Macroscopic superpositions in the presence of phase noise in a Bose Josephson junction

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Work done at : LPMMC, Grenoble

With : Dominique Spehner, Anna Minguzzi, Frank W.J.Hekking

Bose-Einstein Condensation

BEC first observed in 1995 → Nobel prize in 2001 to Ketterle, Wieman, Cornell

Typical parameters

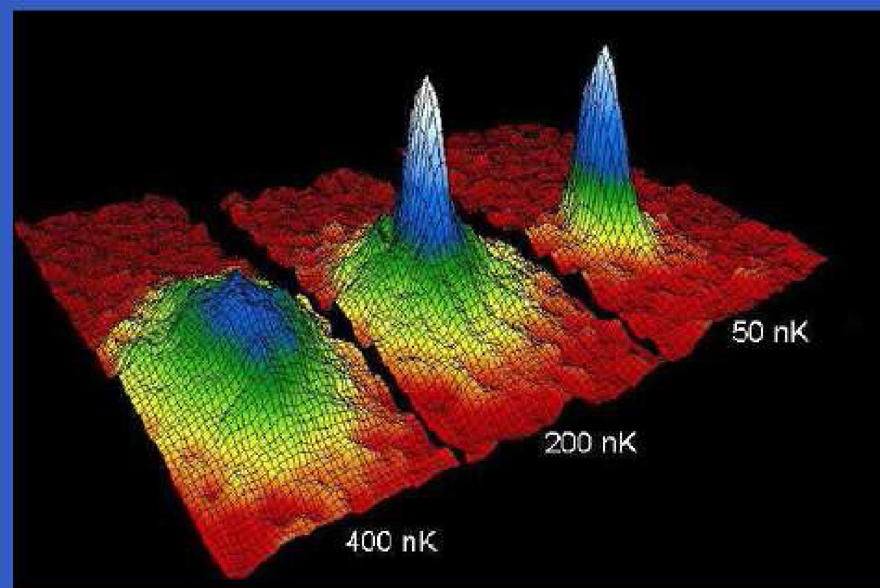
temperature 10 - 100 nK

density 10^{13} - 10^{14} cm^{-3}

number of atoms 10^{3} - 10^{7}

size 10 μm - 1 mm

lifetime 10 s



JILA & MIT 1995

Condensed species

^{87}Rb Na ^{7}Li H ^{85}Rb ^{4}He* ^{41}K ^{133}Cs ^{174}Yb ^{52}Cr ^{39}K ...

Anderson et al., Science '95 Davis et al. PRL '95

Cold atoms

Extremely high degree of control on the experimental parameters

- Interaction strength ;
- Trap geometry ;
- Atomic species...

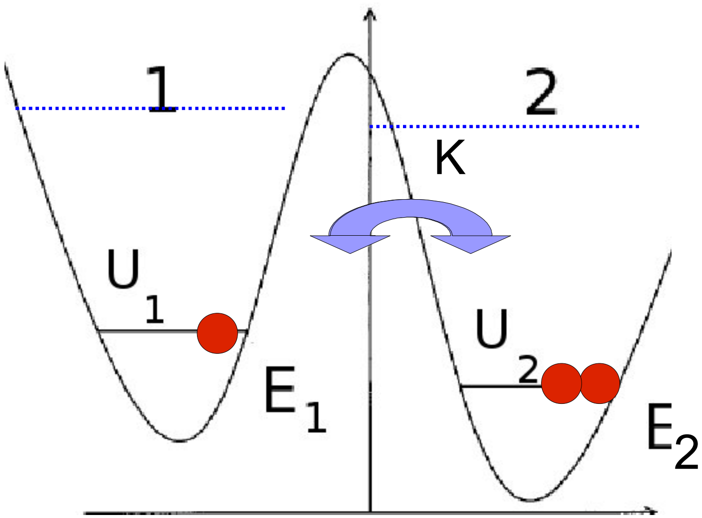
⇒ Applications for :

- Low dimensional regimes ;
- Non-linear physics (solitons...) ;
- Quantum simulators ;
- Quantum metrology

  (atomic clocks, magnetic field sensors, rotation sensors, interferometers...)

The Bose Josephson junction : N bosons in 2 modes

Ultracold atoms in a double-well potential



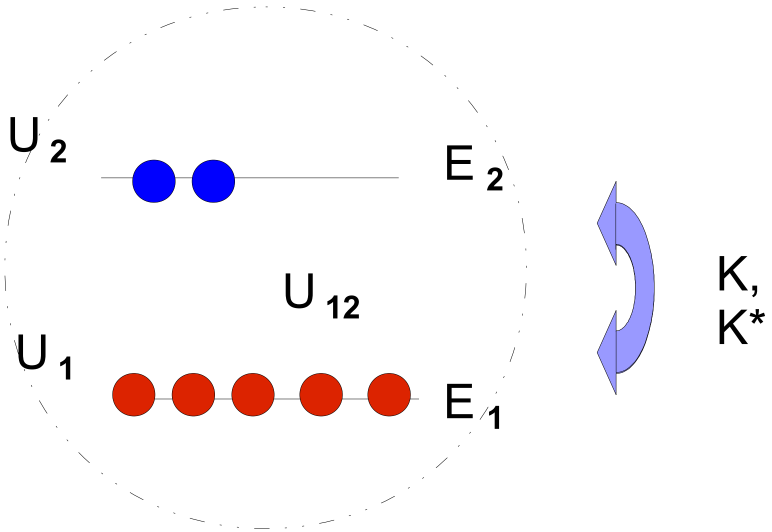
- $E_1 \, , E_2$: onsite energies;
- $U_1 \, , U_2$: interaction energies;
- K : tunneling

Heidelberg [C. Gross, M. Oberthaler];
Haifa [J. Steinhauer];
Cambridge [W. Ketterle, D. E. Pritchard];

EXTERNAL BOSE JOSEPHSON JUNCTION

The Bose Josephson junction : N bosons in 2 modes

...or atoms in two different hyperfine states



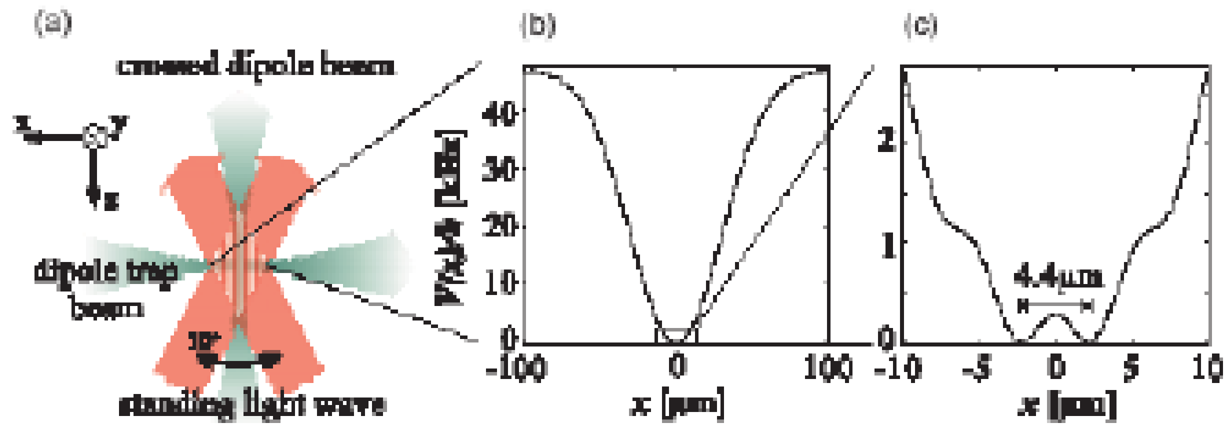
- E_1 , E_2 : hyperfine energies;
- U_1 , U_2 : interaction energies;
- U_{12} : cross-interaction energy ;
- K , K^* : coupling

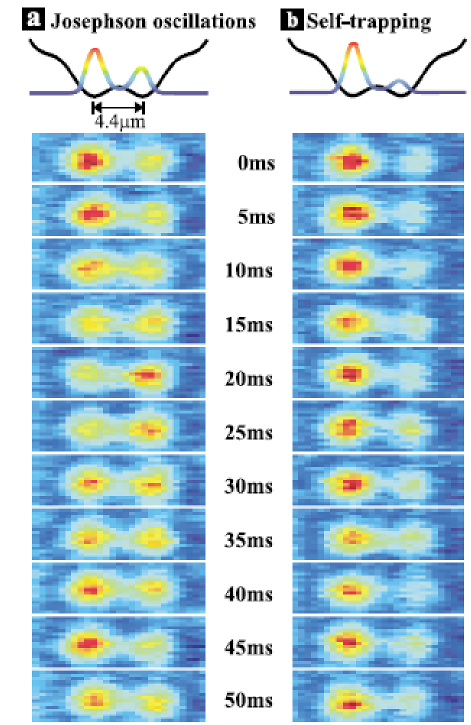
Heidelberg [C. Gross, M. Oberthaler];
Munich + Paris [P. Treutlein, T. Hansch, J. Reichel];
Boulder [C. E. Wieman ; E. A. Cornell];

INTERNAL BOSE JOSEPHSON JUNCTION

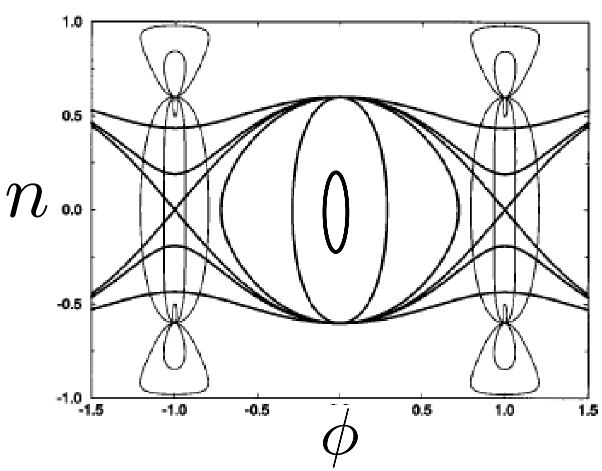
Mean field regime: oscillations and self-trapping

*Experiments : [M. Albiez et al, PRL **95**, 010402 (2005)].*

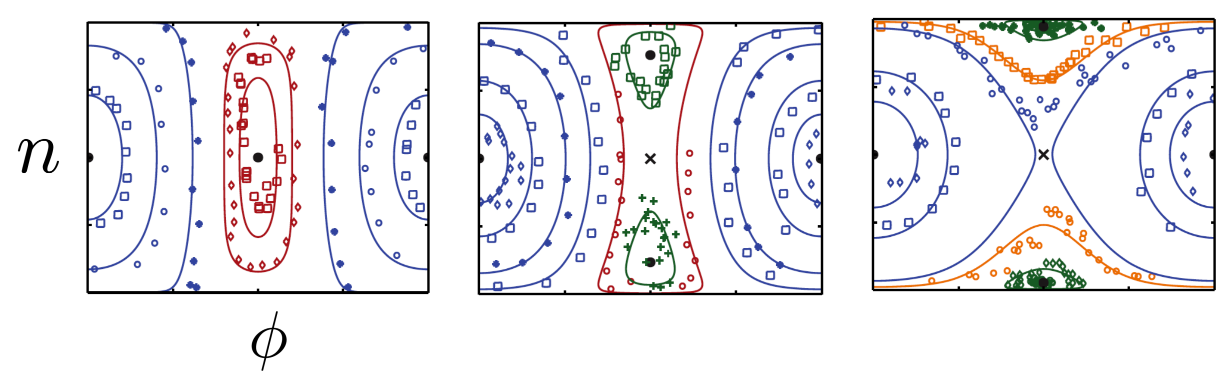




*Theory : [A. Smerzi et al, PRA **79**, 4951 (1997)].*



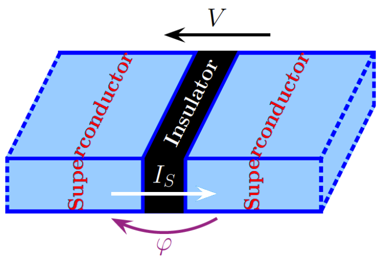
*New experiments: [Zibold et al, PRL **105**, 204101 (2010)]*



Why is this interesting?

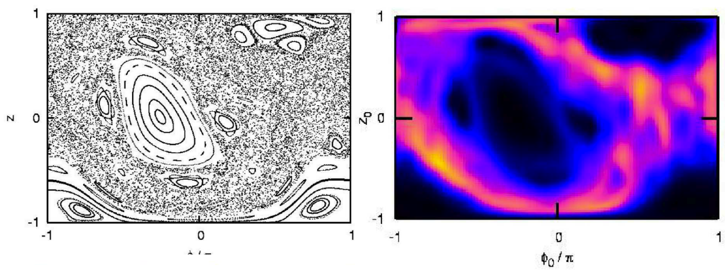
- Analogy with the superconducting Josephson junction

  *[S. Giovanazzi et al, PRA **84**, 4521 (2000),*
  *S. Levy et al, Nature **449**, 06186 (2007)].*



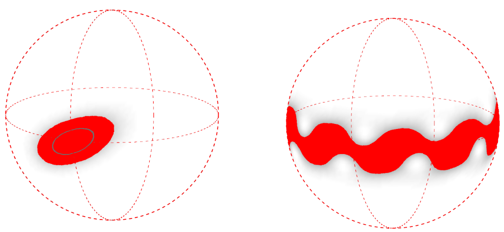
- Transport, chaos & entanglement

  *[C. Weiss et al, PRL **100**, 140408 (2008)]*



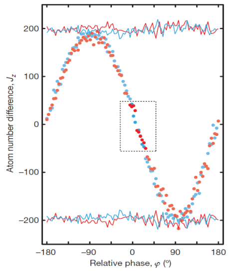
- Generation of many-body entangled states

  *[M. Kitagawa et al, PRL **34**, 3974 (1986),*
  *G. Ferrini et al, PRA **78** 023606 (2008)]*



- Applications to metrology

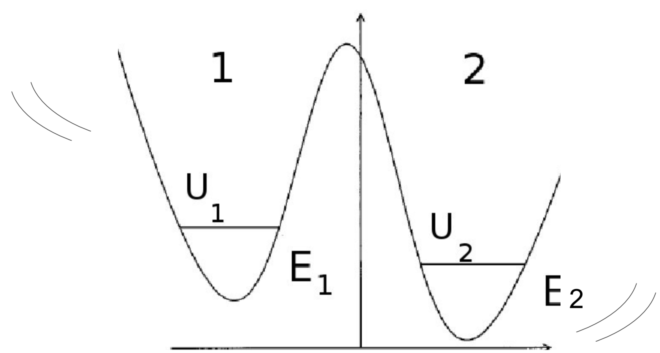
  *[C. Gross et al, Nature **464**, 1165 (2010)*
  *G. Ferrini et al, PRA **84**, 043628 (2011)]*

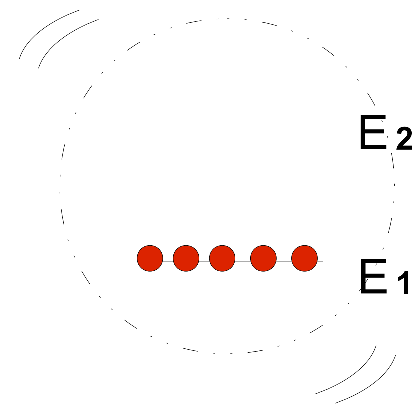


Many sources of noise...

