Université Lille 1 Sciences et Technologies, Lille, France

Laboratoire de Physique des Lasers, Atomes et Molécules

Équipe Chaos Quantique

Nonlinearity and interactions in bosonic systems

Jean-Claude Garreau

GDR « Dynamique Quantique »
Toulouse, 8/2/2012

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Laboratoire de Physique des Lasers, Atomes et Molécules Lille

Benoît Vermersch

Maxence Lepers

Quentin Thommen

Véronique Zehnlé

JCG

Interactions and chaos in quantum systems

Schrödinger

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left(\frac{P^2}{2M} + V(x) \right) \psi(x,t)$$

ψ dynamical variable

x parameter of the potential

$$V(x) \propto x^n \quad n \geq 3$$

is **linear**

No sensitivity to initial conditions → no chaos in the classical sense

Newton

$$M \frac{d^2 x(t)}{dt^2} = -\frac{dV(x(t))}{dx}$$

x is the parameter of the potential ***and*** the dynamical variable

$$V(x) \propto x^n \quad n \geq 3$$

is **nonlinear**

Sensitivity to initial conditions → chaos

n identical particles

$$i\hbar\frac{\partial\Psi(x_1,\ldots x_n,t)}{\partial t}=\left(\frac{{P_1}^2}{2M}+\cdots+\frac{{P_n}^2}{2M}+V(x_1,\ldots x_n)\right)\Psi(x_1,\ldots x_n,t)$$

$$\Psi(x_1,\ldots x_n,t)$$

symmetric (bosons) or asymmetric (fermions) combination of

$$\psi_1(x_1,t)\ldots\ \psi_n(x_n,t)$$

is still **linear,** but very hard to solve

For fermions: no “simple” approximation

For bosons: “mean-field” approach

Model the effect of particle-particle interactions on a "typical" particle by its mean effect as a potential acting on the typical particle

The Gross-Pitaevskii equation

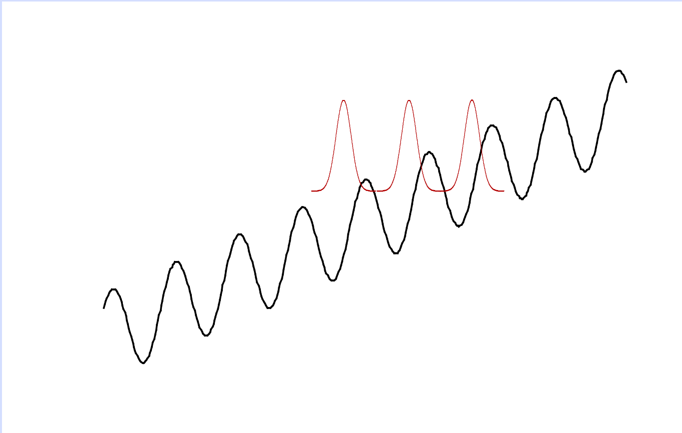
$$i\frac{\partial\psi(x,t)}{\partial t} = \left(\frac{P^2}{2} + V(x) + g|\psi(x,t)|^2\right)\psi(x,t)$$

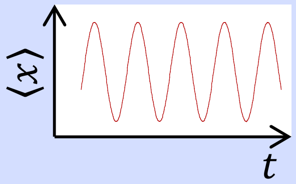
Mean-field term is **nonlinea**r

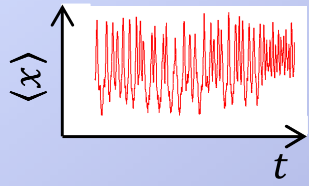
Sensitivity to initial conditions → **chaos** in the classical sense???

