# KERGIN INTERPOLANTS AT THE ROOTS OF UNITY APPROXIMATE $C^2$ FUNCTIONS

By

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**Abstract.** We establish a new formula for Kergin interpolation in the plane and use it to prove that the Kergin interpolation polynomials at the roots of unity of a function of class $C^2$ in a neighborhood of the unit disc $\mathbb{D}$ converge uniformly to the function on $\mathbb{D}$.

## 1. Introduction

A fundamental theorem of constructive real analysis states that the Lagrange interpolation polynomial $L_d(f)$ at the $d$-th Chebyshev points of every Lip1 function $f$ on $[-1, 1]$ converges uniformly to $f$ on $[-1, 1]$ as $d \to \infty$, i.e.

$$\lim_{d\to\infty} \|f - L_d(f)\|_{[-1,1]} = 0.$$

(Recall that the $d$-th Chebyshev points are the roots of the $d$-th Chebyshev polynomial $T_d(x) := \cos d\theta, \quad \cos\theta = x$.) The difficult part of the proof consists in establishing that the Lebesgue constant $\Delta_d$ (i.e., the norm of the operator $L_d$ on $C[-1, 1]$) is equivalent to $(2/\pi)\log d$ as $d \to \infty$ (see, e.g., [R]), for then, if $p_d$ is the best approximation of $f$ among the polynomials of degree at most $d$, one has

$$\|f - L_d(f)\| \le (1 + \Delta_d)\|f - p_d\|;$$

whence the convergence follows by the classical Jackson Theorem.

The purpose of this note is to prove a two-dimensional version of this result in which $[-1, 1]$ is replaced by the unit disc

$$\mathbb{D} := \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 \le 1\};$$

the Chebyshev nodes, by the vertices of the standard regular $d$-polygon (i.e., the $d$-th roots of unity)

$$e^{kd} := (\cos\theta_{kd}, \sin\theta_{kd}) = e^{i\theta_{kd}}, \quad k = 1, \ldots, d \tag{1.1}$$



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