Phase noise *[G.Ferrini et al, PRA **82**, 033621 (2010) ; PRA **84**, 043628 (2011)]*

fluctuations of the energies of the two modes





- Particle losses *[A. Sinatra et al, Eur. Phys. J. D 4, 247 (1998)]*;
- Collisions with thermal atoms *[J. Anglin et al, PRL **79**, 6 (1997)]*;
- ...

EFFECT OF

DECOHERENCE

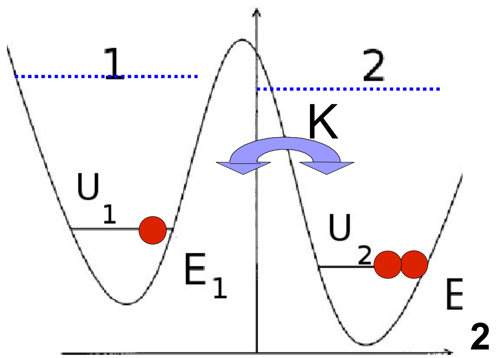
Outlines

- Introduction to the Bose Josephson junction (BJJ) in the quantum regime
- Generation of entangled states
- Decoherence effects induced by phase noise
- Applications in interferometry

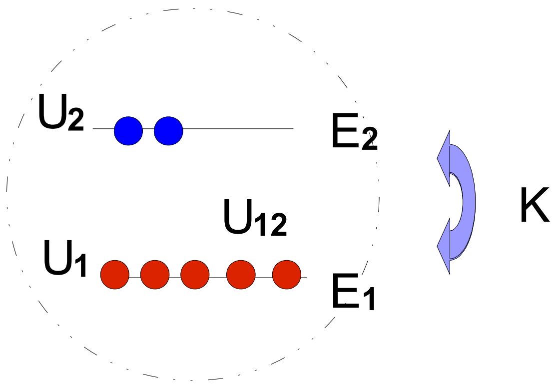
Introduction to the Bose Josephson junction

The Bose Josephson junction

EXTERNAL BOSE JOSEPHSON



INTERNAL BOSE JOSEPHSON



- E_1 ,E_2 : onsite energies;
- U_1 ,U_2 : interaction energies;
- U_{12} : cross interaction energy (only in the internal case) ;
- K : coupling

$$\hat{H}^{(0)} = E_1 \hat{a}_1^\dagger \hat{a}_1 + E_2 \hat{a}_2^\dagger \hat{a}_2 - (K \hat{a}_1^\dagger \hat{a}_2 + K^* \hat{a}_2^\dagger \hat{a}_1)$$
$$+ U_1 \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1 + U_2 \hat{a}_2^\dagger \hat{a}_2^\dagger \hat{a}_2 \hat{a}_2 + U_{12} a_1^\dagger a_1 a_2^\dagger a_2$$

Mapping into a spin problem

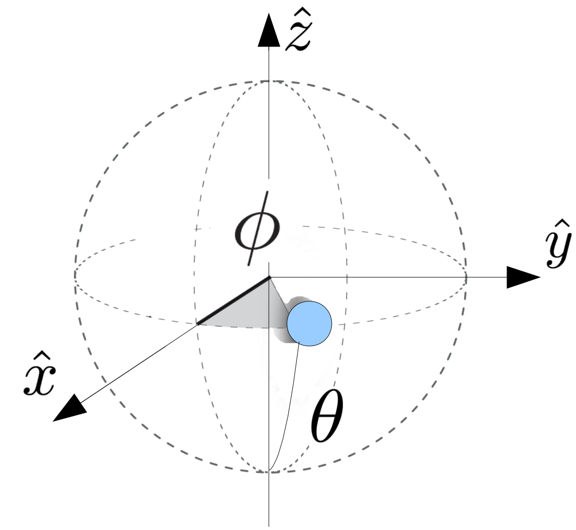
$$N = a_1^\dagger a_1 + a_2^\dagger a_2 \qquad J^2 = N/2(N/2+1)$$

$$\hat{J}_z = \frac{1}{2}(\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2) \equiv \hat{n}$$ *Number imbalance*

$$\hat{J}_y = -i\frac{1}{2}(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1)$$ *Current*

$$\hat{J}_x = \frac{1}{2}(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)$$ *Tunneling*

Bloch sphere



$$\hat{H}^{(0)} = \chi \hat{J}_z^2 - \lambda \hat{J}_z - 2K\hat{J}_x$$

with $\lambda = (E_1 - E_2) + (N-1)\frac{U_1 - U_2}{2}$

$\chi = U_1 + U_2$ external BJJ

$\chi = U_1 + U_2 - 2U_{12}$ internal BJJ

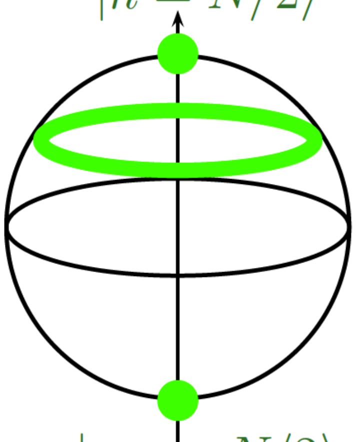
*[G.Milburn et al, PRA **55**, 4318 (1997)].*

Possible basis

- Fock states

$$|N_1, N_2\rangle \equiv |n = \frac{N_1 - N_2}{2}\rangle$$

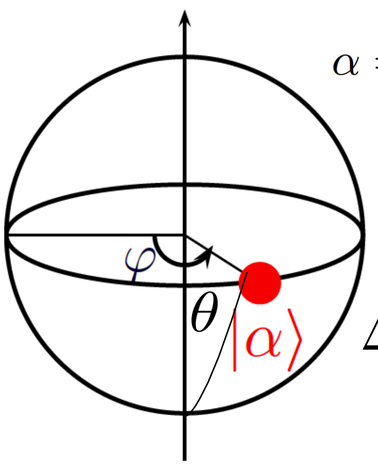
$$\hat{J}_z|n\rangle = n|n\rangle \qquad -N/2 \leq n \leq N/2$$



- SU(2) coherent states

$$|\theta, \phi\rangle = \frac{1}{2^{\frac{N}{2}}} \sum_{n=-N/2}^{N/2} \binom{N}{\frac{N}{2}+n}^{\frac{1}{2}} \alpha^{(\frac{N}{2}+n)}|n\rangle$$

$$= \frac{(\cos\frac{\theta}{2}a_1^\dagger + \sin\frac{\theta}{2}e^{-i\phi}a_2^\dagger)^N}{\sqrt{N!}}|0\rangle \equiv |\alpha\rangle$$



→ phase states : $|\phi\rangle \equiv |\theta = \frac{\pi}{2}, \phi\rangle$ on the equator

Ground state of a Bose Josephson junction

$$\hat{H}^{(0)} = \chi \hat{J}_z^2 - \lambda \hat{J}_z - 2K\hat{J}_x = \chi(\hat{J}_z - \frac{\lambda}{2\chi})^2 - 2K\hat{J}_x$$

$\chi \gg KN$ Fock regime

$\chi N \ll K$ Rabi regime

$|\psi_{GS}\rangle = |n\rangle$ **Fock state**

$n = \mathrm{Int}\,[\lambda/(2\chi)]$

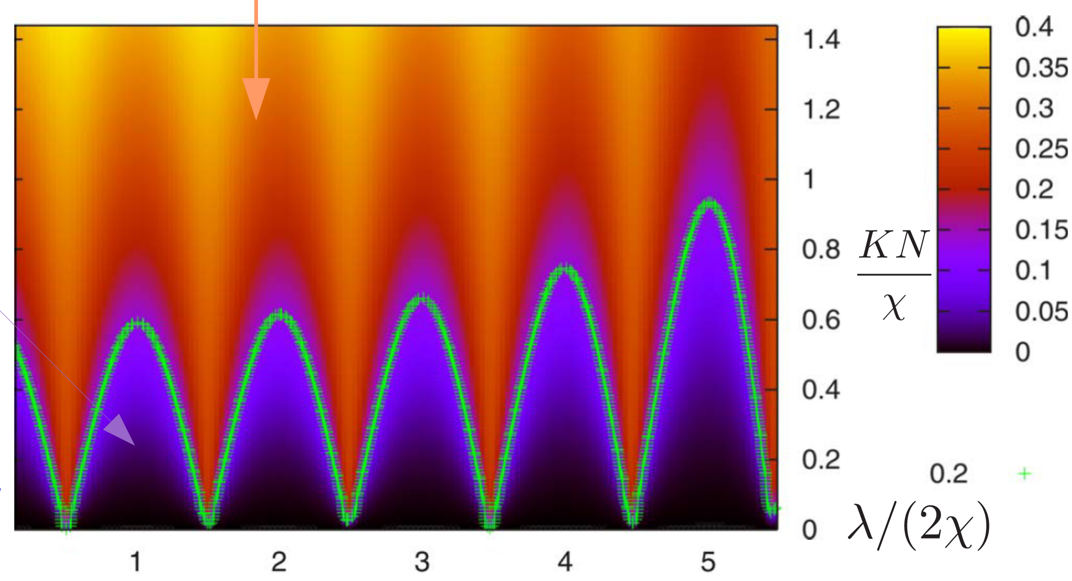
$|\psi_{GS}\rangle = |\phi = 0\rangle$ phase state

*[A. J. Leggett, Rev Mod Phys **73**, 307 (2001)]*

number fluctuations diagram

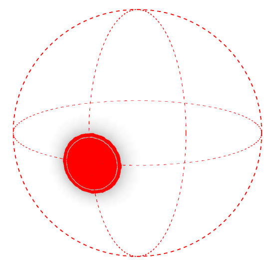
$$\langle \psi_{GS} | \Delta^2 n | \psi_{GS} \rangle$$

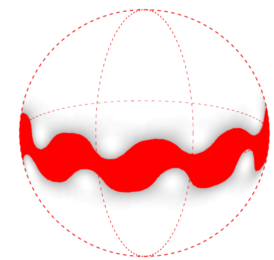
*[G.Ferrini et al, PRA **78**, 023606 (2008)].*



Entangled states in a BJJ

From a coherent state to...



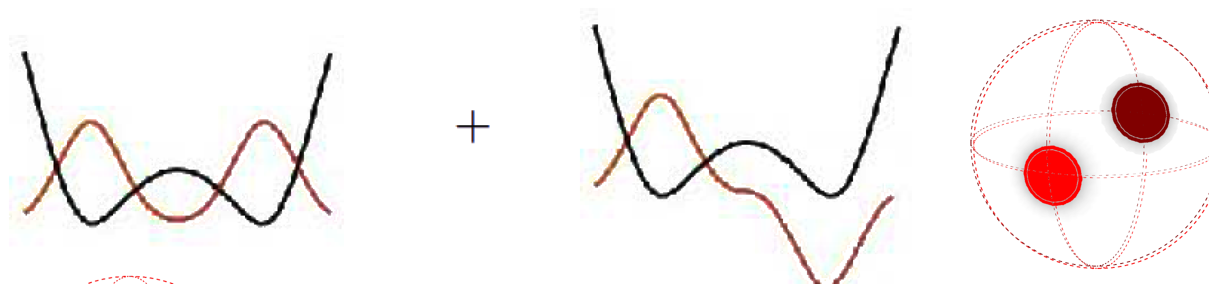


- Macroscopic superpositions of coherent states

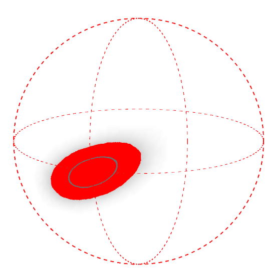
  e.g.

Phase-cat state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\varphi = 0\rangle + e^{i\gamma}|\varphi = \pi\rangle\right)$$



- Squeezed states



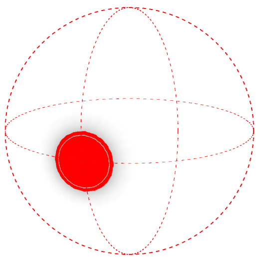
Generally speaking : ρ entangled if it cannot be written as

$$\rho = \sum_k P_k \rho_k^{(1)} \otimes \rho_k^{(2)} \otimes \ldots \otimes \rho_k^{(N)}$$

Creation of entangled states

Quenched dynamics

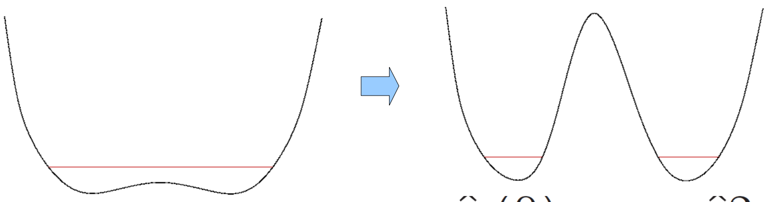
- Initial state : phase state $|\psi(t=0)\rangle = |\phi\rangle$



- Quench : set K = 0

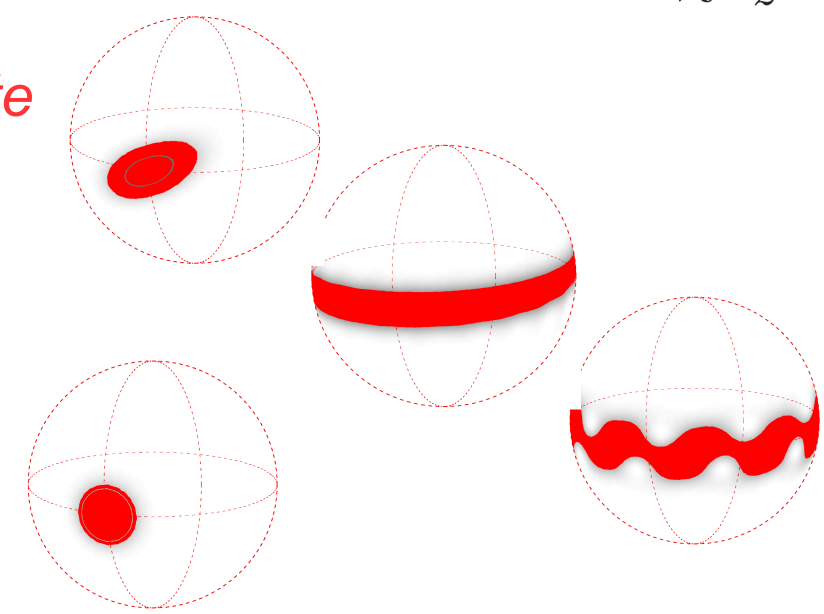
  → Internal BJJ : switch-off the coupling

  → External BJJ : suddenly raise the barrier



$$\hat{H}^{(0)} = \chi \hat{J}_z^2$$

- Short times $t \sim 1/(\chi N^{\frac{2}{3}})$: *squeezed state*
  [M. Kitagawa et al, PRL ***34****, 3974 (1986)].*

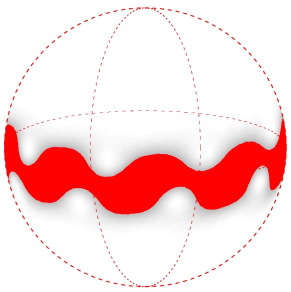


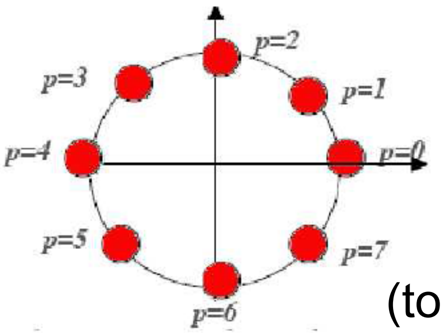
- $t = T = 2\pi/\chi$: initial coherent state

.