Classical chaos in a tilted lattice

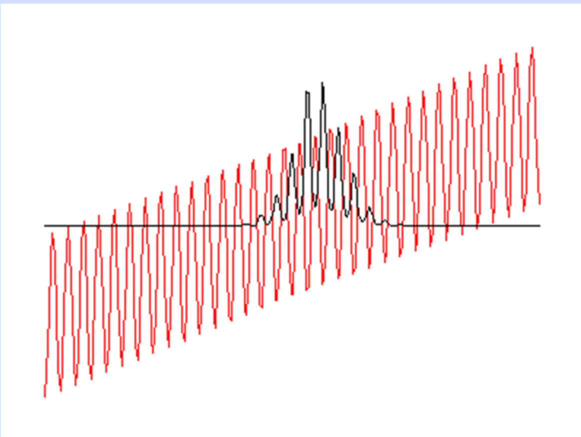
Simple(st?) system





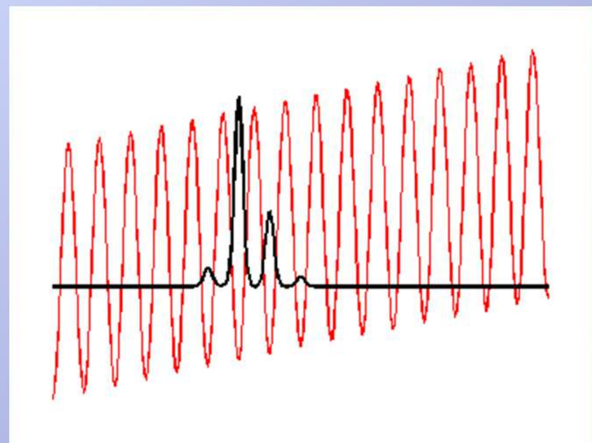


Linear



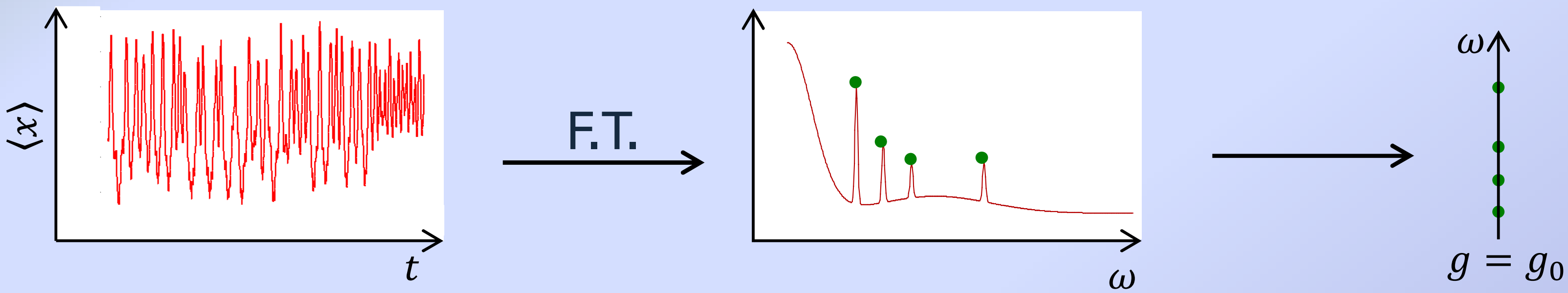
Bloch oscillation

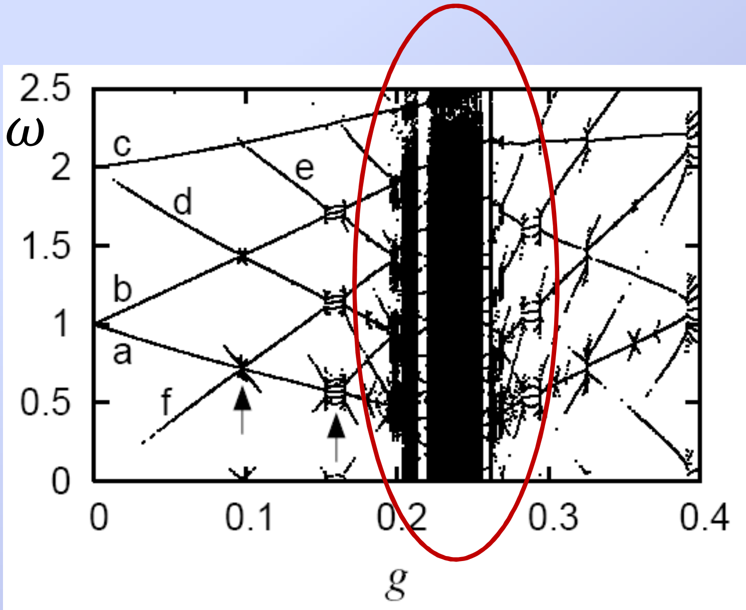
Nonlinear



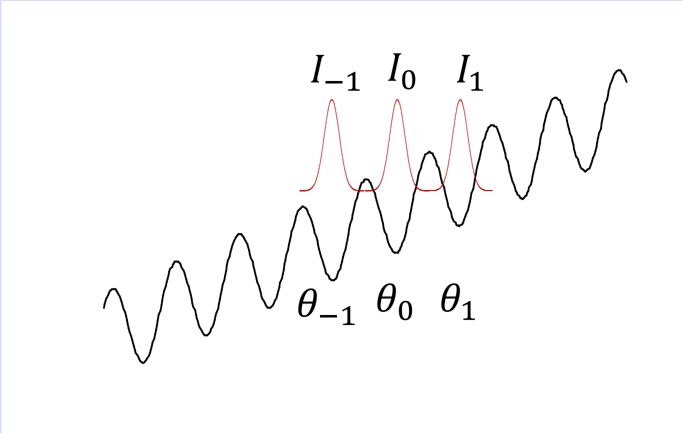
Chaos ?

Is it really chaos?

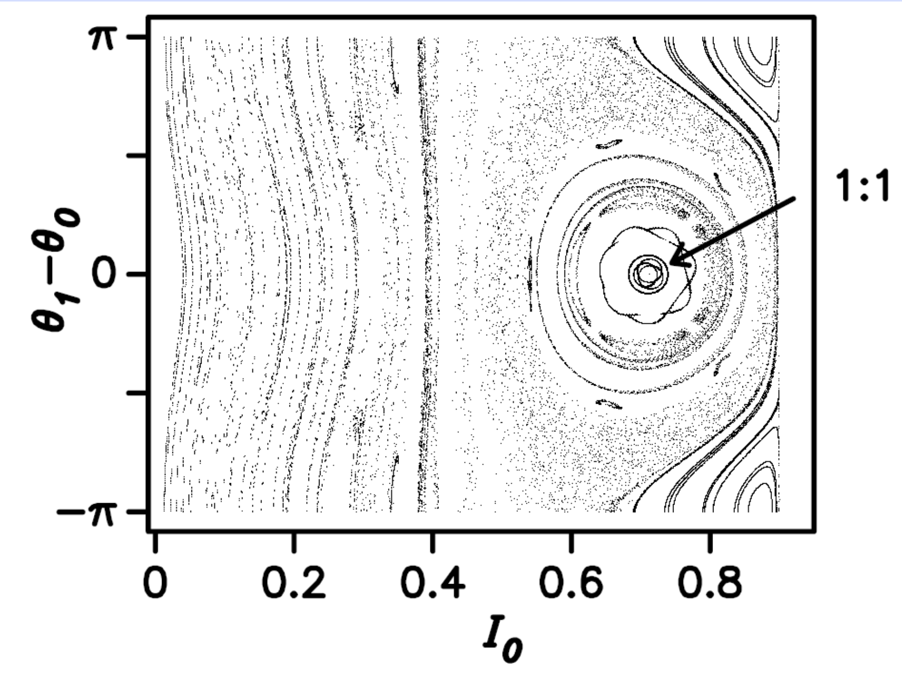




Poincaré section



$I_{-1} = 0.1; \theta_{-1} = \theta_0$



$$H = \frac{P^2}{2} + V_0 \cos x + Fx + g|\psi|^2$$

$$H = H_0(I) + \epsilon \sum_n (V_n(I) + \cos(\theta_{n+1} - \theta_n))$$

- "KAM" form

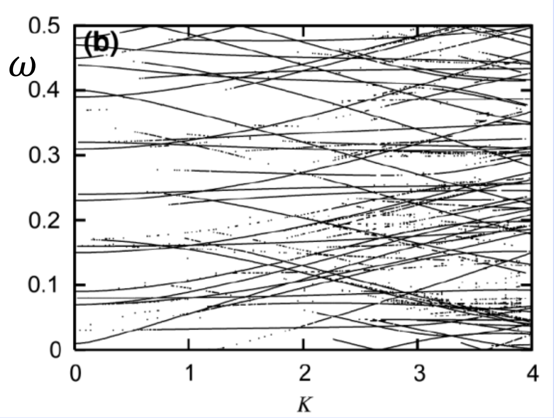
"Quasi-classical" chaos

Q. Thommen *et al., Classical Chaos with Bose-Einstein Condensates in Tilted Optical Lattices,* Phys. Rev. Lett. **91**, 210405 (2003)

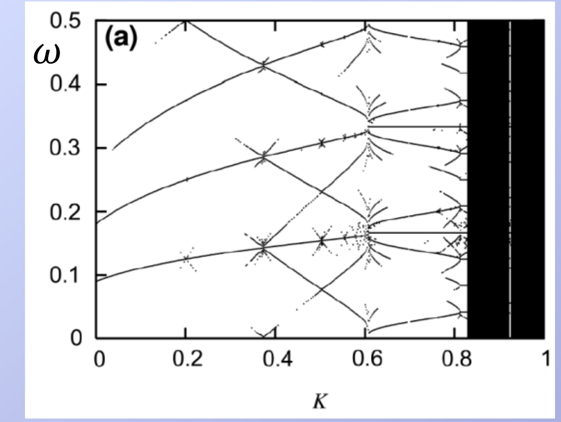
"Quantum chaos" x classical chaos

"Traditional" definition of quantum chaos: *Quantum* systems whose *classical* counterpart is chaotic
→ no S.I.C. (no "real" chaos)

