

> restart;

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Solutions des exercices 6 à 9 de la page 8 du polycopié sur la transformation de Laplace  
L3 CBPS UPS 2011

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### Exercice 6

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E2:

> ode :=  $\frac{d^2}{dx^2} y(x) = \text{diff}(y(x), x) + x^2$ ;

$$\text{ode} := \frac{d^2}{dx^2} y(x) = \frac{d}{dx} y(x) + x^2 \quad (1)$$

> dsolve(ode, y(x), method = laplace);

$$y(x) = -2 + 2 e^x - \frac{1}{3} x^3 + y(0) - x^2 - 2 x + D(y)(0) (-1 + e^x) \quad (2)$$

> ics := y(0) = 2, D(y)(0) = 3;

$$\text{ics} := y(0) = 2, D(y)(0) = 3 \quad (3)$$

> dsolve({ode, ics});

$$y(x) = -x^2 - \frac{1}{3} x^3 + 5 e^x - 2 x - 3 \quad (4)$$

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### Exercice 7

E1

> ode7 :=  $\frac{d^2}{dx^2} y(x) - 2 \text{diff}(y(x), x) + y(x) = x$

$$\text{ode7} := \frac{d^2}{dx^2} y(x) - 2 \left( \frac{d}{dx} y(x) \right) + y(x) = x \quad (5)$$

> dsolve(ode7, y(x), method = laplace);

$$y(x) = 2 + x + e^x (-2 + y(0) + x D(y)(0) + x - x y(0)) \quad (6)$$

> ics := D(y)(0) = 0, y(0) = 0

$$\text{ics} := D(y)(0) = 0, y(0) = 0 \quad (7)$$

> dsolve({ode7, ics});

$$y(x) = -2 e^x + e^x x + 2 + x \quad (8)$$

> collect(%, exp(x));

$$y(x) = (-2 + x) e^x + 2 + x \quad (9)$$

E2



> subs(y(0) = 0, %);

$$y(t) = -\frac{1}{10} \cos(2t) - \frac{1}{5} \sin(2t) + \frac{1}{15} (-5 D(y)(0) - 1) e^{-t} + \frac{1}{6} e^{2t} (2 D(y)(0) + 1) \quad (23)$$

> subs(D(y)(0) = 0, %);

$$y(t) = -\frac{1}{10} \cos(2t) - \frac{1}{5} \sin(2t) - \frac{1}{15} e^{-t} + \frac{1}{6} e^{2t} \quad (24)$$

E4

> oode(-8, 19, 0, 0, 0)

$$D^{(2)}(y)(t) - 8 D(y)(t) + 19 y(t) = 0 \quad (25)$$

> ics := y(0) = 3, D(y)(0) = 19

$$ics := y(0) = 3, D(y)(0) = 19 \quad (26)$$

> dsolve({%%, ics});

$$y(t) = \frac{7}{3} \sqrt{3} e^{4t} \sin(\sqrt{3} t) + 3 e^{4t} \cos(\sqrt{3} t) \quad (27)$$

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E5

> od5 := D(D(D(y)))(t) + 2·D(D(y))(t) - D(y)(t) - 2·y(t) = 6 + t

$$od5 := D^{(3)}(y)(t) + 2 D^{(2)}(y)(t) - D(y)(t) - 2 y(t) = 6 + t \quad (28)$$

> ics := y(0) = 0, D(y)(0) = 0, D(D(y))(0) = 3

$$ics := y(0) = 0, D(y)(0) = 0, D^{(2)}(y)(0) = 3 \quad (29)$$

> dsolve(od5, y(t), method = laplace);

$$y(t) = -\frac{11}{4} + \frac{7}{6} e^t + \frac{5}{2} e^{-t} - \frac{1}{2} t - \frac{11}{12} e^{-2t} + \frac{1}{3} y(0) (e^t - e^{-2t} + 3 e^{-t}) + \frac{1}{2} D(y)(0) (e^t - e^{-t}) + \frac{1}{6} D^{(2)}(y)(0) (e^t + 2 e^{-2t} - 3 e^{-t}) \quad (30)$$

> dsolve({od5, ics}, y(t), method = laplace);

$$y(t) = \frac{8}{3} \cosh(t) + \frac{2}{3} \sinh(t) - \frac{11}{4} - \frac{1}{2} t + \frac{1}{12} e^{-2t} \quad (31)$$

E6

> oode(-1, 1, 1, 0, -1);

$$D^{(2)}(y)(t) - D(y)(t) + y(t) = e^{-t} \quad (32)$$

> ics := y(0) = 0, D(y)(0)

$$ics := y(0) = 0, D(y)(0) \quad (33)$$

> dsolve({%%, ics})

$$y(t) = \frac{1}{3} e^{\frac{1}{2} t} \sin\left(\frac{1}{2} \sqrt{3} t\right) \sqrt{3} - \frac{1}{3} e^{\frac{1}{2} t} \cos\left(\frac{1}{2} \sqrt{3} t\right) + \frac{1}{3} e^{-t} \quad (34)$$

> collect(%, exp( $\frac{t}{2}$ ));

$$y(t) = \left( \frac{1}{3} \sin\left(\frac{1}{2} \sqrt{3} t\right) \sqrt{3} - \frac{1}{3} \cos\left(\frac{1}{2} \sqrt{3} t\right) \right) e^{\frac{1}{2} t} + \frac{1}{3} e^{-t} \quad (35)$$

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Ex 9

E2

$$\begin{aligned} > \text{sysode} := D(x)(t) = -x(t) + 2 \cdot y(t), D(y)(t) = -x(t) - 4 \cdot y(t) \\ & \quad \text{sysode} := D(x)(t) = -x(t) + 2 y(t), D(y)(t) = -x(t) - 4 y(t) \end{aligned} \quad (36)$$

$$\begin{aligned} > \text{ics} := x(0) = 1, y(0) = 1 \\ & \quad \text{ics} := x(0) = 1, y(0) = 1 \end{aligned} \quad (37)$$

$$\begin{aligned} > \text{dsolve}([\text{sysode}, \text{ics}], \{x(t), y(t)\}, \text{method} = \text{laplace}); \\ & \quad \{x(t) = -3 e^{-3t} + 4 e^{-2t}, y(t) = 3 e^{-3t} - 2 e^{-2t}\} \end{aligned} \quad (38)$$

$$\begin{aligned} > \text{dsolve}([\text{sysode}, \text{ics}]) \\ & \quad \{x(t) = -3 e^{-3t} + 4 e^{-2t}, y(t) = 3 e^{-3t} - 2 e^{-2t}\} \end{aligned} \quad (39)$$

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