Creation of macroscopic superpositions

- $t_q \equiv \frac{\pi}{\chi q} \equiv \frac{T}{2q}$:





(top view)

$$|\psi^{(0)}(t_q)\rangle = \sum_{k=0}^{q-1} c_k |\phi_k\rangle$$

$$c_k = 1/q \sum_{q-1}^{m=0} e^{i\frac{\pi k(N+k)}{q}} e^{-i\frac{\pi m^2}{q}}$$

q-component *macroscopic superposition*

The first one: $q_{\max} \sim \frac{2\pi(N/2)}{\sqrt{N}/2} \sim \sqrt{N}$ components, at time: $t_{q_{max}} \sim \frac{T}{\sqrt{N}}$

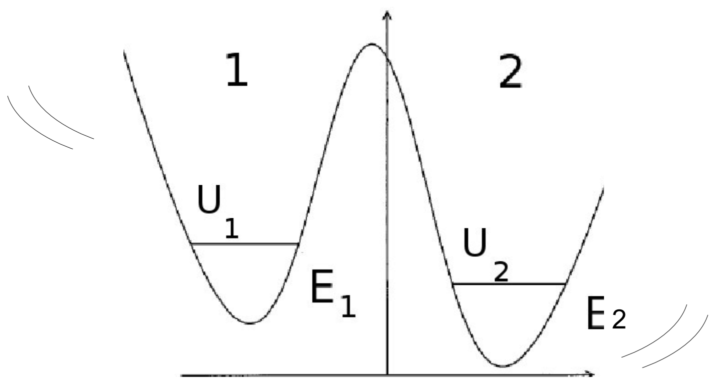
*[G. Ferrini et al, PRA **78** 023606 (2008) ; F. Piazza et al, PRA **78** 051601 (2008)].*

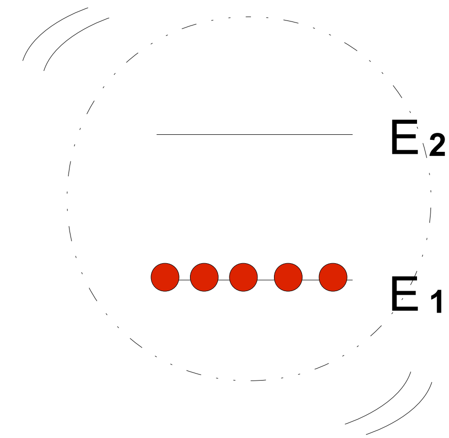
Decoherence

What if noise is disturbing the quenched dynamics?

Phase noise: fluctuations of the energies of the two modes

$$\hat{H} = \chi \hat{J}_z^2 - \lambda(t)\hat{J}_z$$





$U_1 = U_2$

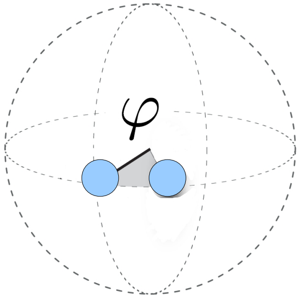
$\lambda = E_1 - E_2$, randomly fluctuating

noise hamiltonian commutes with the unitary part → *WE CAN SOLVE IT EXACTLY!*

Phase noise during the quenched dynamics

At each realization

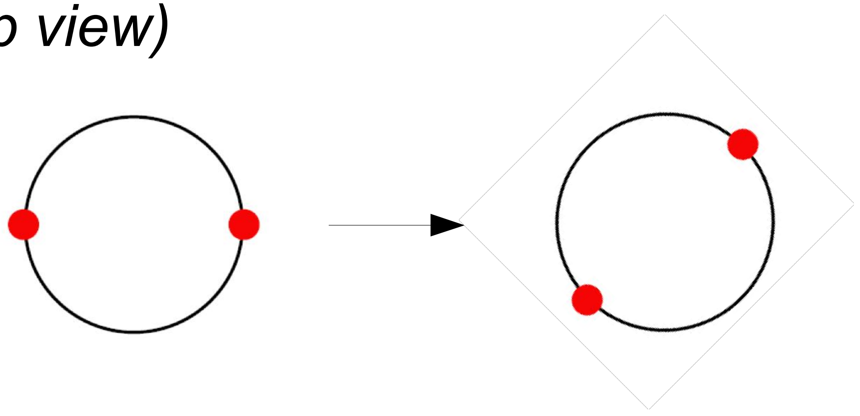
$$|\psi(t)\rangle = e^{i\int_0^t d\tau\lambda(\tau)\hat{J}_z}|\psi^{(0)}(t)\rangle = e^{-i\phi(t)\hat{J}_z}|\psi^{(0)}(t)\rangle$$



➡ *Rotation of the quantum state by* $\phi(t) = -\int_0^t d\tau\lambda(\tau)$

(top view)

e.g: t=T/4 (two-component superposition)

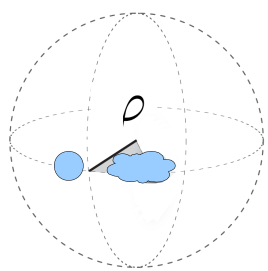


Phase noise during the quenched dynamics

Then: average over the realizations

$$\hat{\rho}(t) = \overline{|\psi(t)\rangle\langle\psi(t)|} = \int dP\,[\lambda]\,|\psi(t)\rangle\langle\psi(t)|$$

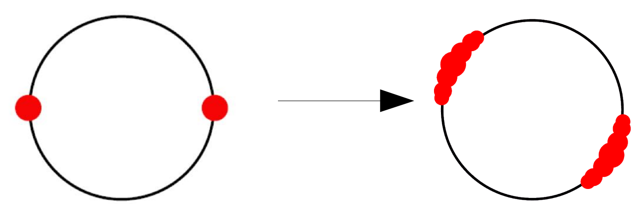
⇨ *Rotation + spreading*



Variance:

$$a^2(t) = \int_0^t d\tau \int_0^t d\tau' \left(\overline{\lambda(\tau)\lambda(\tau')} - \overline{\lambda}^2\right)$$

e.g: t=T/4 (two-component superposition)



(top view)

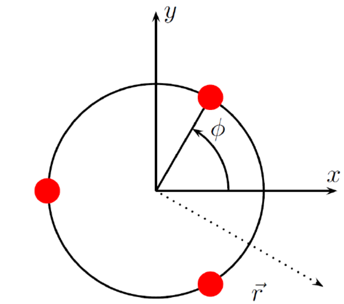
Projecting onto the Fock basis

$$\langle n|\hat{\rho}(t)|n'\rangle = e^{-\frac{a^2(t)(n-n')^2}{2}} e^{i\overline{\lambda}t(n-n')} \langle n|\hat{\rho}^{(0)}(t)|n'\rangle$$

*[G.Ferrini et al, PRA **82**, 033621 (2010)]*

Density matrix of a macroscopic superposition

q-component macroscopic superposition $$|\psi^{(0)}(t_q)\rangle = \sum_{k=0}^{q-1} c_k |\phi_k\rangle$$

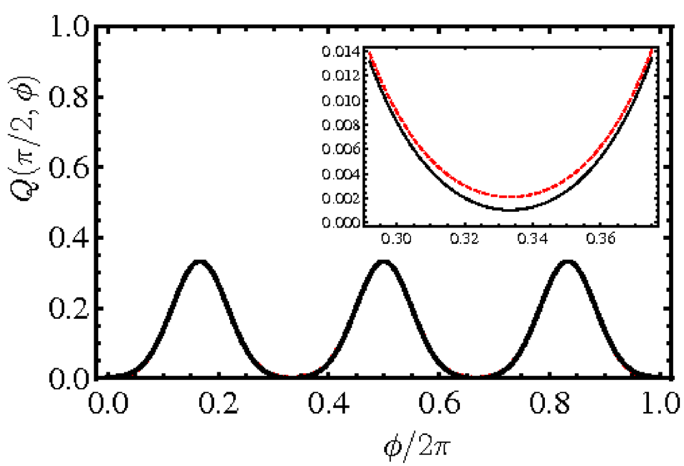


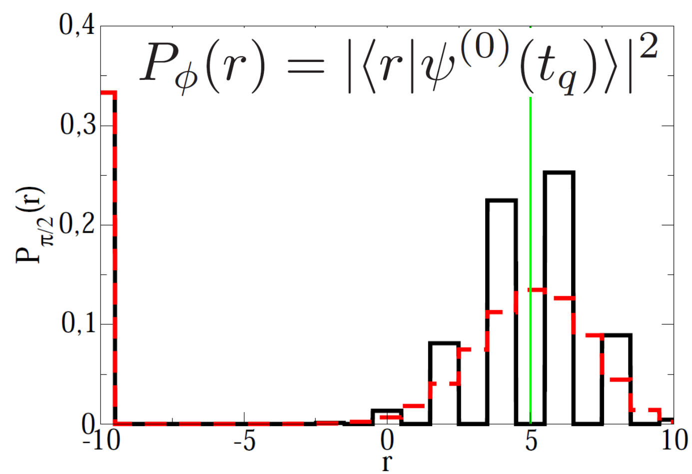
$$\hat{\rho}^{(0)}(t_q) = \sum_{k=0}^{q-1} |c_k|^2 |\phi_k\rangle\langle\phi_k| + \sum_{k\neq k'0}^{q-1} c_k c_k'^{*} |\phi_k\rangle\langle\phi'_k|$$

$$\hat{J}_r = \hat{J}_x \sin\phi - \hat{J}_y \cos\phi$$

Incoherent mixture of phase states: phase profile

Correlations: interference effects; **quantum correlations**





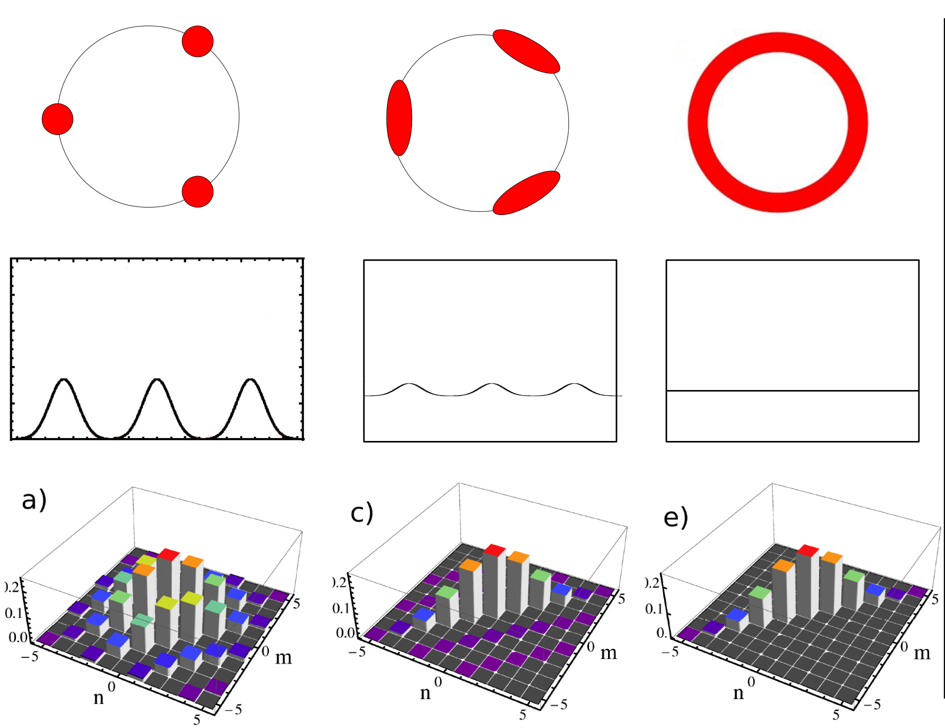
$$\hat{J}_r|r\rangle = r|r\rangle$$

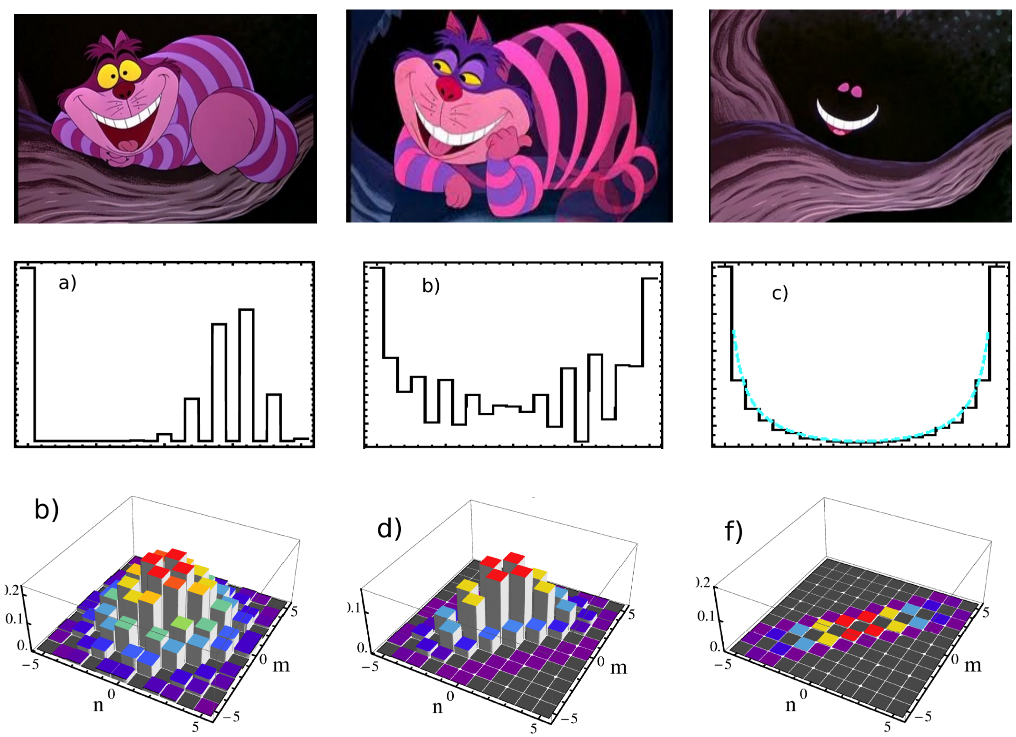
*[G.Ferrini et al, PRA **80**, 043628 (2009)]*