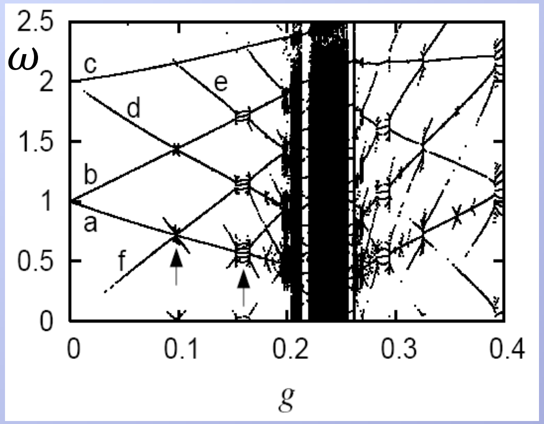
Quantum "chaotic" kicked rotor



classical chaos (KAM) in the kicked rotor

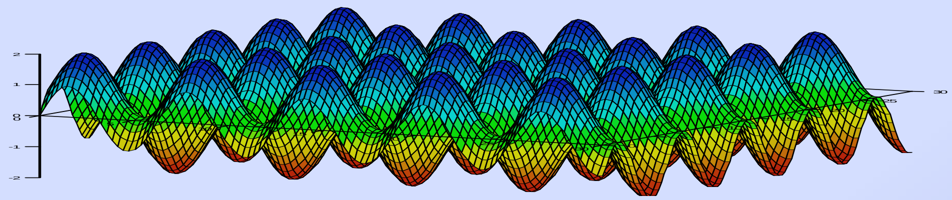


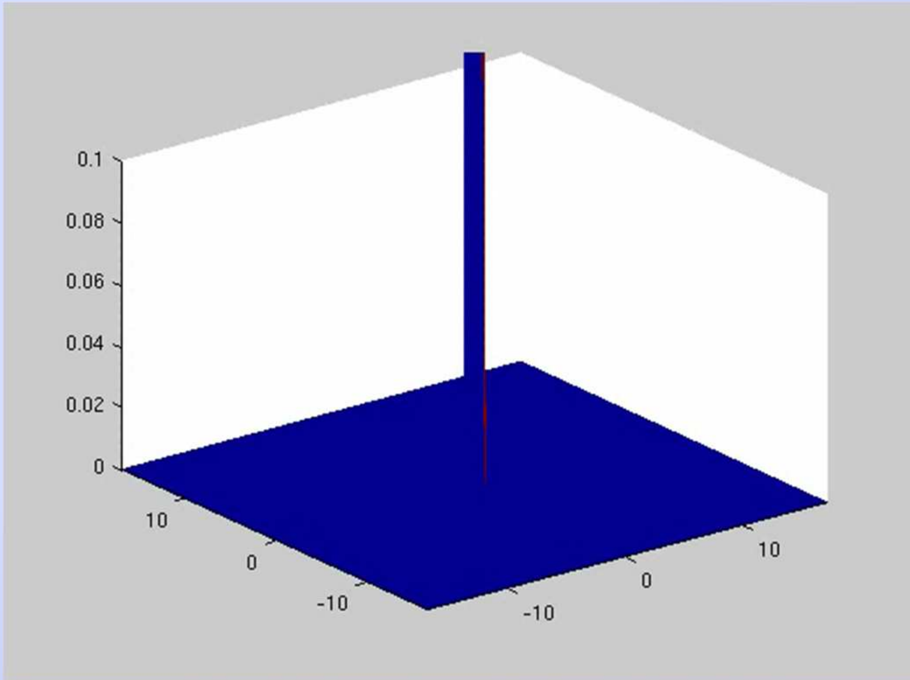
KAM chaos in a (quantum) BEC



Interactions and disorder in quantum systems

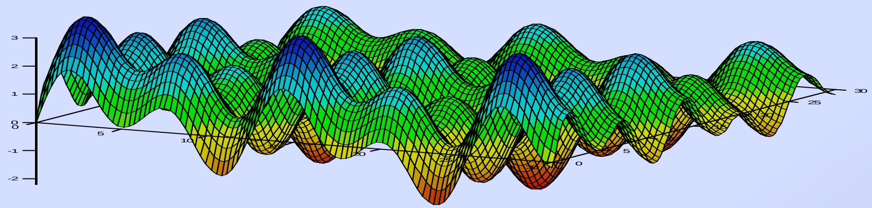
Perfect crystal: Delocalized Bloch waves → diffusive dynamics

