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A CONVERGENCE PROBLEM FOR KERGIN INTERPOLATION

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Let E, F, G be three compact sets in $\mathbb{C}^n$. We say that (E,F,G) holds if for any choice of an interpolating array in F and of an analytic function f on G, the Kergin interpolation polynomial of f exists and converges to f on E. Given two of the three sets, we study how to construct the third in order that (E,F,G) holds.

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1. Formulating the problem

Let us first recall some basic facts for Kergin interpolation. Let Ω be a $\mathbb{C}$-convex domain in $\mathbb{C}^n$, i.e. for each complex line $l \subset \mathbb{C}^n$, $l \cap \Omega$ is empty or simply connected. Denote by $H(\Omega)$ the space of holomorphic functions on Ω and $P_d(\mathbb{C}^n)$ the space of polynomials whose degree does not exceed d.

Let $A = \{a_0, a_1, \ldots, a_d\}$ be a subset of $d+1$ (nonnecessarily distinct) points in Ω, then there exists a unique continuous linear map:

$$K_A\colon H(\Omega) \to P_d(\mathbb{C}^n)$$

with the following properties.

(K1) For $i = 0, 1, \ldots, d$ and $f \in H(\Omega)$, $K_A(f)(a_i) = f(a_i)$.

(K2) If $g \in H(\Omega)$ is of the form $g = f \circ u$ with u an affine map from $\mathbb{C}^n$ to $\mathbb{C}^m$ and $f \in H(u(\Omega))$ then

$$K_A(g) = K_{u(A)}(f) \circ u$$

where $u(A) = \{u(a_0), u(a_1), \ldots, u(a_d)\}$. Thus if $m = 1$

$$K_A(g) = L_{u(A)}(f) \circ u$$

where $L_{u(A)}(f)$ is the usual Lagrange Hermite interpolation polynomial of the one variable function f with respect to the points $u(a_0), \ldots, u(a_d)$.

(K3) When all the points a_0, $a_1, \ldots, a_d$ coincide, $K_A(f)$ is the Taylor expansion of f at the point $a(= a_0, a_1, \ldots, a_d)$ and of degree d.