With noise: → *Phase relaxation* → *Decoherence*

Effect of noise on macroscopic superpositions

$$\hat{\rho}^{(0)}(t_q) = \sum_{k=0}^{q-1} |c_k|^2 |\phi_k\rangle\langle\phi_k| + \sum_{k\neq k'0}^{q-1} c_k c_k'^{*} |\phi_k\rangle\langle\phi'_k|$$



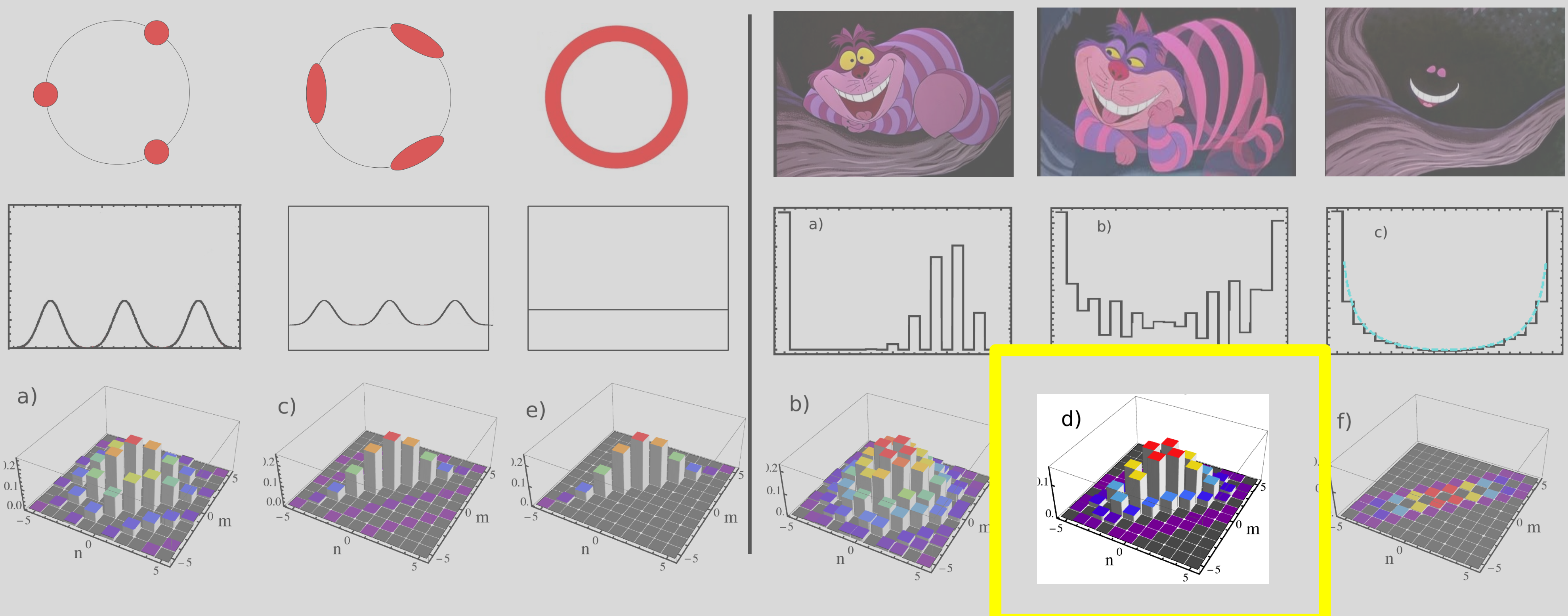


Phase relaxation → **SAME RATE!** ← *Decoherence*

$\overline{\lambda} = 0$ **Independent on *N*** *[G.Ferrini et al, PRA **82**, 033621 (2010)]*

Effect of noise on macroscopic superpositions

$$\hat{\rho}^{(0)}(t_q) = \sum_{k=0}^{q-1} |c_k|^2 |\phi_k\rangle\langle\phi_k| + \sum_{k\neq k'0}^{q-1} c_k c_k'^{*} |\phi_k\rangle\langle\phi_k'|$$



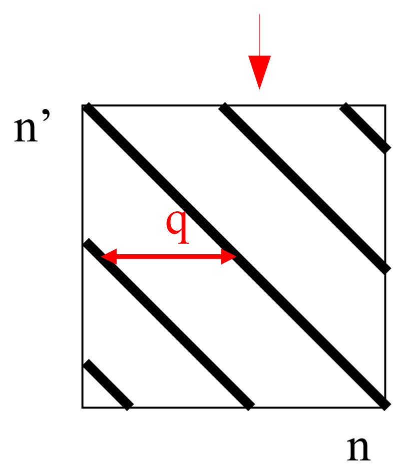
Phase relaxation → **SAME RATE!** ← *Decoherence*

$\overline{\lambda} = 0$ **Independent on *N*** *[G.Ferrini et al, PRA **82**, 033621 (2010)]*

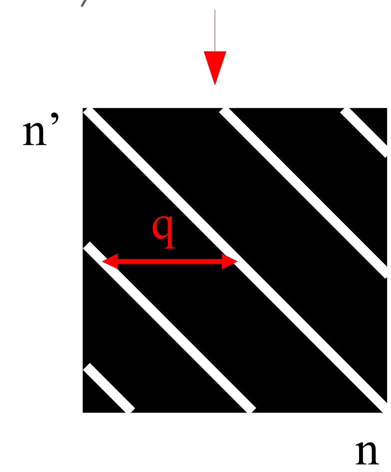
Why the «same» rate for decoherence and relaxation?

- Mathematical interpretation: $\hat{\rho}^{(0)}(t_q) = \sum_{k=0}^{q-1} |c_k|^2 |\phi_k\rangle\langle\phi_k| + \sum_{k\neq k'0}^{q-1} c_k c_k'^{*} |\phi_k\rangle\langle\phi_k'|$

$$\langle n'|\hat{\rho}(t)|n\rangle = e^{-\frac{a^2(t)(n-n')^2}{2}} \langle n'|\hat{\rho}^{(0)}(t)|n\rangle$$



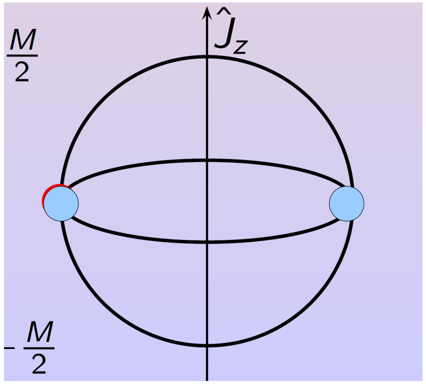
phase relaxation for aq >1



Decoherence for a > 1

- Pictorial interpretation: $\hat{H}_{\text{int}} = -\lambda(t)\hat{J}_z$

Noise acts perpendicularly to the plane of the superpositions



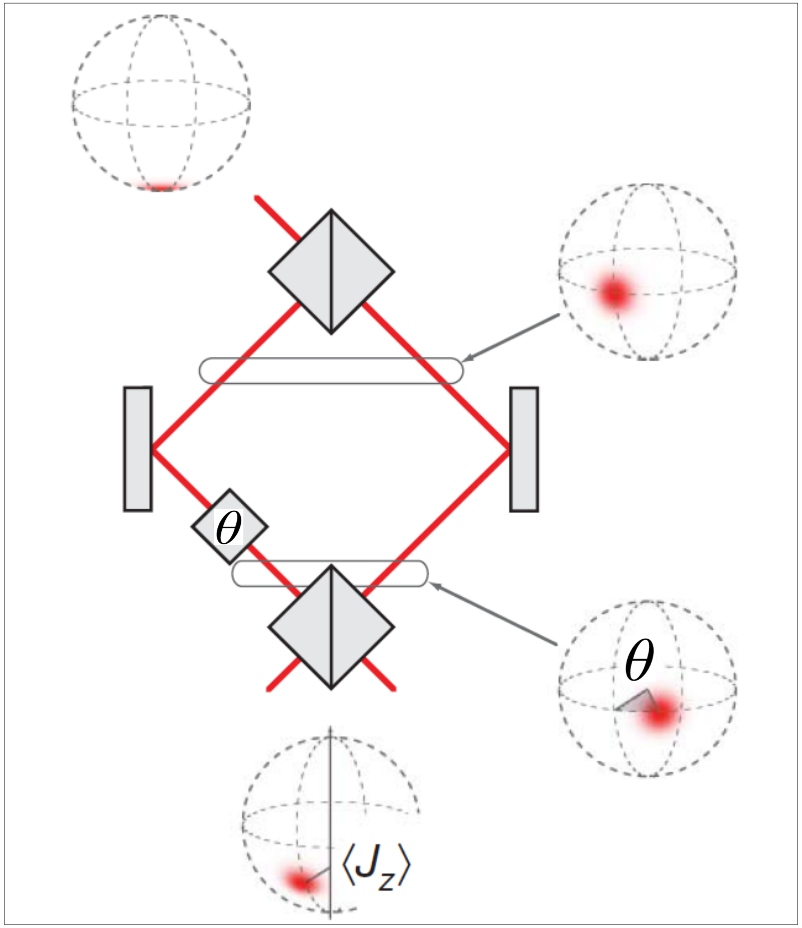
See also : *[R. Chaves et al, arXiv:1112.2645 (2011)]*

Consequences for interferometry

BJJ as an interferometer

Goal: to estimate a phase shift ϴ with the highest possible precision.

Interferometer = *ROTATION OF THE INPUT STATE*



$$|\psi\rangle_{out} = e^{-iJ_y\frac{\pi}{2}} e^{iJ_z\theta} e^{iJ_y\frac{\pi}{2}} |n = -\frac{N}{2}\rangle$$

$$= e^{-iJ_x\theta} |n = -\frac{N}{2}\rangle$$

Estimate ϴ from: $\langle \hat{J}_z \rangle_{out} = \frac{N}{2}\cos\theta$

Ramsey fringes

Limits on the precision and useful states

General interferometer: $|\psi\rangle_{out} = e^{-i\theta J_{\vec{n}}} |\psi\rangle_{in}$

General bound on the phase sensitivity : $\Delta\theta \geq \frac{1}{\sqrt{F_Q[|\psi\rangle_{in}, \hat{J}_n]}} \equiv \Delta\theta_{\text{best}}$

Cramer-Rao lower bound (single measure)

$F_Q[\psi, \hat{J}_n] = 4\langle\psi|\Delta^2 \hat{J}_{\vec{n}}|\psi\rangle$ (pure states) *QUANTUM FISHER INFORMATION*

defines the degree of **usefulness of a quantum states for interferometry**

$\Delta\theta\langle\Delta\hat{J}_{\vec{n}}\rangle \geq 1/2$ *generalized uncertainty principle*

$|\psi\rangle = |\phi\rangle$ $F_Q = N$ $\Longrightarrow$ $\Delta\theta_{\text{best}} = 1/\sqrt{N} \equiv \Delta\theta_{\text{SN}}$

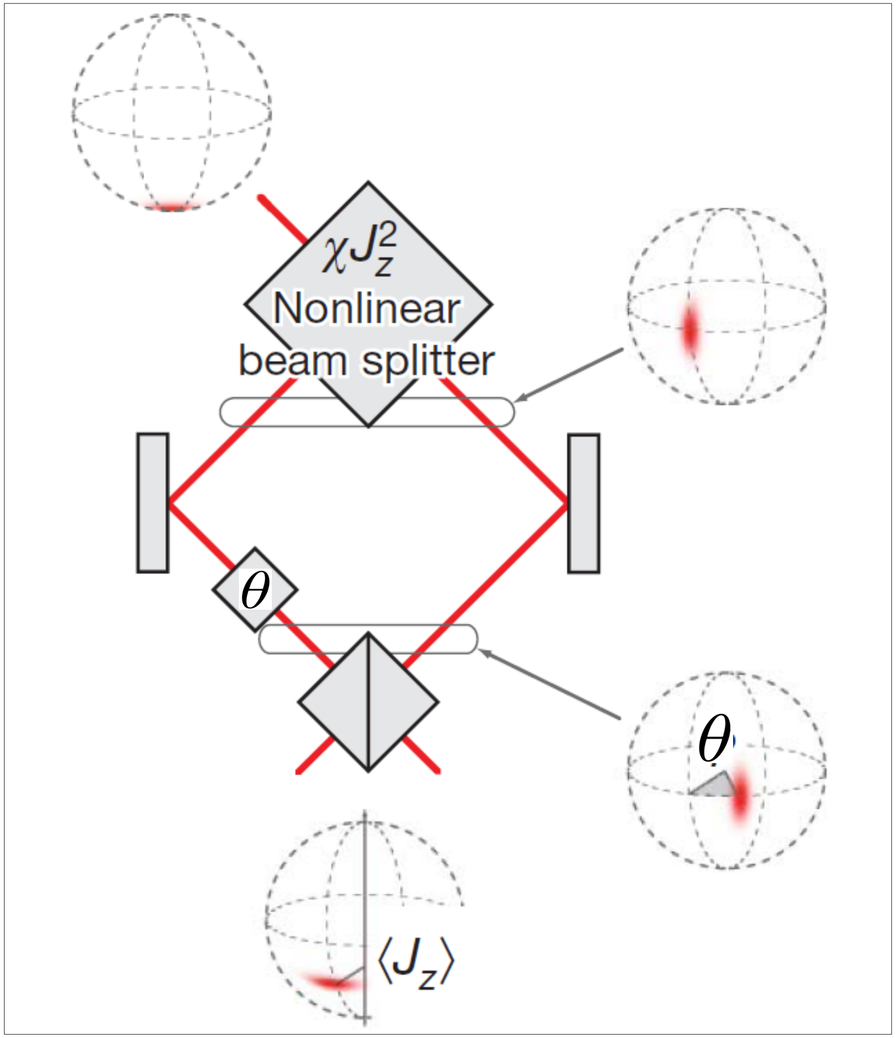
Shot noise limit

$|\psi\rangle = (|\phi\rangle + |-\phi\rangle)/\sqrt{2}$ $F_Q = N^2$ $\Longrightarrow$ $\Delta\theta_{\text{best}} = 1/N \equiv \Delta\theta_{\text{HL}}$