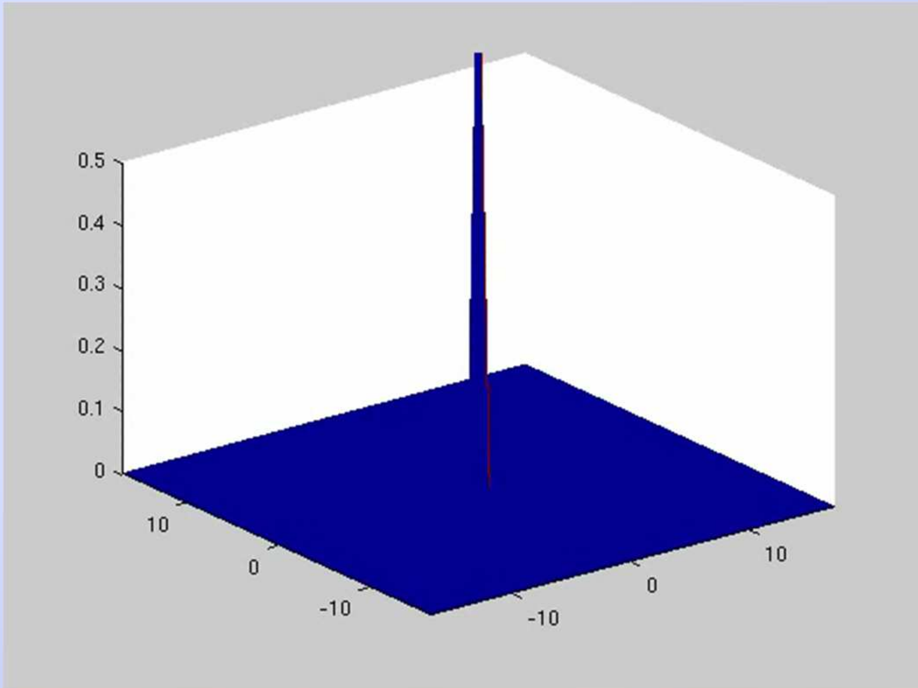




Conducteur

Disordered crystal





Insulator

Anderson model of a disordered lattice

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

"Tight-binding" model of a quantum lattice

$$Hu_n = V_n u_n + T_n u_{n+1} + T_n u_{n-1}$$

"Diagonal" disorder

$$-\frac{W}{2} < V_n < \frac{W}{2}$$

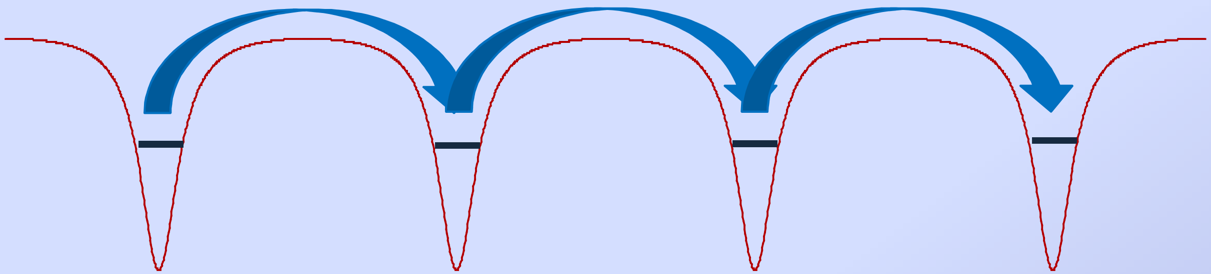
Consequences of the Anderson model

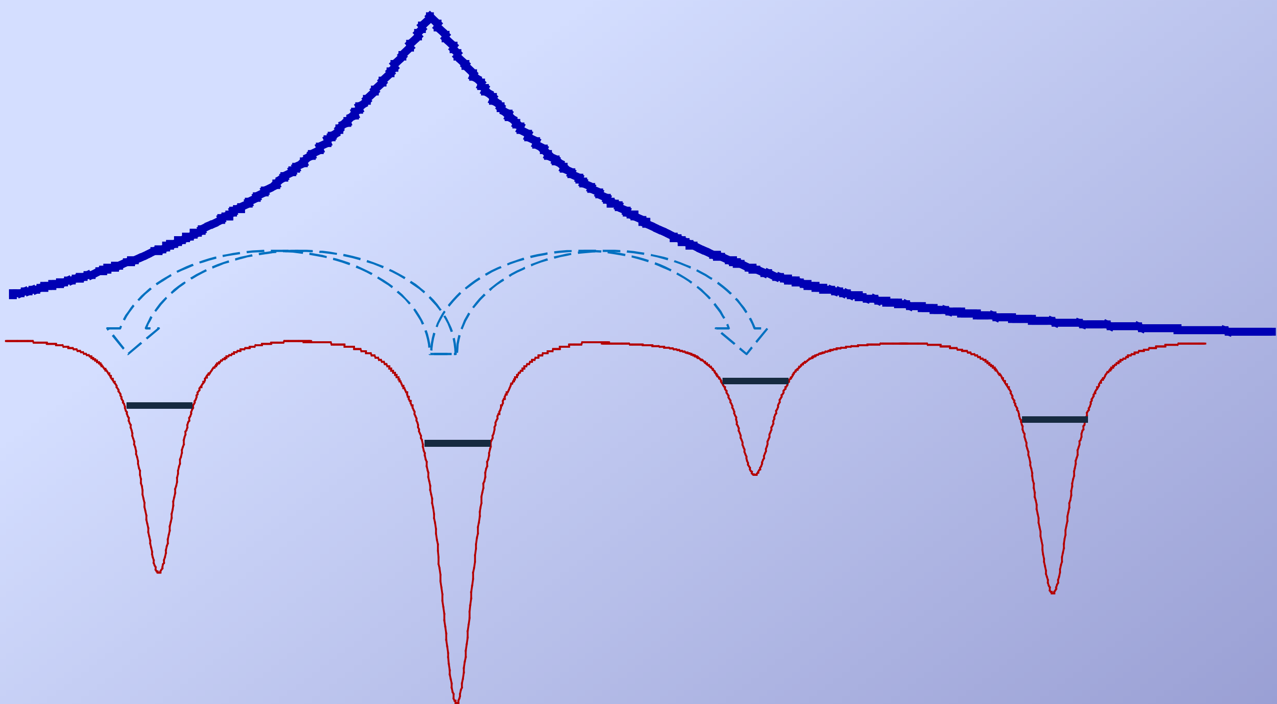
- 1D : Exponential localization of the eigenfunctions $\psi \sim \exp(|x - x_0|/\ell)$
- Suppression of the diffusion → Insulator
- 3D → « Mobility edge » → Metal-insulator transition

Limitations of the Anderson model

- "One-particle" model → No particle interactions
- Zero-temperature
- Oversimplified description of a crystal lattice

Simple picture of Anderson dynamics





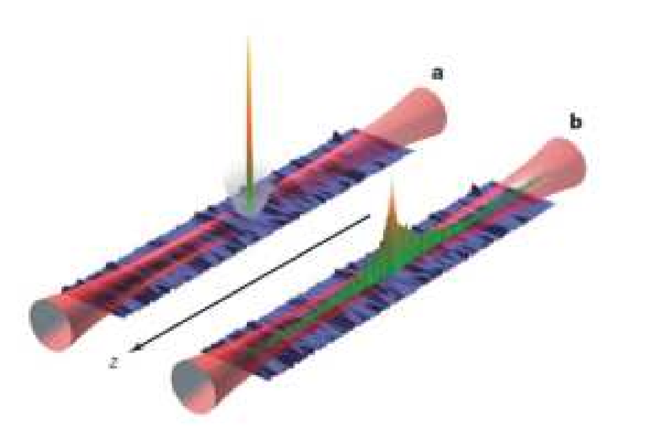
Condensed matter

- Decoherence (ill-defined quantum phases)
- No access to the wave function
- Electron-electron coulombian interactions

Ultracold atoms

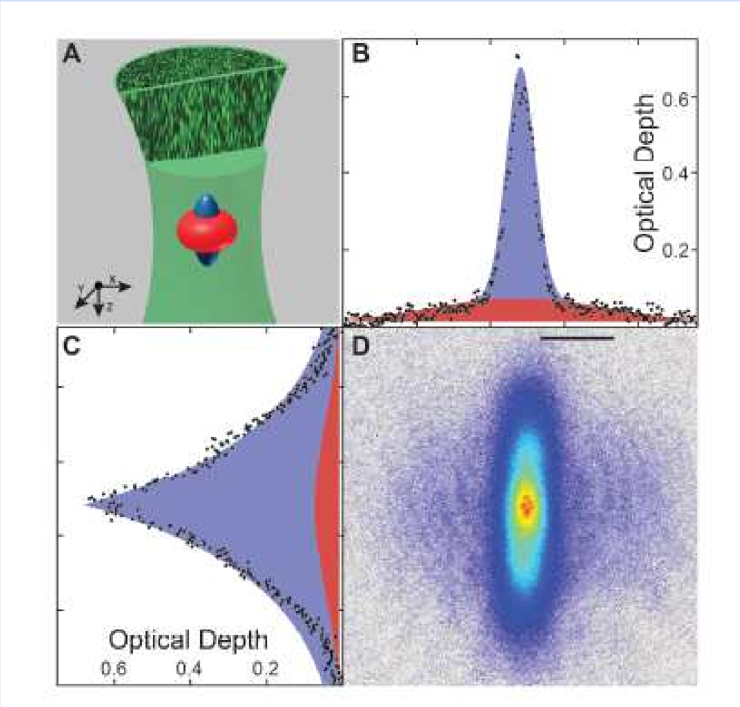
- Control of decoherence
- Access to probability distributions (and even the full wavefunction)
- Control of interactions (Feschbach resonance)

Experiments with ultracold atoms



1D: J. Billy *et al.*, *Direct observation of Anderson localization of matter-waves in a controlled disorder*, Nature 453, 891 (2008)

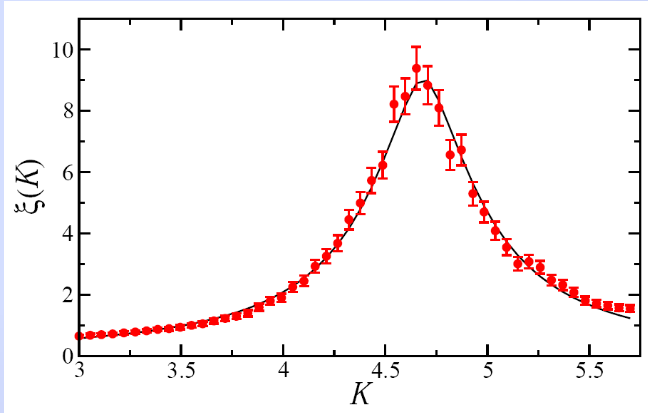
"3D" : F. Jendrzejewski *et al.*, *Three-dimensional localization of ultracold atoms in an optical disordered potential*, arXiv/1108.0137 (2011)



