)$ given by the 2-fluids model compared to those given by the quasi-neutral model: values observed between the cathode $x = 0$ and the interface at times $t = 0.04\tau$, $t = 0.08\tau$, $t = 0.12\tau$.

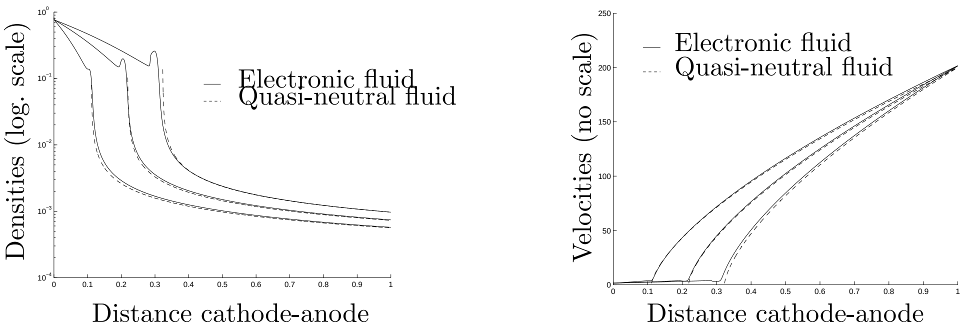

FIGURE 3. Densities and speeds of the electronic fluid given by the 2-fluids model compared to those given by the Child-Langmuir model: values observed between the interface and the anode $x = 1$ at times $t = 0.04\tau$, $t = 0.08\tau$, $t = 0.12\tau$.

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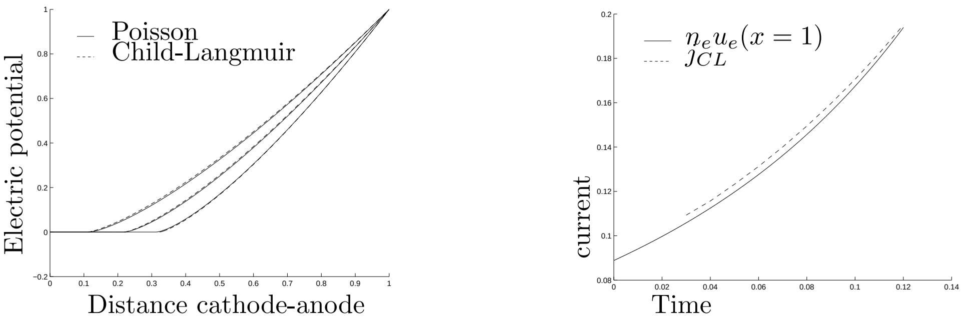

FIGURE 4. Electric potential computed on the one hand by the Poisson equation, on the other hand by the Child-Langmuir model: values observed between the interface and the anode $x = 1$ at times $t = 0.04\tau$, $t = 0.08\tau$, $t = 0.12\tau$ (left picture). Electronic current at the anode given by the Child-Langmuir model and by the 2-fluids model (right picture)

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