Heisenberg limit

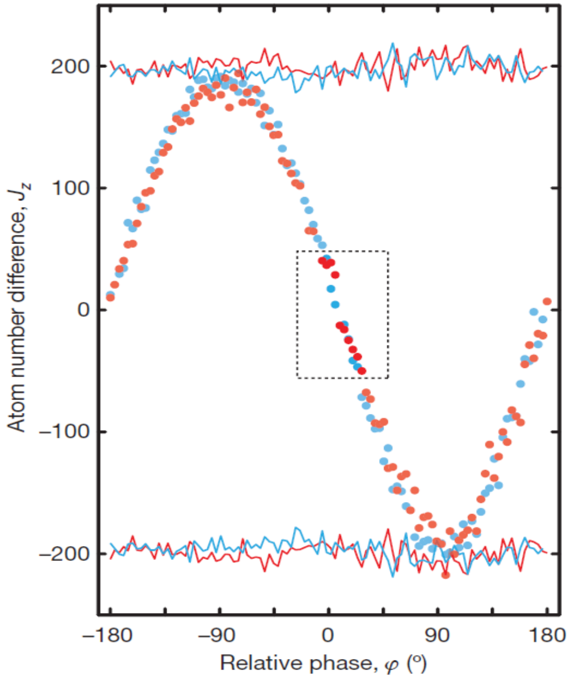
*[Braunstein et al, PRL **72** 3439 (1994), L. Pezzé et al, PRL **102** 100401 (2009)].*

Squeezed states as input states of an interferometer



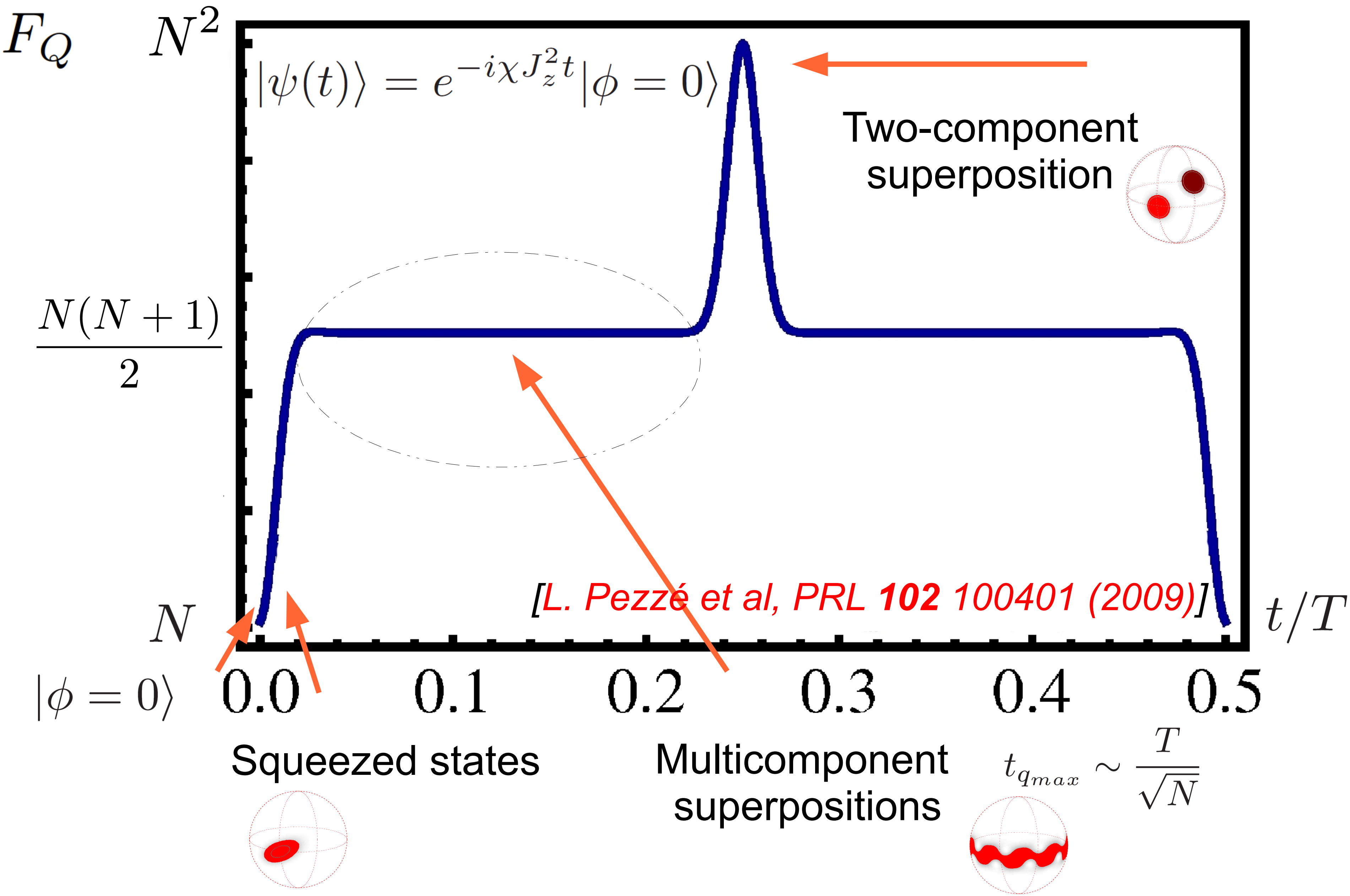
$$F_Q \sim N^{5/3} \quad \Rightarrow \quad \Delta\theta_{\text{best}} < \Delta\theta_{SN}$$

Recent experiments beating the shot noise limit with squeezed states

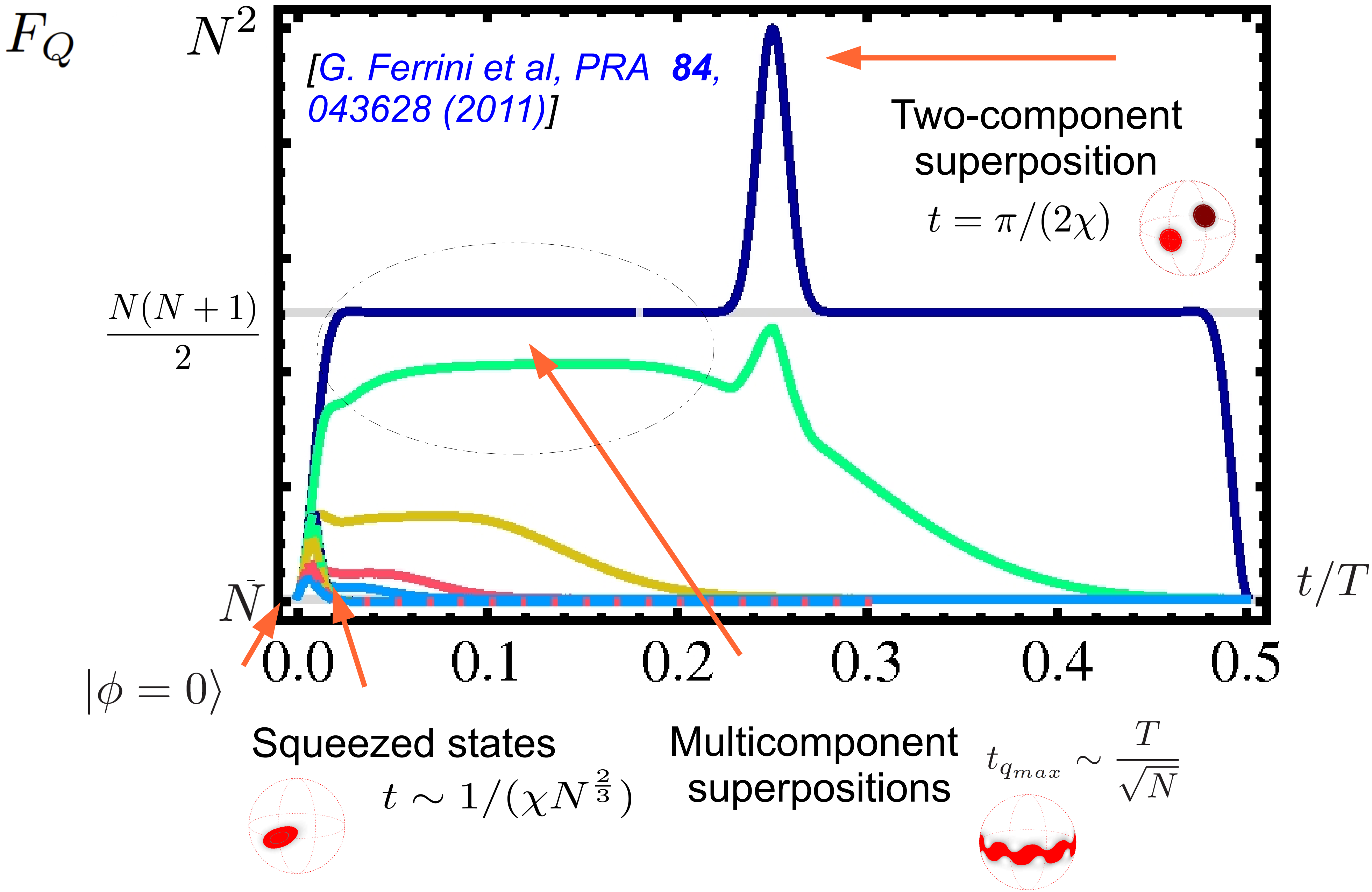


[C. Gross et al, Nature **464**, 1165 (2010)]

Fisher information during the quenched dynamics

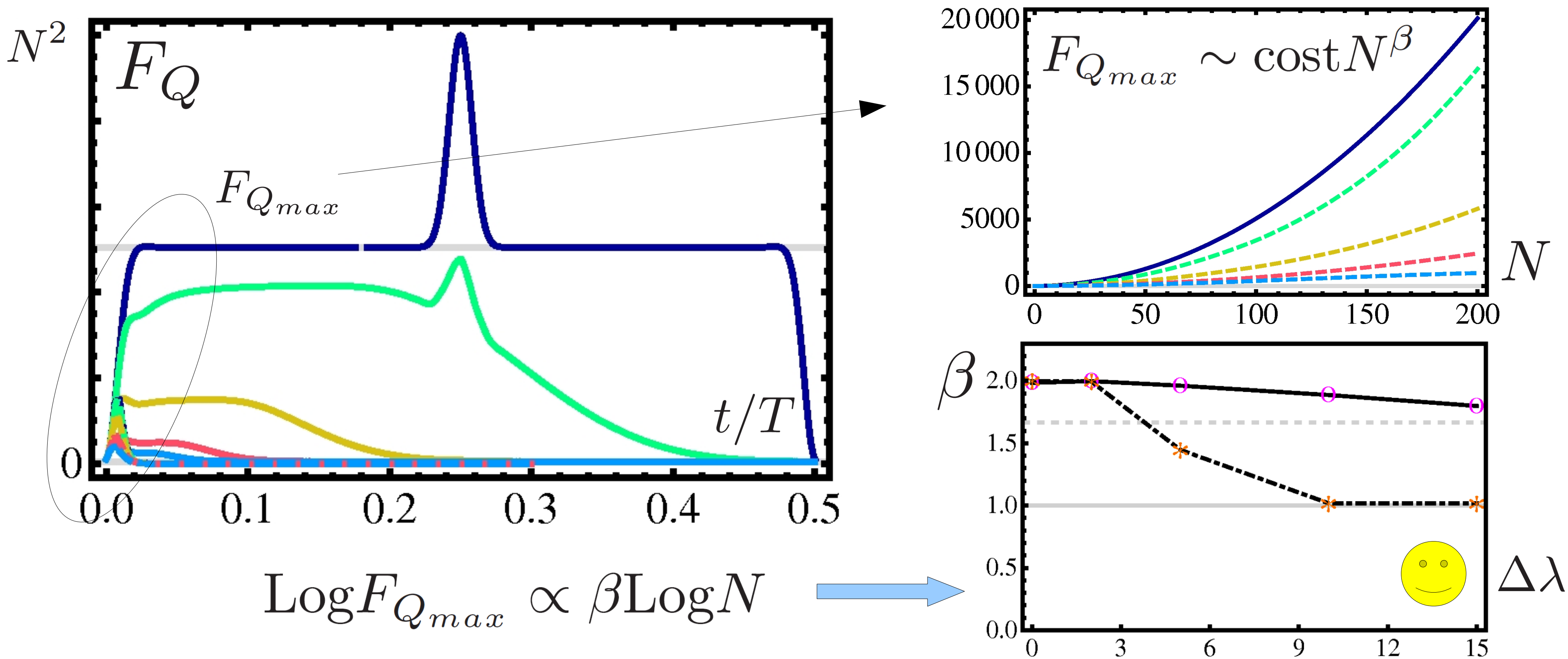


Fisher information during the noisy quenched dynamics



Effect of phase noise on the Fisher information

Scaling of $F_Q(t_{q_{max}} \sim T/\sqrt{N})$



$$\text{Log} F_{Q_{max}} \propto \beta \text{Log} N$$

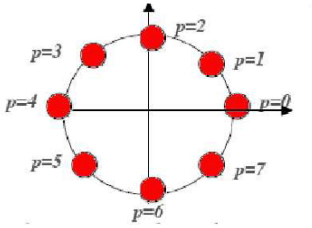
Better than the shot noise limit at intermediate noise strength

$$0 < \Delta\lambda \lesssim 10\text{Hz} \quad (\chi = \pi\text{Hz})$$

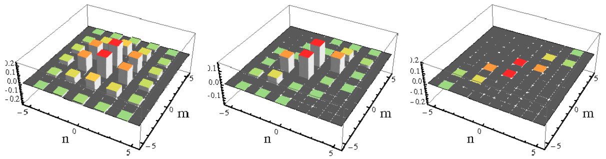
$\Delta\lambda = 0\text{Hz}$
$\Delta\lambda = 2\text{Hz}$
$\Delta\lambda = 5\text{Hz}$
$\Delta\lambda = 10\text{Hz}$
$\Delta\lambda = 15\text{Hz}$