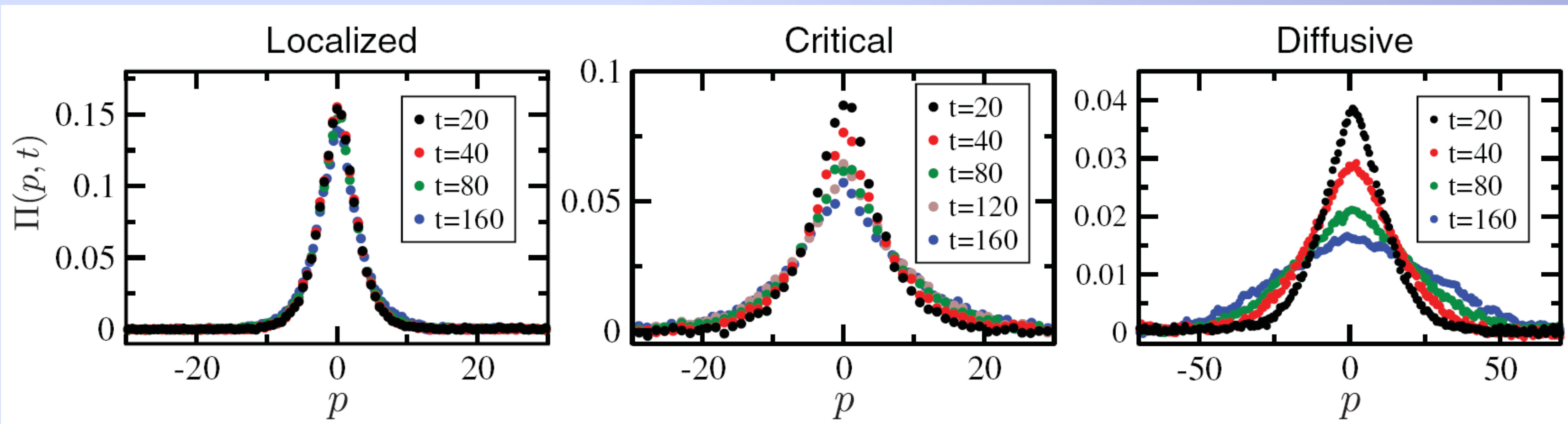
"3D": S. S. Kondov *et al.*, *Three-Dimensional Anderson Localization of Ultracold Fermionic Matter*, Science 334, 66 (2011)

The quasiperiodic kicked rotor is a "quantum simulator" for the Anderson model

Measurement of the critical exponent



Direct measurement of probability distributions



J. Chabé *et al., Experimental Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves*, Phys. Rev. Lett. **101**, 255702 (2008)
G. Lemarié *et al., Critical State of the Anderson Transition: Between a Metal and an Insulator*, Phys. Rev. Lett. **105**, 090601 (2010)
M. Lopez *et al., Experimental Test of Universality of the Anderson Transition*, arXiv/1108.0630v1 (2011) (in press)

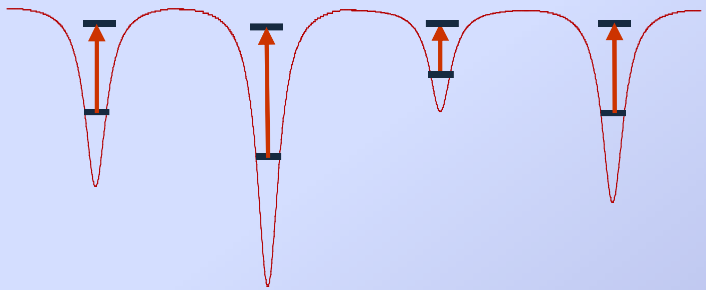
Adding particle-particle interactions

$$i\frac{\partial u_n}{\partial t} = u_{n+1} + u_{n-1} + V_n u_n + g|u_n|^2 u_n$$

$$(V_n + g|u_n|^2)u_n$$

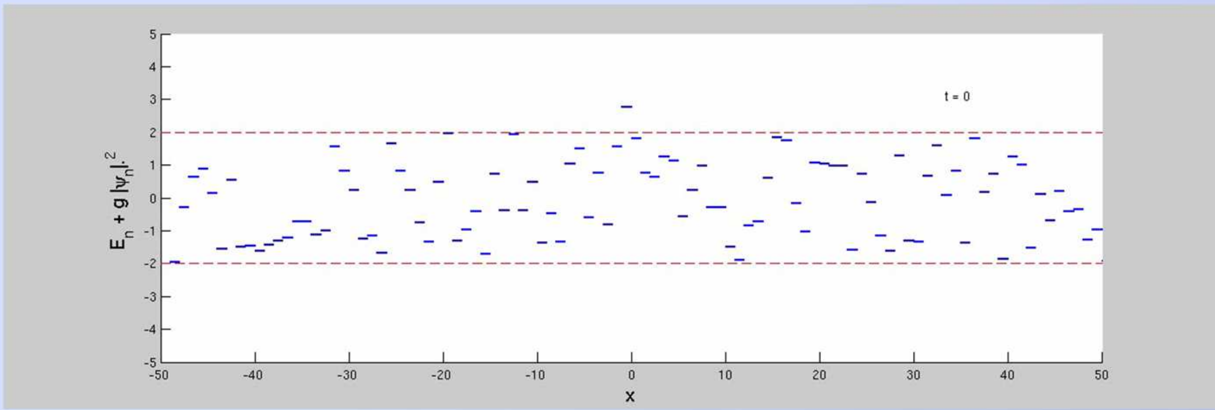
« DANSE » : Discrete Anderson Nonlinear Schrödinger Equation

$g \sim W$ $\qquad$ $(V_n + g|u_n|^2)u_n$

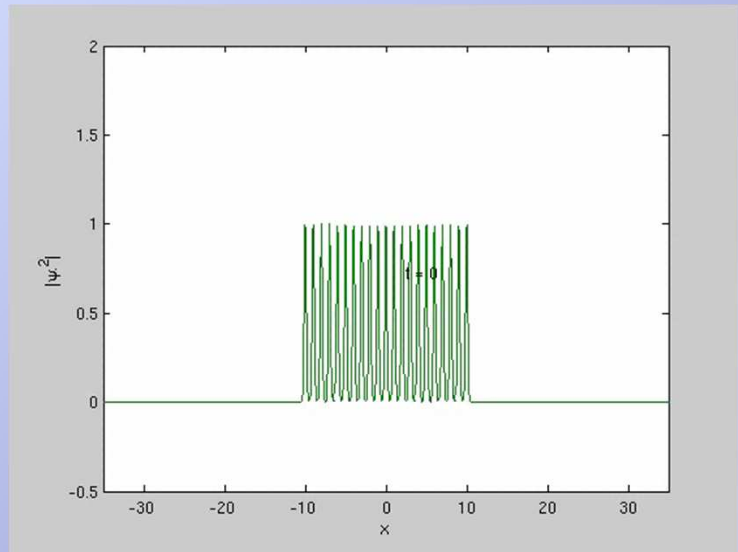


- Nonlinearity and disorder can (partially) compensate each other!

