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Intertwining unisolvent arrays for multivariate Lagrange interpolation

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Generalizing a classical idea of Biermann, we study a way of constructing a unisolvent array for Lagrange interpolation in $\mathbb{C}^{n+m}$ out of two *suitably ordered* unisolvent arrays respectively in $\mathbb{C}^n$ and $\mathbb{C}^m$. For this new array, important objects of Lagrange interpolation theory (fundamental Lagrange polynomials, Newton polynomials, divided difference operator, vandermondian, etc.) are computed.

Keywords: multivariate polynomials, Lagrange interpolation, unisolvent arrays

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1. Introduction

Let $\mathcal{P}_d(\mathbb{C}^n)$ denote the space of polynomials of degree at most d in n complex variables. A subset X of $\mathbb{C}^n$ – we will usually speak of configuration or array – is said to be *unisolvent* for $\mathcal{P}_d(\mathbb{C}^n)$ (or simply unisolvent of degree d) if, for every function f defined on X there exists a unique polynomial $P \in \mathcal{P}_d(\mathbb{C}^n)$ such that $P(x) = f(x)$ for every $x \in X$. This polynomial is called the *Lagrange interpolation polynomial* of f at (the points of) X and is denoted by $\mathbf{L}_X[f]$. A necessary condition for X to be unisolvent of degree d is that its cardinality coincide with the dimension of $\mathcal{P}_d(\mathbb{C}^n)$, that is, $\sharp X = t_d(n)$ where $t_d(n) := \binom{n+d}{d}$ (we will often abbreviate $t_d(n)$ to t_d). This condition is not sufficient as soon as $n > 1$. Apart from containing $t_d(n)$ points, it is required that X not be included in an algebraic hypersurface of degree smaller or equal to d. If $X = \{x_i,\ i = 1, 2, \dots, t_d\}$, the condition translates in

$$\mathrm{VDM}(x_1, x_2, \dots, x_{t_d}) \neq 0 \tag{1.1}$$

where the left-hand term stands for the *Vandermonde determinant* (or *vandermondian*), that is,

$$\mathrm{VDM}(x_1, x_2, \dots, x_{t_d}) = \det\bigl[\mathbf{e}^i(x_j)\bigr]_{i,j=1,\dots,t_d}, \tag{1.2}$$