Conclusions

- *Creation of macroscopic superpositions by the quenched dynamics of a BJJ*



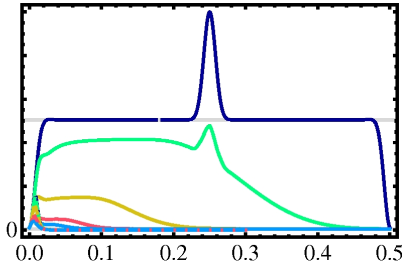
- *Phase cat states are robust under phase noise!*



Decoherence rate does not depend on the number of particles

- *Quantum Fisher information during the quenched dynamics*

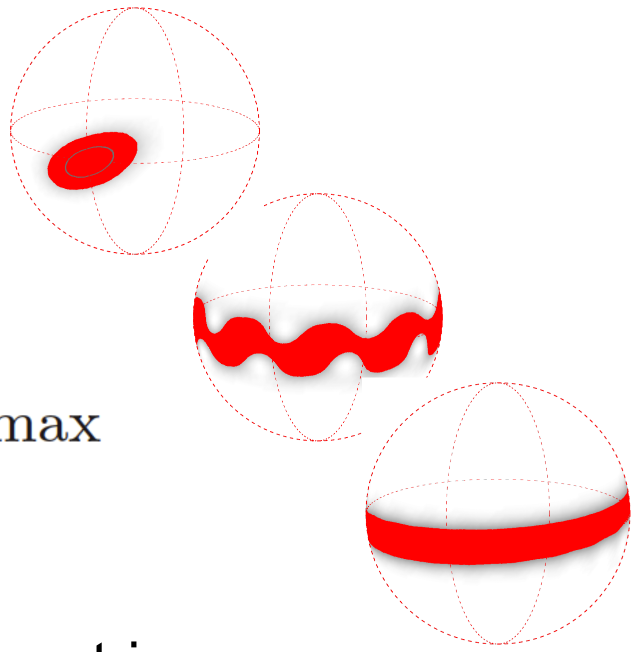
→ Application to interferometry : at intermediate noise strength *quantum correlations useful for interferometry survive BEYOND THE SPIN SQUEEZING REGIME*



THANK YOU FOR YOUR ATTENTION!

Usefulness of the quantum state during the quenched dynamics

To assign an intrinsic degree of useful correlations :
optimized quantum Fisher information



$$F_Q\left[\hat{\rho}_{\rm in}\right] \equiv \max_{\hat{n}} F_Q\left[\hat{\rho}_{\rm in}, \hat{J}_n\right] = 4\gamma_{\max}$$

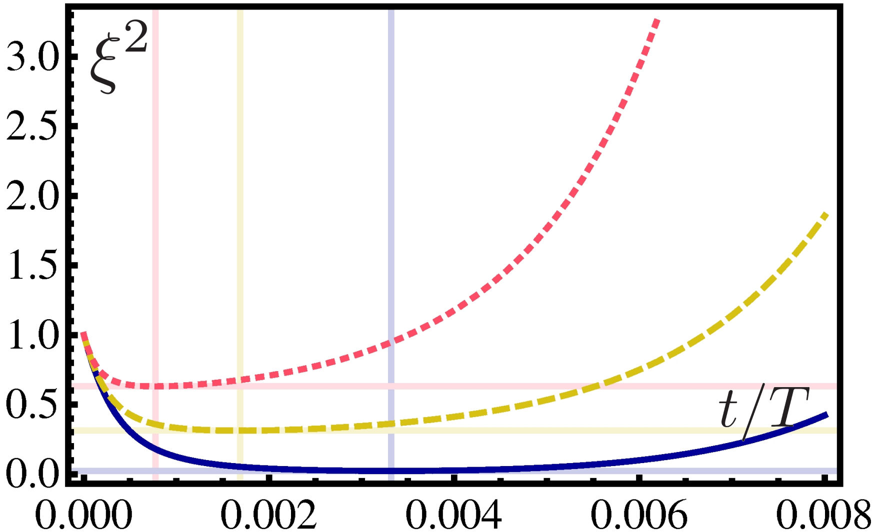
with $\gamma_{\max}$ the largest eigenvalue of the covariance matrix

$$\gamma_{ij}\left[\hat{\rho}_{\rm in}\right] = \frac{1}{2}\sum_{l,m,p_l+p_m>0}\frac{(p_l-p_m)^2}{p_l+p_m}\Re\mathrm{e}\left[\langle l|\hat{J}_i|m\rangle\langle m|\hat{J}_j|l\rangle\right]$$

[P. Hyllus et al, PRA ***82*** *012337 (2010)].*

Effect of phase noise on spin squeezing

$$\langle n|\hat{\rho}(t)|n'\rangle = e^{-\frac{a^2(t)(n-n')^2}{2}} e^{i\overline{\lambda}t(n-n')} \langle n|\hat{\rho}^{(0)}(t)|n'\rangle$$



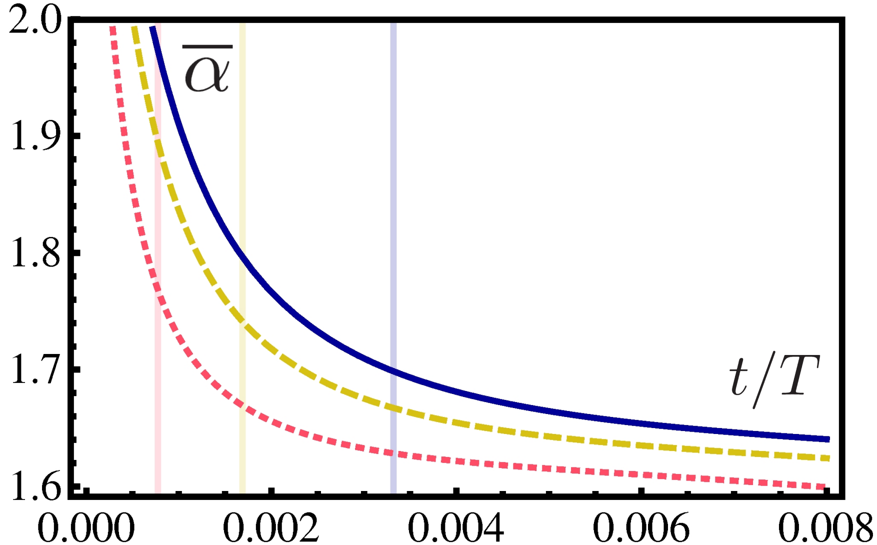
Short time approximation: $t \ll t_c$

$$a^2(t) = \int_0^t d\tau \int_0^t d\tau' \left(\overline{\lambda(\tau)\lambda(\tau')} - \overline{\lambda}^2\right) \sim \Delta^2\lambda\, t^2$$

Optimum squeezing as a function of time

$$\xi^2 = \min_{\vec{n}} \frac{N\Delta^2 \hat{J}_{\vec{n}}}{\langle \hat{J}_{\vec{p_1}}\rangle^2 + \langle \hat{J}_{\vec{p_2}}\rangle^2}$$

$$\hat{J}_{\vec{n}} = \cos\alpha \hat{J}_y + \sin\alpha \hat{J}_z$$



Angle of rotation of the optimum direction

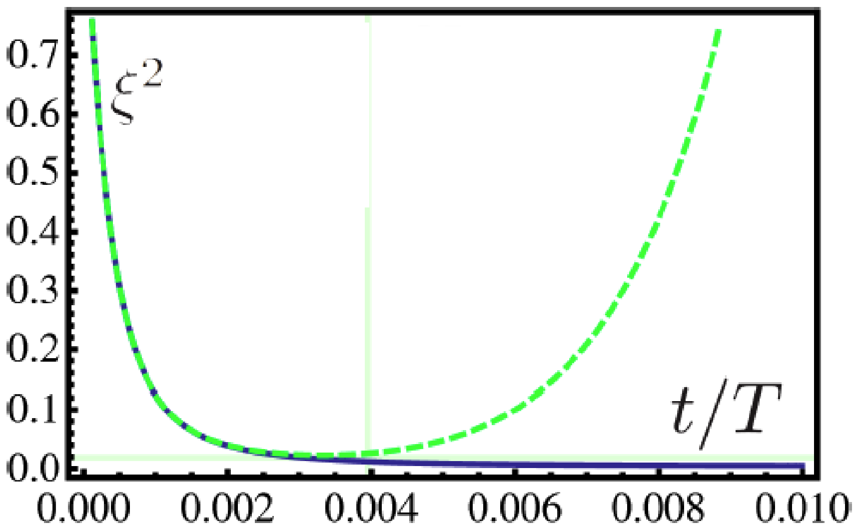
$$\overline{\alpha} = \frac{1}{2}\arctan\frac{\mathrm{Tr}\left[\hat{\rho}\left\{\hat{J}_y, \hat{J}_z\right\}\right]}{\mathrm{Tr}\left[\hat{\rho}\hat{J}_y^2\right] - \mathrm{Tr}\left[\hat{\rho}\hat{J}_z^2\right]} + k\frac{\pi}{2}$$

The presence of noise changes the optimum time for squeezing and the angle of rotation.

$N = 400; \quad \chi = 0.13\pi\mathrm{Hz}$

$\Delta\lambda = 0, 5, 10\ \mathrm{Hz} \qquad \overline{\lambda} = 0$

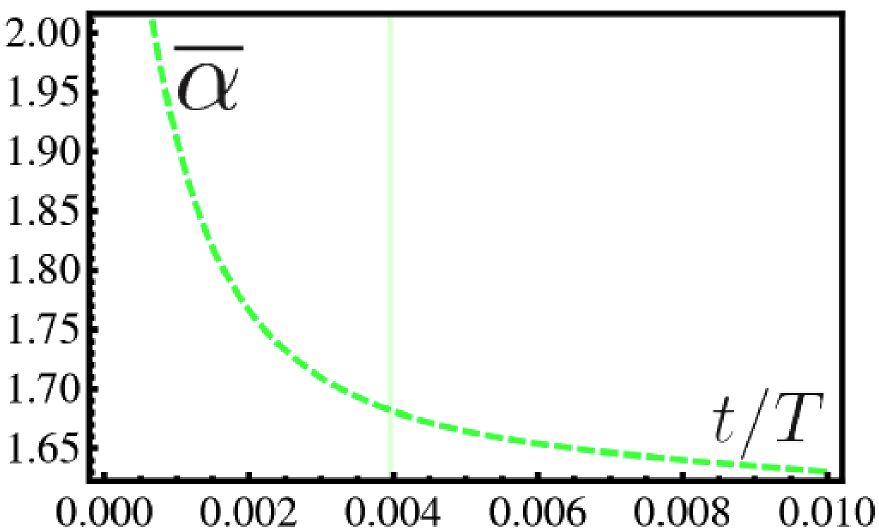
Which states produced during the quenched dynamics are useful for interferometry ?



Squeezing as a function of time

$$t \sim 1/(\chi N^{\frac{2}{3}}) \Rightarrow \xi^2 \approx \frac{1}{2}\frac{3}{N}^{\frac{2}{3}}$$

[Kitagawa et al, PRA **47**, 5138 (1993)]



Angle of rotation of the optimizing direction

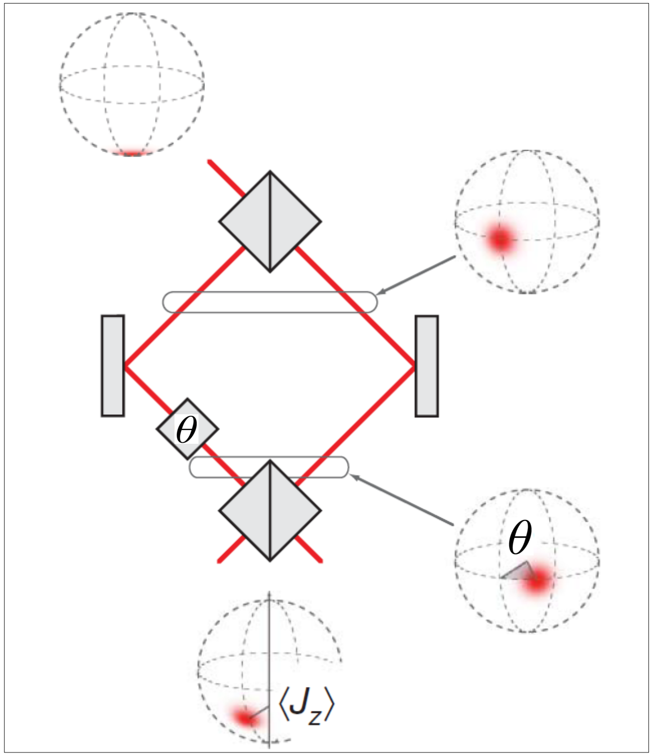
$$\hat{J}_{\vec{n}} = \cos\alpha \hat{J}_y + \sin\alpha \hat{J}_z$$

$$\overline{\alpha} = \frac{1}{2}\arctan\frac{\langle \hat{J}_y, \hat{J}_z \rangle}{\langle \hat{J}_y^2 \rangle - \langle \hat{J}_z^2 \rangle} + k\frac{\pi}{2}$$

$$N = 400; \quad \chi = 0.13\pi \text{Hz}$$

BJJ as an interferometer

Goal: to estimate a phase shift ϴ with the highest possible precision.



