A Convergence Problem

for Kergin Interpolation II

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Abstract. Let E, F, G be three compact sets in $\mathbb{C}^n$. We say that (E, F, G) holds if for any choice of an interpolating array in F and of an analytic function f on G, the corresponding Kergin interpolation polynomial of f exists and converges to f on E. Given G, regular $\mathbb{C}$-convex and E (resp F) we construct optimally F (resp E) so that (E, F, G) holds.

§1. Introduction

Let E, F and G be three compact sets in $\mathbb{C}^n$, the first two of which are included in the third. One says that the property (E, F, G) holds if for any triangular array $A = \{a_{di}, i = 0, \ldots, d; d = 0, 1, 2, \ldots\}$ included in F and any function f holomorphic in a neighborhood of G the Kergin interpolation polynomial of f with respect to the points in the d-th row of A, i.e $K_d(f) := K[a_{d0}, a_{d1}, \ldots, a_{dd}](f)$ exists and converges uniformly to f on E as d approaches ∞. The three problems we wish to study are the following: given two of the three compact sets above, find optimally the third in order that (E, F, G) holds. Here optimality has the following meaning, if E (resp. F) and G are given, we have to find the largest F (resp. E) such that (E, F, G) holds while if E and F are given it is the smallest G we must find.

The one dimensional case of these problems, in which Kergin interpolation reduces to the Lagrange-Hermite interpolation, has been posed and entirely solved by Smirnov and Lebedev, [8]. The sets solving the problems are certain metric envelopes of the given sets, for example E and G being given, the best F is the sets of points z which are the centre of a closed disc containing E and included in G. Of course, this set may be empty.

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