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Level sets and extrema of random processes and fields. (English)

Hoboken, NJ: John Wiley & Sons. xi, 393 p. EUR 91.70; £ 73.50 (2009). ISBN 978-0-470-40933-6/hbk

Given a stochastic process $\mathcal{X} = \{X(t) : t \in T\}$ with regular paths indexed by a parameter set T the authors investigate mainly the two following problems: firstly the study of the properties of the level sets $\{t \in T : X(t) = u\}$ for a given u in the state space of $\mathcal{X}$, and secondly to compute, when the set T is an Euclidian space, the distribution function $F_{M_T(u)} = P(M_T \leq u)$ of the supremum $M_T = \sup_{t\in T} X(t)$ of the field $\mathcal{X}$. The main tools used to deal with the former problem are the Rice formulas which allow one to express the factorial moments of the number of roots of $X(t) = u$ as an integral of a function on the joint distribution of the process and its derivative when T is a Borel set in $\mathbb{R}^d$ and $\mathcal{X}$ takes its values in $\mathbb{R}^{d'}$. When the preceeding dimensions differ, more general geometric arguments have to enter into the analysis (the Chapter 6 is devoted to the derivation of different Rice formulas whether $d = d'$ or not). In Chapter 5, similar quantities, namely the Rice series, are used to express the tail of the distribution of the maximum of the process, establishing a bridge between the distribution of the maximum on an interval of a one-parameter process and the factorial moments of the up-crossings of the paths. This leads to the second problem: to investigate the behavior and the properties of the distribution of the supremum. For this the authors present several methods, giving particular attention to the Gaussian framework. The authors explain elegantly that a general methodology is difficult to provide even in the Gaussian context, and that most of the techniques used in the literature are tailored to the process under consideration. However, general lower and upper bounds can be provided and this is presented in Chapter 2. These bounds are refined in Chapters 4, 5, 8 and 9. In addition several asymptotic analyses of the behavior of $F_{M_T(u)}$ are provided (for example in Chapter 8 when u tends to infinity). The regularity of the function $u \mapsto F_{M_T(u)}$ is given in Chapter 7.
It is worth mentioning that the theoretical results presented in this book are applied to various examples including some statistical applications (Chapter 4), genomics applications (Chapter 4 Section 4.4), and modeling sea waves (Chapter 11).
Finally, the book is pleasant to read and is written in a very comprehensive way. The main theoretical results are illustrated with examples and applications, including precise and rich bibliographical comments. It is worth noting that each chapter ends with a series of comprehensible and interesting exercises making this book suitable for researchers and for post-graduated students with a basic background in probability theory.

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Keywords : random fields; level sets; extrema of random fields; Rice formula
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∗60G60 Random fields
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62M40 Statistics of random fields