Interferometer = *ROTATION OF THE INPUT STATE*

$$|\psi\rangle_{out} = e^{-iJ_y\frac{\pi}{2}} e^{iJ_z\theta} e^{iJ_y\frac{\pi}{2}} |n = -\frac{N}{2}\rangle = e^{-iJ_x\theta}|n = -\frac{N}{2}\rangle$$

Estimate ϴ from: $\langle \hat{J}_z \rangle_{out} = \frac{N}{2}\cos\theta$

With which precision? $\Delta\theta = \frac{\Delta \hat{J}_{z_{out}}}{|\partial \langle \hat{J}_{z_{out}} \rangle / \partial\theta|}$

input *COHERENT STATE*

$$\Delta \hat{J}_{z\,out} = \sqrt{N}/2$$

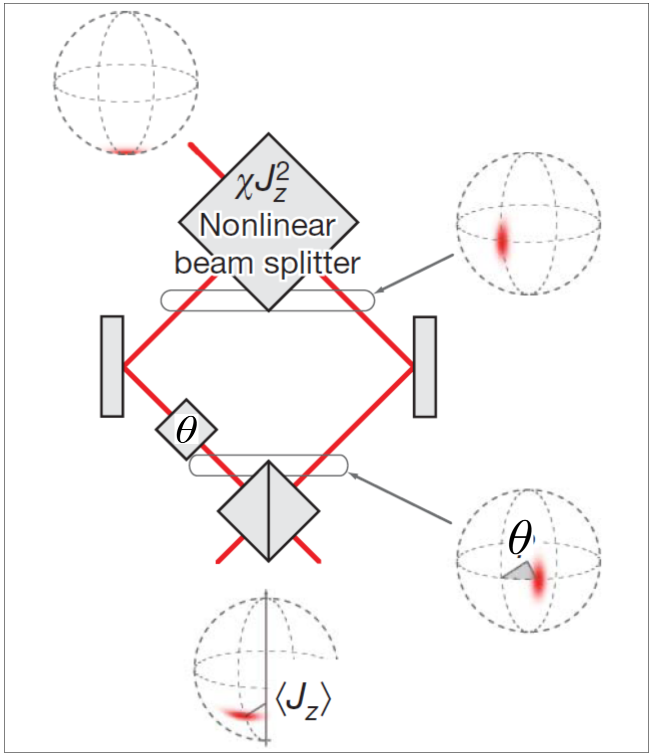
Best possible sensitivity:

$$\Delta\theta_{\text{best}} = 1/\sqrt{N} \equiv \Delta\theta_{SN}$$

Shot noise limit

BJJ as an interferometer

Goal: to estimate a phase shift ϴ with the highest possible precision.



Interferometer = *ROTATION OF THE INPUT STATE*

$$|\psi\rangle_{out} = e^{-iJ_y\frac{\pi}{2}} e^{iJ_z\theta} e^{iJ_y\frac{\pi}{2}} |n = -\frac{N}{2}\rangle = e^{-iJ_x\theta}|n = -\frac{N}{2}\rangle$$

Estimate ϴ from: $\langle \hat{J}_z \rangle_{out} = \frac{N}{2}\cos\theta$

With which precision? $\Delta\theta = \frac{\Delta \hat{J}_{z_{out}}}{|\partial \langle \hat{J}_{z_{out}} \rangle / \partial\theta|}$

input *SQUEEZED STATE* →

$$\Delta \hat{J}_{z\,out} < \sqrt{N}/2$$

*[Wineland et al, PRA **50** 67 (1994)].*

Best possible sensitivity:

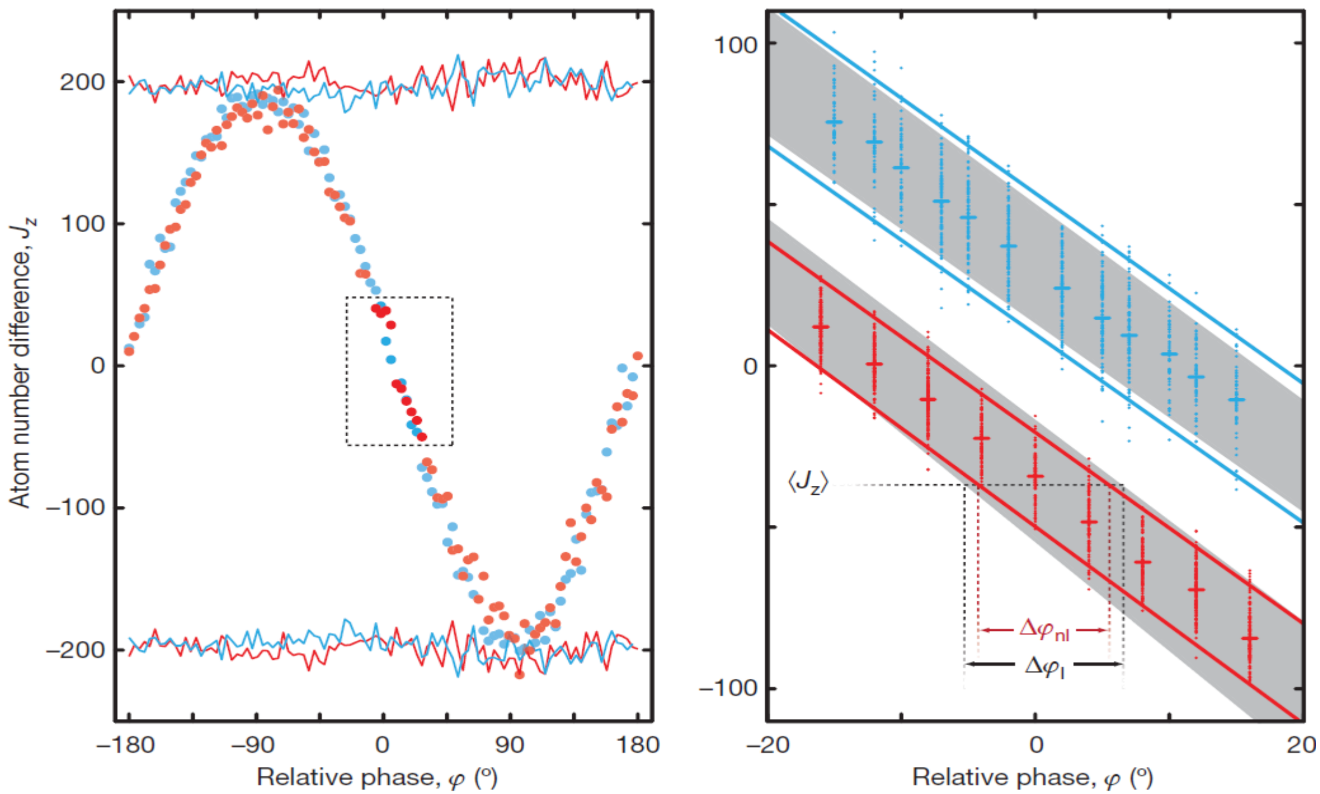
$$\Delta\theta_{\text{best}} < \Delta\theta_{SN}$$

Better that the shot noise limit !

BJJ as an interferometer : experiment

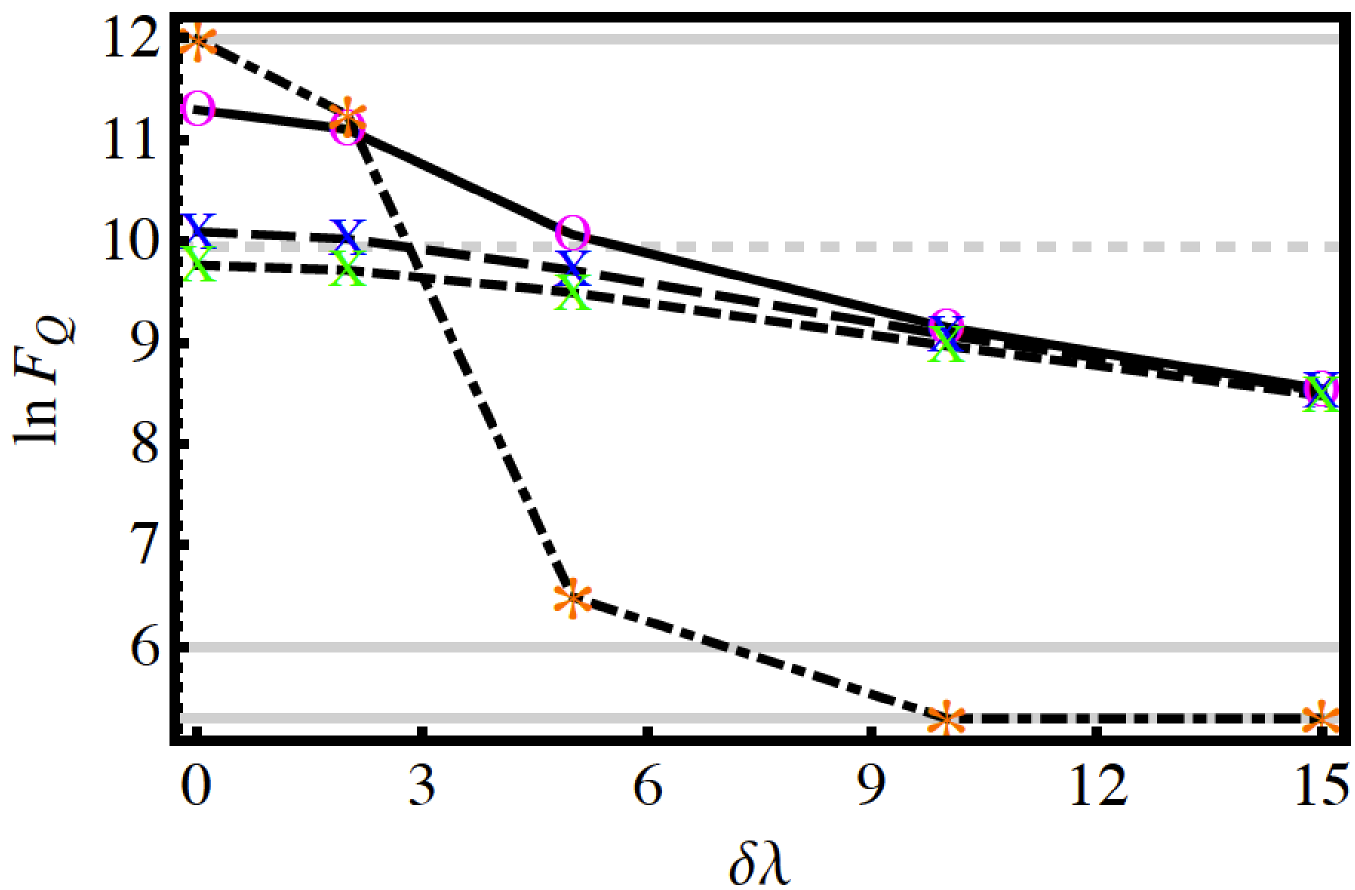
SQUEEZED STATE: $\xi \equiv \Delta\theta/\Delta\theta_{SN} \simeq 0.87$ (15% precision enhancement)