Site energy + nonlinear correction



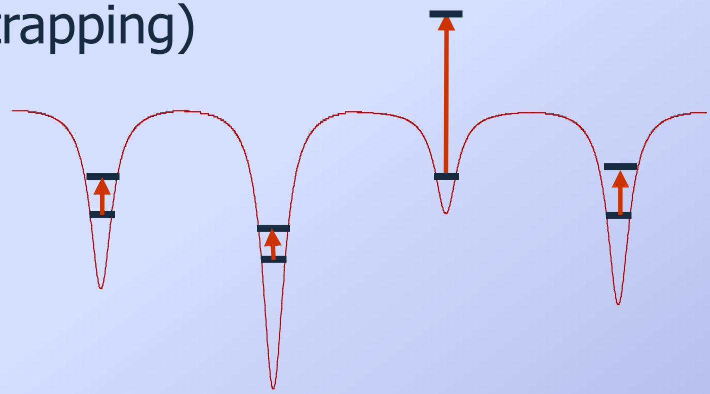
Lattice site



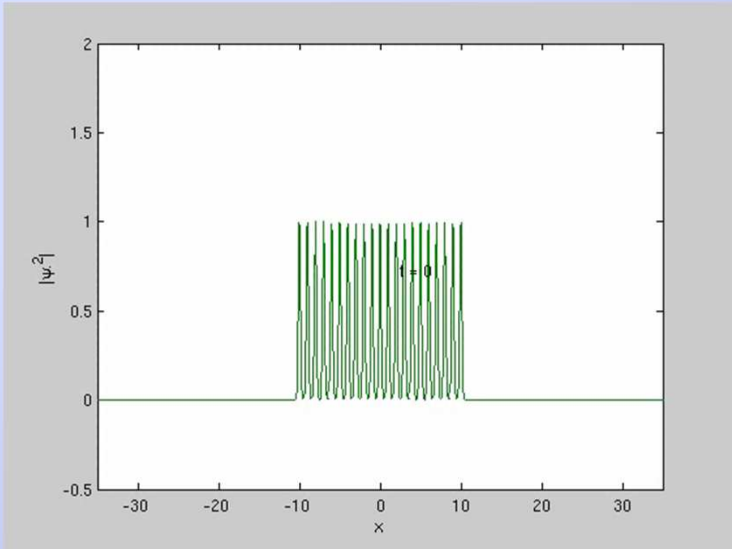
- (Sub-)diffusion induced by interactions

Effects of the nonlinearity: Nonlinear trapping (self-trapping)

$g \gg W \qquad (V_n + g|u_n|^2)u_n$



- Large population difference decouples neighbor sites



- Diffusion inhibited by interactions

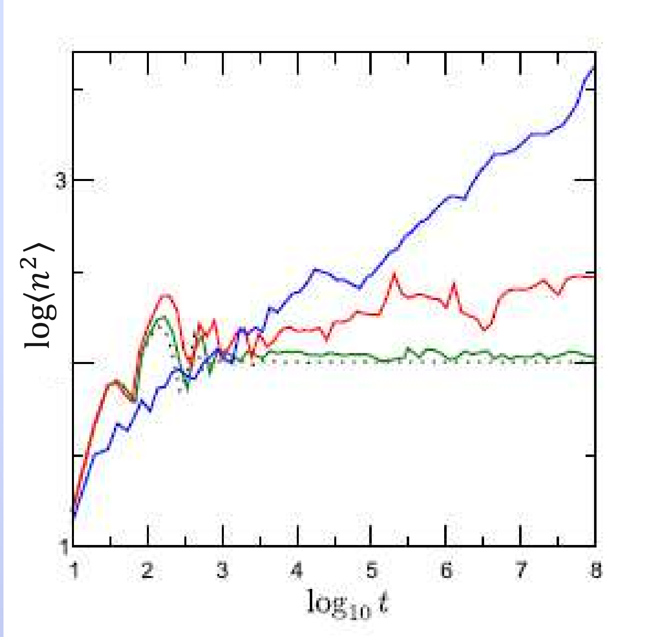
Literature ≥ 2008

PRL **100,** 094101 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Destruction of Anderson Localization by a Weak Nonlinearity

A. S. Pikovsky[1] and D. L. Shepelyansky[2,1]

[1]*Department of Physics, University of Potsdam, Am Neuen Palais 10, D-14469, Potsdam, Germany*
[2]*Laboratoire de Physique Théorique, UMR 5152 du CNRS, Université Toulouse III, 31062 Toulouse, France*
(Received 24 August 2007; published 4 March 2008)

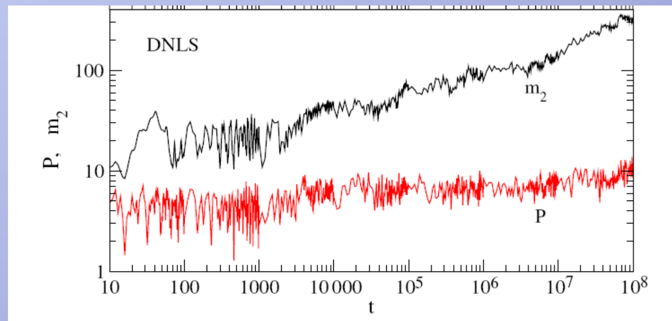


PRL **100,** 084103 (2008) PHYSICAL REVIEW LETTERS week ending 29 FEBRUARY 2008

Absence of Wave Packet Diffusion in Disordered Nonlinear Systems

G. Kopidakis,[1,2] S. Komineas,[1] S. Flach,[1] and S. Aubry[1,3]

[1]*Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Strasse 38, D-01187 Dresden, Germany*
[2]*Department of Materials Science and Technology, University of Crete, 71003 Heraklion, Greece*
[3]*Laboratoire Léon Brillouin (CEA-CNRS), CEA Saclay, 91191-Gif-sur-Yvette, France*
(Received 11 October 2007; published 27 February 2008)



Literature ≥ 2008

PRL **102,** 024101 (2009) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2009

Universal Spreading of Wave Packets in Disordered Nonlinear Systems

S. Flach, D. O. Krimer, and Ch. Skokos

Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Strasse 38, D-01187 Dresden, Germany

(Received 30 May 2008; published 14 January 2009)

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August 2010

EPL, **91** (2010) 30001 www.epljournal.org

doi: 10.1209/0295-5075/91/30001

The crossover from strong to weak chaos for nonlinear waves in disordered systems

T. V. Laptyeva, J. D. Bodyfelt[(a)], D. O. Krimer, Ch. Skokos and S. Flach

Max Planck Institute for the Physics of Complex Systems - Nöthnitzer Straße 38, D-01187 Dresden, Germany, EU

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published online 16 August 2010

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The crossover from strong to weak chaos for nonlinear waves in disordered systems

