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CONSTRUCTIVE APPROXIMATION

Kergin Interpolants of Holomorphic Functions

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Abstract. Let D be a $\mathbf{C}$-convex domain in $\mathbf{C}^n$. Let $\{A_{dj}\}$, $j = 0, \ldots, d$, and $d = 0, 1, 2, \ldots$, be an array of points in a compact set $K \subset D$. Let f be holomorphic on $\overline{D}$ and let $K_d(f)$ denote the Kergin interpolating polynomial to f at $A_{d0}, \ldots, A_{dd}$. We give conditions on the array and D such that $\lim_{d\to\infty} \|K_d(f) - f\|_K = 0$. The conditions are, in an appropriate sense, optimal.

This result generalizes classical one variable results on the convergence of Lagrange–Hermite interpolants of analytic functions.

1. Introduction

The procedure known as Kergin interpolation (see below) is a multivariate counterpart of the classical Lagrange–Hermite interpolation. It was introduced in 1978 by Kergin [Ke] for sufficiently differentiable functions on a convex open set in $\mathbf{R}^n$. A little later, a constructive approach was given by Micchelli [M], see also [MM]. In the complex case the work of Andersson and Passare, see [AP1] and [AP2], showed the crucial role played by $\mathbf{C}$-convexity.

The main properties of convergence are known for the case of entire functions, see [B1] and [AP1]. In this paper, we are concerned with convergence problems for nonentire functions. More precisely, we will study the

Problem 1.1. Let $A = (A_{dj})$ be a triangular array of (not necessarily distinct) points in a compact set K in $\mathbf{C}^n$, find a domain D in $\mathbf{C}^n$ as small as possible (or a compact set K_1) such that for every function f holomorphic on D (or in a neighborhood of K_1) the Kergin interpolation polynomial of f at the points $A_{d0}, \ldots, A_{dd}$ exists and converges to f uniformly on K as d approaches ∞.

Note that the existence requirement is not superfluous or straightforward for the definition of the Kergin operator for functions holomorphic on a domain D needs a strong geometric condition, namely, D must be $\mathbf{C}$-convex (see below again).

As it turns out, the problem is closely related to the distribution of the points (A_{dj}),

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