Ramsey fringes : $\langle \hat{J}_z \rangle_{out} = \frac{N}{2} \cos\theta$



*[C. Gross et al, Nature **464**, 1165 (2010)]*

Effect of phase noise on the Fisher information

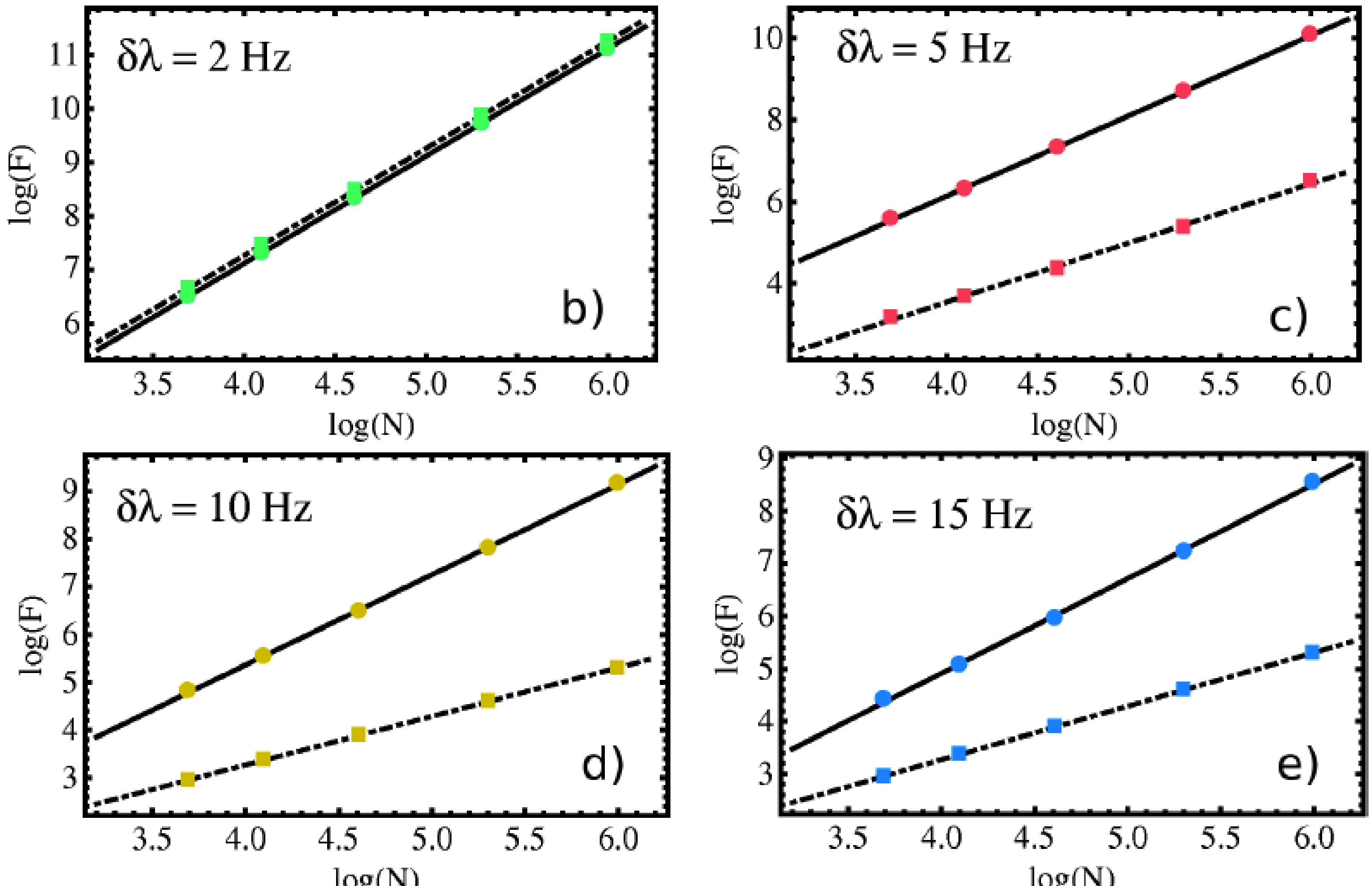


* Two component cat state $t = t_2 = T/4$

O Multicomponent cat states $t = t_{\max} \simeq T/\sqrt{N}$

X Squeezed states $t = t_{\rm squ} \sim T/N^{2/3}$

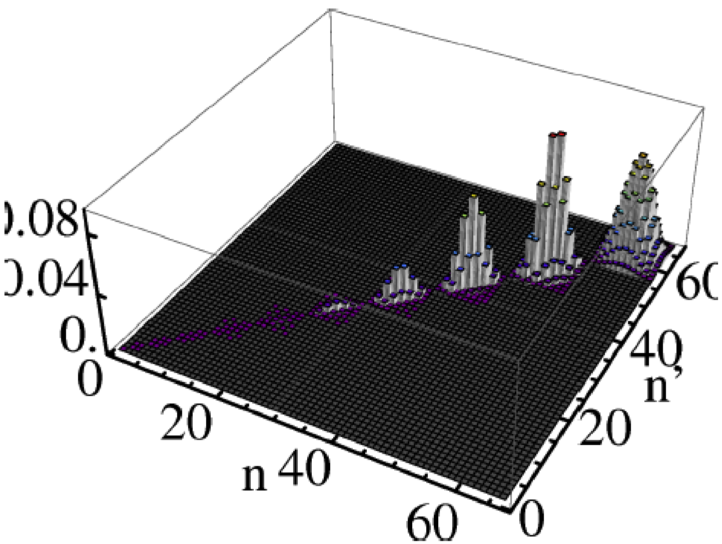
Linearized maximum Fisher information

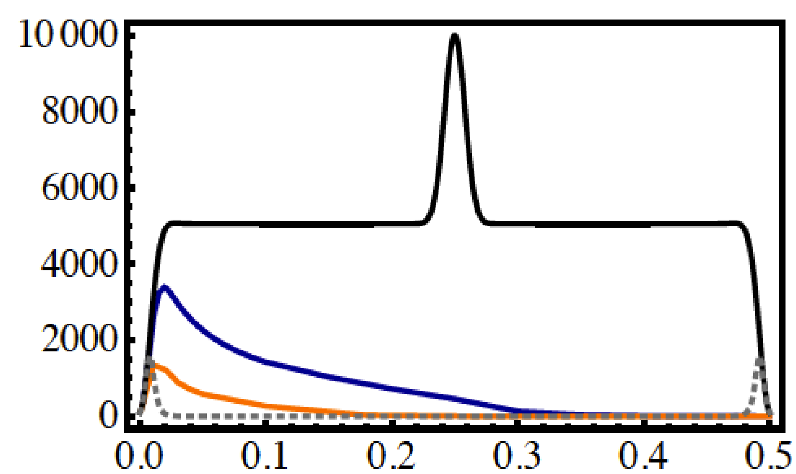


Particle losses

Master equation : $$\partial_t \hat{\rho} = -\frac{i}{\hbar}\left[H^{(0)}, \hat{\rho}\right] + \gamma \sum_{k=1}^{2} \left(\left[\hat{a}_k, \hat{\rho}\hat{a}_k^\dagger\right] + \left[\hat{a}_k\hat{\rho}, \hat{a}_k^\dagger\right] \right)$$

Solution : following *K. Pawlovski et al, PRA **81** 013620 (2010)*

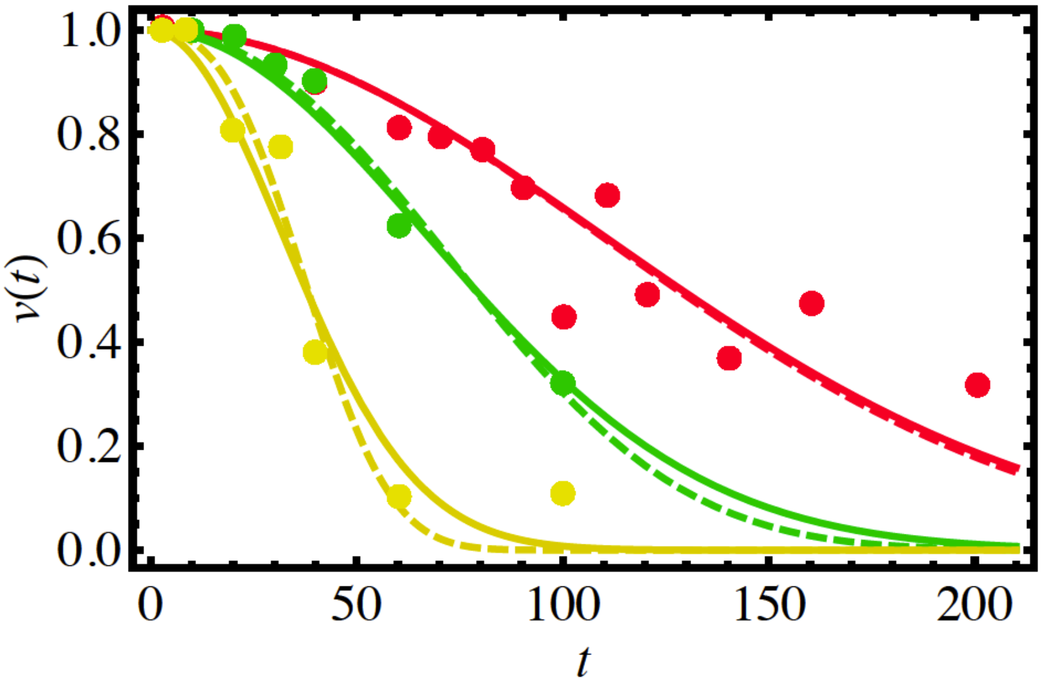




Particle losses + phase noise :

$$\hat{\rho}_{k,m-k}^{k+r,m-(k+r)}(t) = e^{i\bar{\Lambda}rt} e^{-\frac{a(t)^2 r^2}{2}} \frac{N! e^{-2\gamma m t}}{2^{(N-m)}(N-m)!} \left[\frac{1 - e^{-2\gamma t} e^{iU_1 rt}}{1 - iU_1 r/(2\gamma)} + \frac{1 - e^{-2\gamma t} e^{-iU_2 rt}}{1 + iU_2 r/(2\gamma)} \right]^{(N-m)} \times \hat{\rho}_{k,m-k}^{(0)\ k+r,m-(k+r)}(t),$$

Visibility decay in the presence of noise



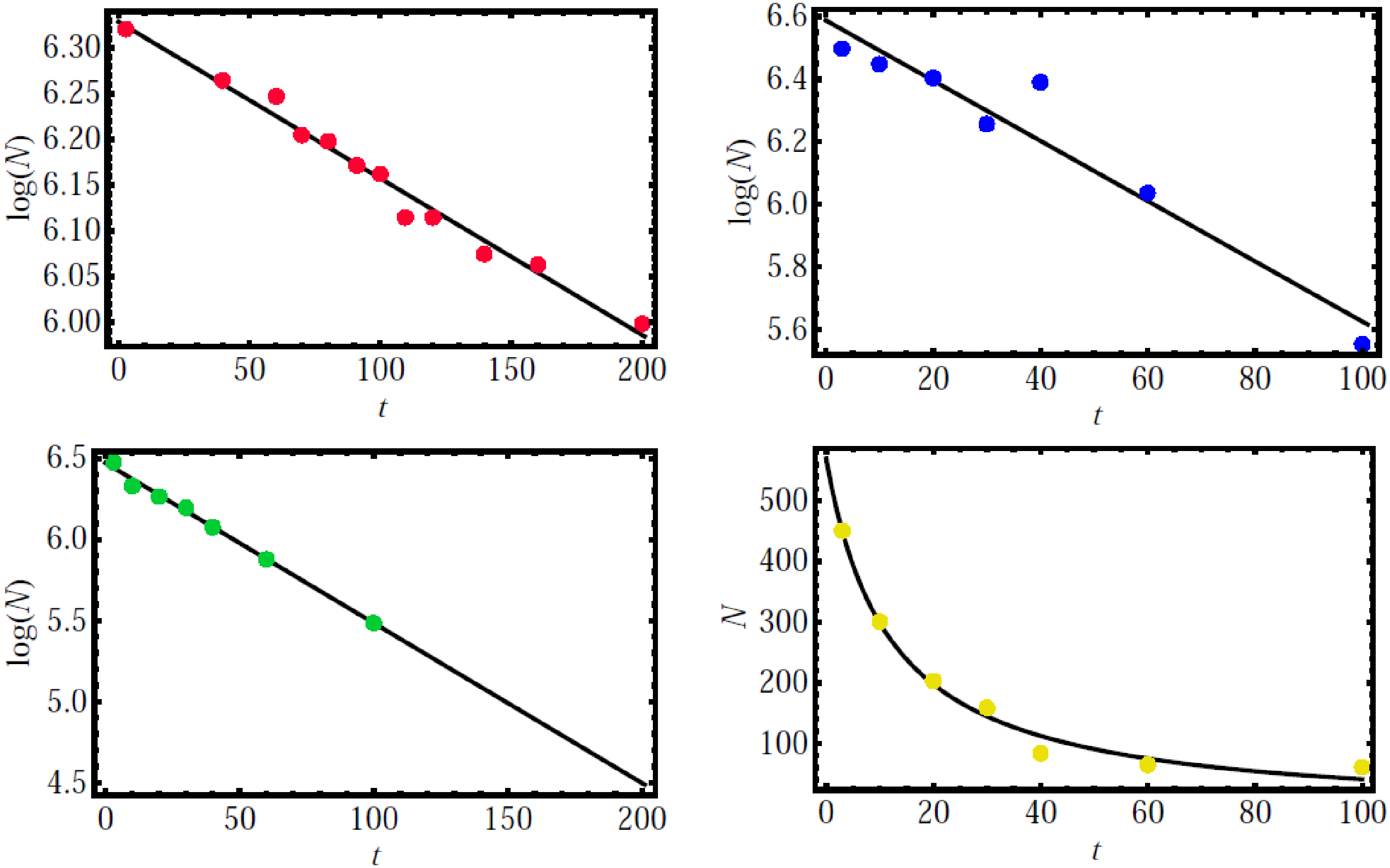
$$\nu^{(0)}(t) = \frac{\langle \hat{J}_x^{(0)} \rangle}{N/2} = \cos^{N-1}(\chi t) \simeq e^{-(N-1)\frac{1}{2}\chi^2 t^2}$$

$$\nu(t) = e^{-\Delta E(0)^2 t^2/2} \nu^{(0)}(t) = e^{-\left(\Delta E(0)^2 + (N-1)\chi^2\right) t^2/2} \equiv e^{-\frac{t^2}{2\tau_{deph}^2}}$$

$$\tau_{noise} \equiv \frac{1}{\Delta E(0)} = \left(\frac{1}{\tau_{deph}^2} - (N-1)\chi^2 \right)^{-\frac{1}{2}}$$

χ (Hz)	τ_{deph} (ms)	τ_{noise} (ms)
$\pi \cdot 0.05$ (B = 9.2G)	109.573	119.769
$\pi \cdot 0.075$ (B = 9.16G)	73.7729	82.4899
$\pi \cdot 0.13$ (B = 9.13G)	66.9390	93.2656
$\pi \cdot 0.25$ (B = 9.11G)	32.0688	37.0688

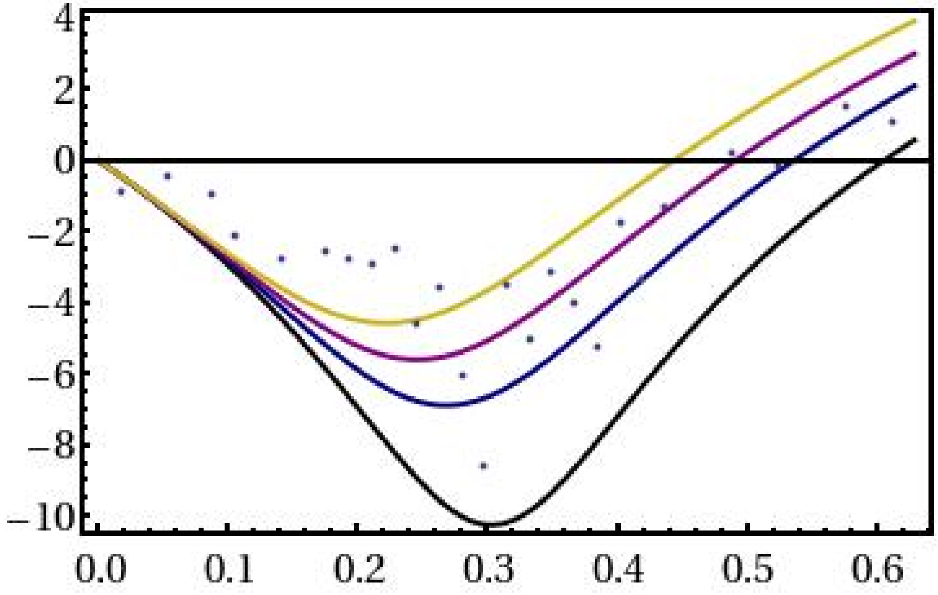
Particle losses

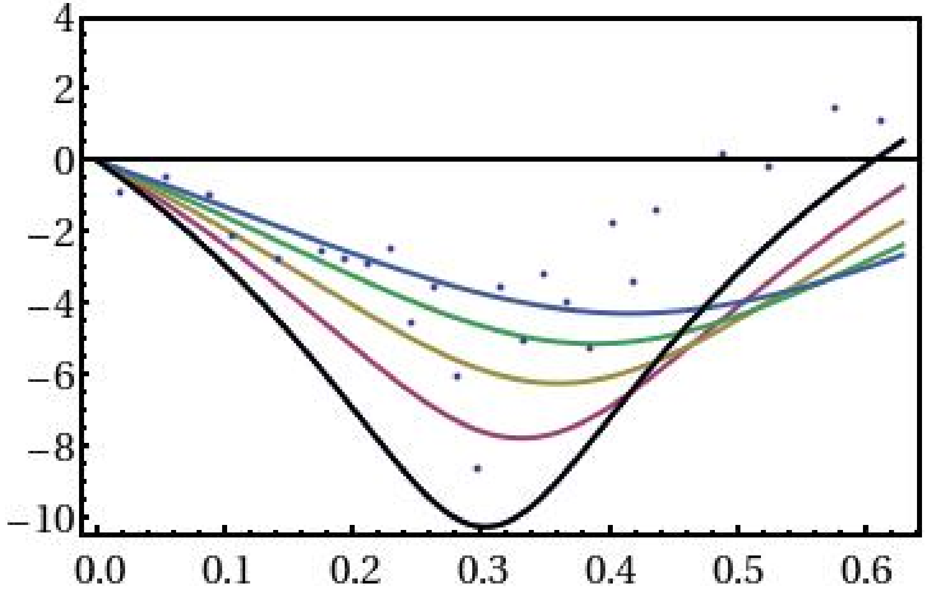


χ (Hz)	$\gamma^{(1)}$ $(\mathrm{ms}^{(-1)})$	τ_{loss} (ms)
$\pi \cdot 0.05$ (B = 9.2G)	0.0017227	503.864
$\pi \cdot 0.075$ (B = 9.16G)	0.0096339	203.71
$\pi \cdot 0.13$ (B = 9.13G)	0.0098743	141.002

Classical noise vs particle losses

Fit of experimental data





Fit of experimental data with Sinatra-Castin model for losses of particles

Fit of experimental data our model for the phase noise

N = 400;