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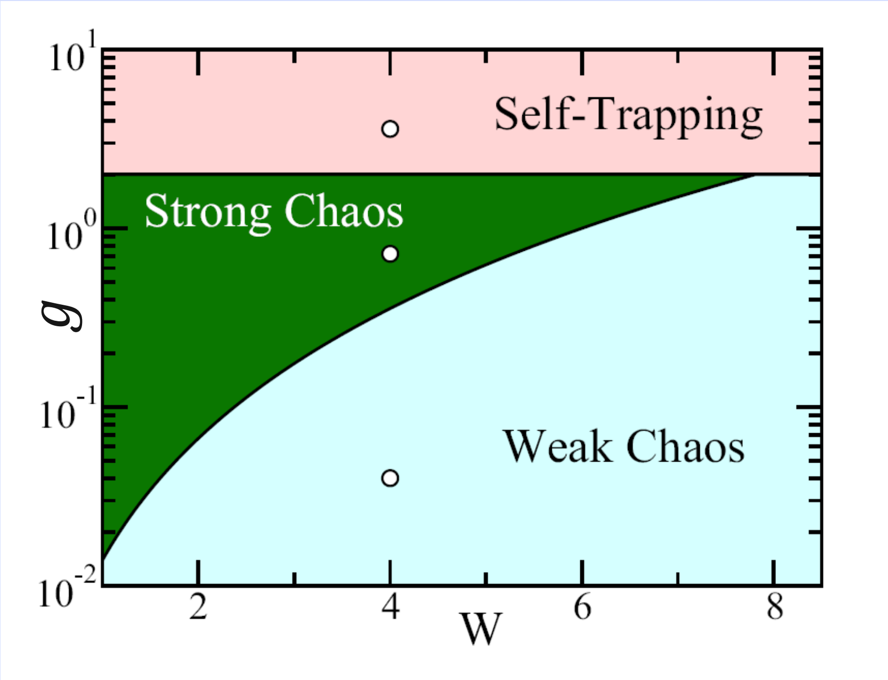
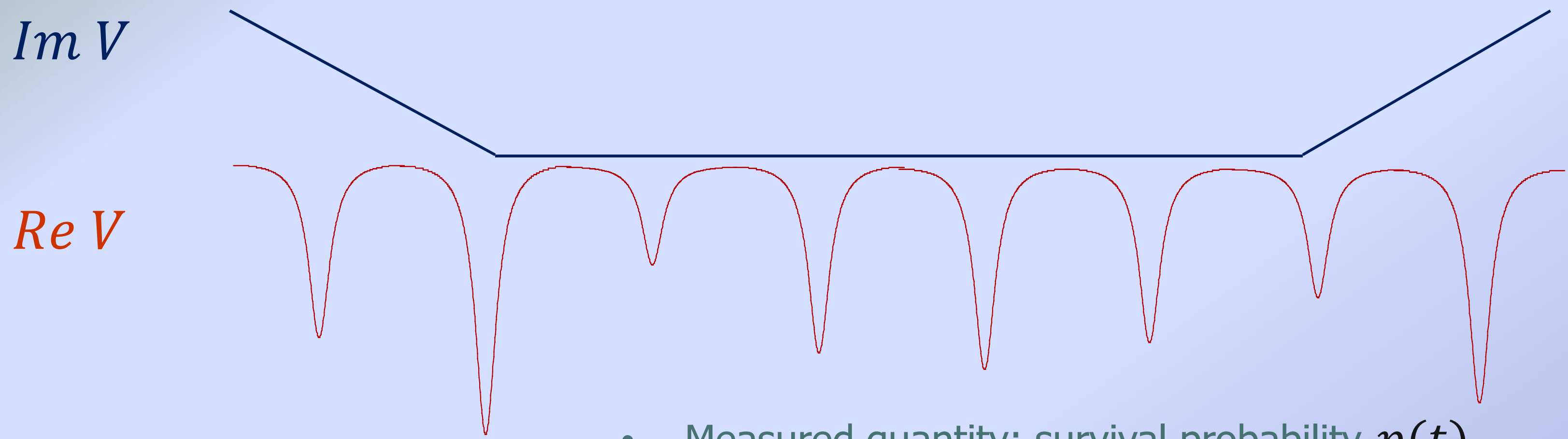


Valid for a given initial state

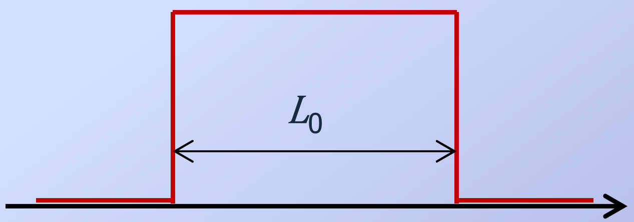
Nonlinearity → sensitivity to the initial state

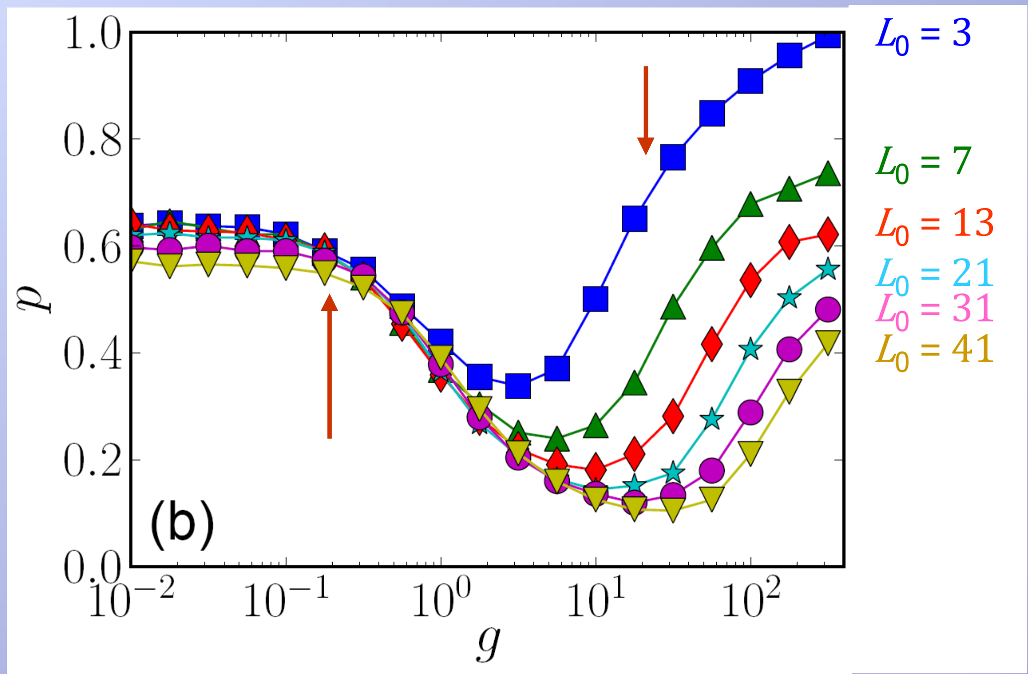
- Smooth initial state favors chaos and thus diffusion
- Peaked initial state favors self-trapping



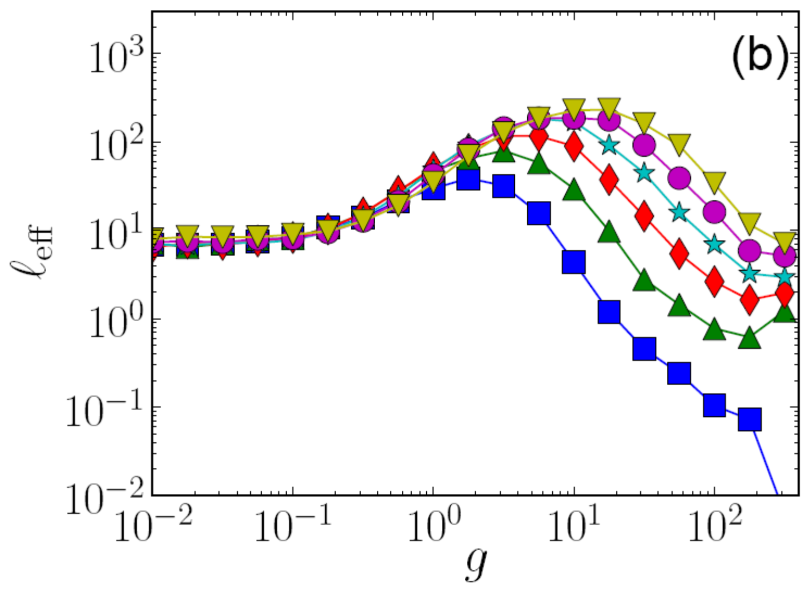
- Measured quantity: survival probability $p(t)$
- Faster computation!

Initial state occupies uniformly L_0 sites with random phases

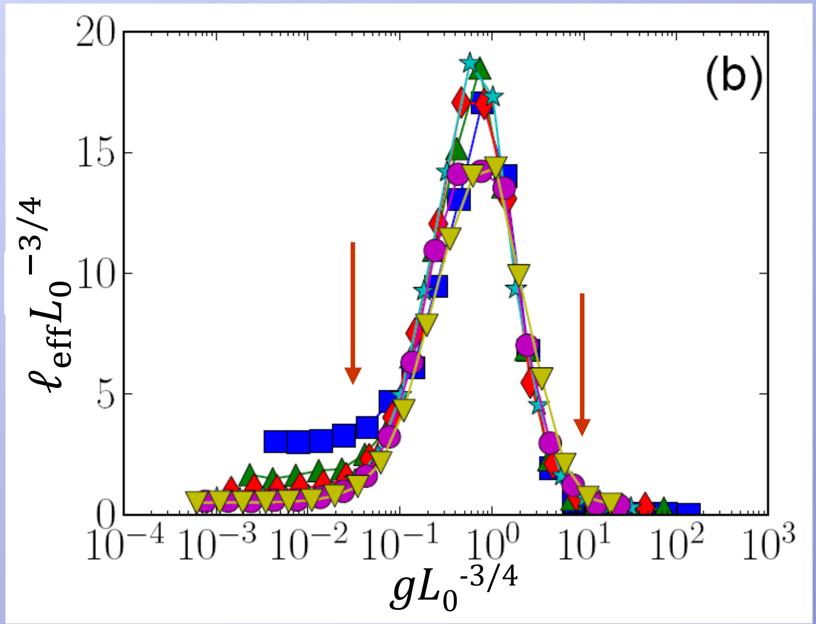




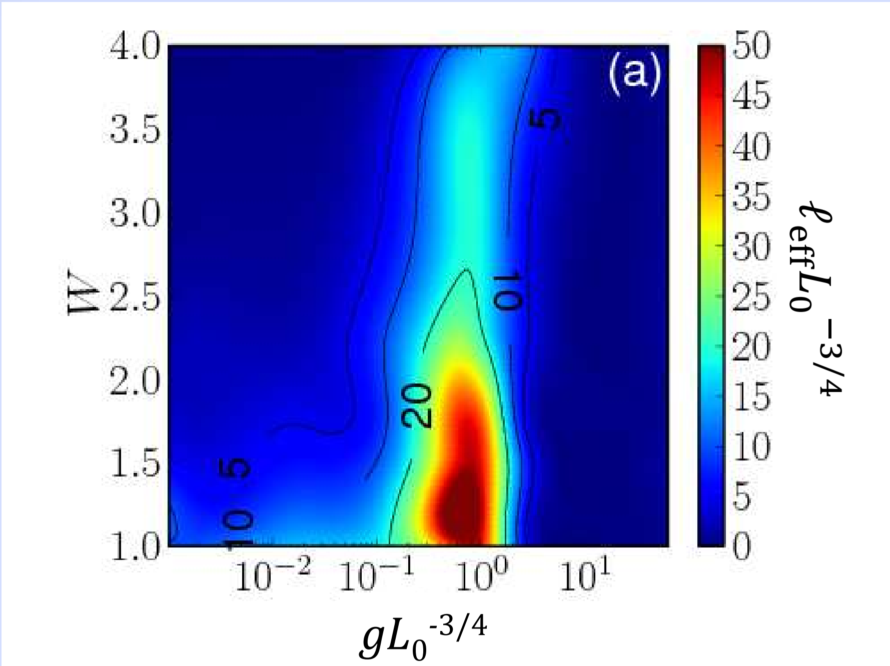
For moderate values of the disorder, one can find an "effective localization length"…



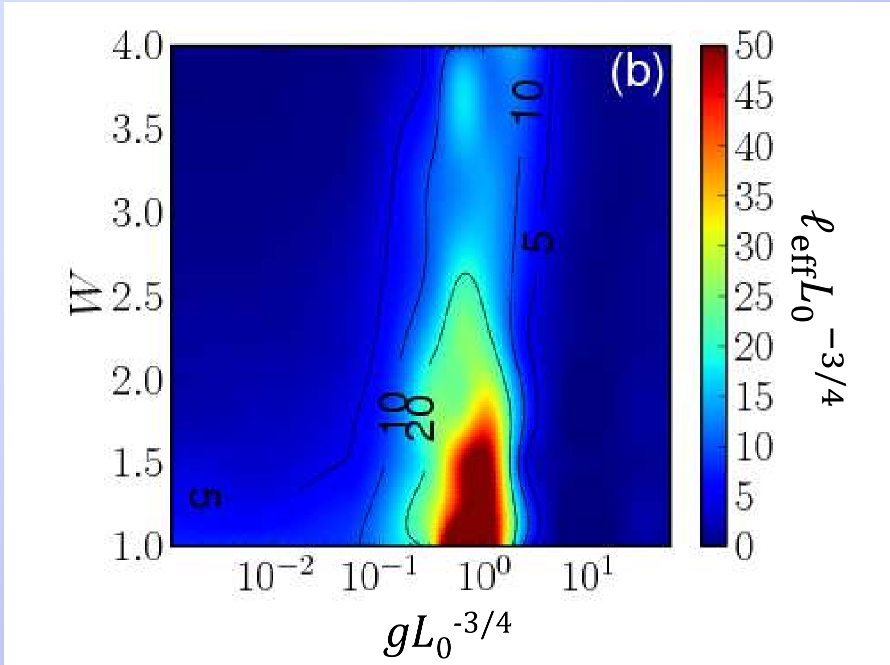
… which obeys a scaling law



$L_0 = 21$



$L_0 = 41$

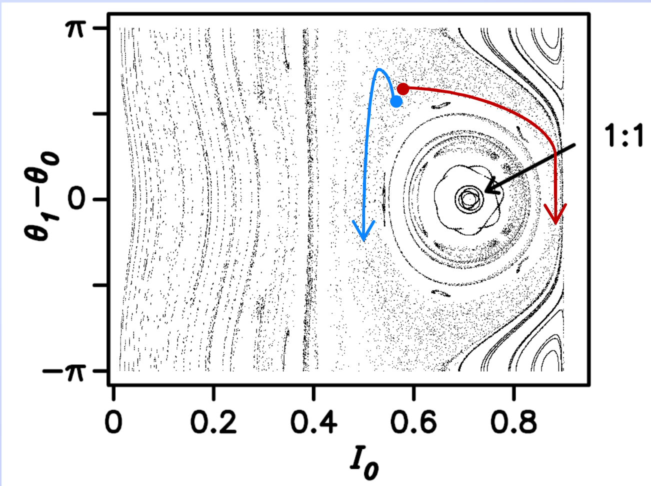


~ Independent of the initial state!

B. Vermersch and J. C. Garreau, *Sensitivity to the initial state of interacting ultracold bosons in disordered lattices*, arXiv/1111.4081 (2011)

Conclusion

- Bose-Einstein condensation opens the way to experimentally realizable nonlinear quantum systems
- Quantum systems displaying sensitivity to initial conditions → "quasiclassical" chaos
- Dynamics much more dependent on the initial state than in the linear case
- We probably need some "rewording" to adapt to nonlinear quantum dynamics (*fundamental principles are **not** challenged*)
- Combination of non local quantum effects and quasiclassical chaos



$$\Psi(t=0) = \psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)$$

$$\Psi(t) = ?$$

How entanglement evolves under chaotic dynamics?